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STUDIES REGARDING THE KINEMATIC AND FUNCTIONING OF UNCONVENTIONAL WORM GEAR

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Abstract: The aim of the paper is the kinematic analysis of an unconventional worm gear, namely the one where the teeth of the worm gear are actually bearings. The analysis of such unconventional gearing is relatively difficult to achieve because of the unknown variable parameters that may occur. Thus, in the paper this gear will be kinematically analyzed by fixing certain parameters. The study carried out is an important and mandatory step in establishing a mathematical model of this type of gear, a model that also takes into account dynamic aspects.
Key words: *unconventional worm gear, kinematically gear.*

1. INTRODUCTION

The aim of the paper is to determine the kinematic parameters of an unconventional worm gear.

Gears are machine parts [1], [4], [6], [7], [8] having importance in industry. It is continuously aimed at eliminating the disadvantages arising in operation and creating new types of gears, such as the one analyzed in the paper, respectively the unconventional worm gear. The main advantage of these gears is high efficiency. The high manufacturing costs can be mentioned as a disadvantage.

The sliding friction between the worm spiral and the worm gear teeth from the classic gear is replaced by the rolling friction between the worm spiral and the driven wheel bearings. It is known that the rolling friction coefficient (s) is about ten times lower than the sliding friction coefficient (μ). The problem is that the engagement should be carried out without backlash and on the largest possible surfaces of the flanks of the teeth.

So, the non-conventional worm gear is so called because the driven wheel has bearings instead of teeth. Reiterating the importance of rolling friction compared to sliding, all these changes are made with the aim of reducing the friction in the respective mechanism, to reduce the backlash in it by making the worm and driven wheel in such a way that between the flank of the worm and the bearings on the driven wheel to have a minimum play or a minimum tension, thus eliminating as much as possible the blacklash in the gear. This modification does not simplify the gearing but eliminates the major disadvantages. In areas where precision is the most important factor and where we also need the known advantages of the worm gear, the proposed gear will become cost-effective.

So, ultimately the most important factor that needs to be improved in the gear is its performance, namely reducing the backlash and replacing the sliding friction moment with a rolling friction moment.

The specialized literature presents us the studies regarding the nonconventional gear designs where these disadvantages have been largely reduced or eliminated, but these studies are still in their initial stages. It should be noted that there is still no mathematical model that reproduces their calculation or dimensioning so that their use is profitable from the point of view of eliminating the backlash between the flanks of the teeth.

If the backlash between the teeth could be permanently eliminated, the accuracy and lifetime of the gear would increase, but this does not simplify the costs, which still remain high,

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which is why their use is profitable only in some areas of the industry.

2. KINEMATICAL STUDY

2.1 The worm geometry

Mathematical modeling [3], [4], [10], [11] can start from the kinematics of the globoidal worm gear, with which this unconventional gear studied in the paper is most similar (Fig.1, [9]). It will be marked with 1 – the worm and with 2 – the driven wheel.

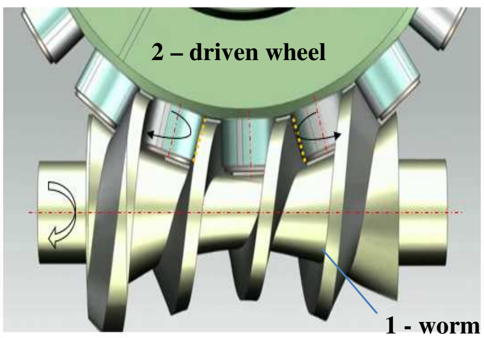

Fig. 1 The unconventional worm gear.

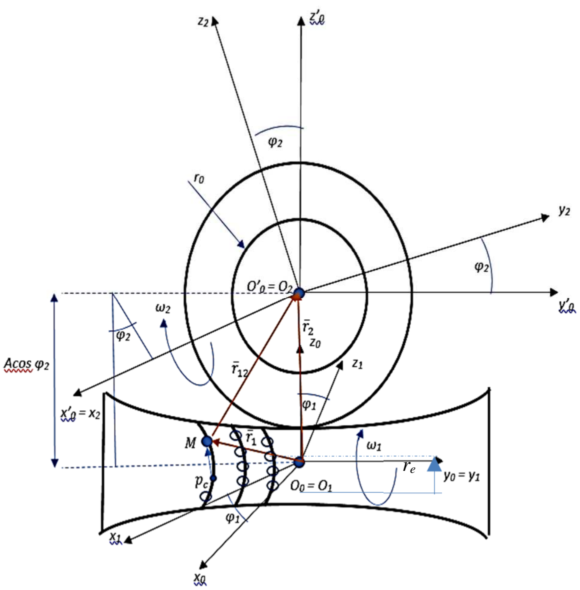

Fig. 2 Axial sketch of the gear and functional (geometric) parameters.

Mathematical modeling of gear kinematics [13] can include:

Definition of meshing worm;

Defining the flanks of the worm wheel by kinematic generation;

Limits on the generation of flanks of worm gear teeth, etc.

To mathematically model the gear (Fig.2), it will be sketched in the axial plane, representing the reference systems and their geometric parameters.

Let M(x_1,y_1,z_1) be a point belonging to the worm.

We want to find the trajectory of point M in relation to the driven wheel, respectively $\bar{r}_{12}(x_{12}y_{12}z_{12})$ [9], [10].

Fixed and mobile reference systems $x_0y_0z_0$ are considered; $x_1y_1z_1$ related to the worm and $x'_0y'_0z'_0$ fixed, related to the driven wheel and initially coinciding in direction with the system $x_0y_0z_0$ and $x_2y_2z_2$ the mobile system related to the driven wheel.

If we refer to the transmission ratio of the gear [5], [6], depending on the parameters φ1 and φ_2 we will have:

$$i = i_{12} = \frac{1}{i_{21}} = \frac{\omega_1}{\omega_2} = \frac{\varphi 1}{\varphi 2} = const. \qquad (1)$$

We are explicitly interested in the kinematics of the worm relative to the driven wheel and the general expression of the driven wheel relative to the worm. So, we consider the position of point M with respect to the system $x_1y_1z_1$:

$$\bar{r}_1 = R(y_0, \varphi_1) \cdot \bar{r}_0 \qquad (2)$$

where: $\bar{r}_1, \bar{r}_0$ – position vectors and $R(y_0, \varphi_1)$ – the rotation matrix of system $x_1y_1z_1$ around axis Oy_1.

$$\begin{bmatrix} x_1 \\ y_1 \\ z_1 \\ 1 \end{bmatrix} = \begin{bmatrix} cos\varphi_1 & 0 & -sin\varphi_1 \\ 0 & 1 & 0 \\ sin\varphi_1 & 0 & cos\varphi_1 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_0 \\ y_0 \\ z_0 \\ 1 \end{bmatrix} \qquad (3)$$

From relation (3) result the parametrical coordonates of the point *M* reporting at the sistem $x_1y_1z_1$, such as:

$$\begin{cases} x_1 = x_0 cos\varphi_1 - z_0 sin\varphi_1 \\ y_1 = y_0 \\ z_1 = x_0 sin\varphi_1 + z_0 cos\varphi_1 \end{cases} \qquad (4)$$

If we are referring to the point M position regarding system x2y2z2 of the driven wheel, this will have it:

$$\bar{r}_2 = R(x'_0, \varphi_2) \cdot \overline{r'}_0 \quad (5)$$

where: $\bar{r}_2, \overline{r'}_0$ – position vectors and $R(x'_0, \varphi_2)$ – is the rotation matrix of the system $x_2y_2z_2$ around the axis $O'x_2$.

$$\begin{bmatrix} x_2 \\ y_2 \\ z_2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\varphi_2 & -sin\varphi_2 \\ 0 & sin\varphi_2 & cos\varphi_2 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} x'_0 \\ y'_0 \\ z'_0 \\ 1 \end{bmatrix} \quad (6)$$

From the relation (6) results that the parametrical coordinates of the M point reported at the system $x_2y_2z_2$ are:

$$\begin{cases} x_2 = x'_0 \\ y_2 = y'_0 cos\varphi_2 - z'_0 sin\varphi_2 \\ z_2 = y'_0 sin\varphi_2 + z'_0 cos\varphi_2 \end{cases} \quad (7)$$

How the initial reference system $x_0y_0z_0$ concide with $x'_0y'_0z'_0$ this could be replace with $x_0 = x'_0$, $y_0 = y'_0$ si $z_0 = z'_0$ such as, the move equations of the worm-wheel gear will have the next position vectors: $\bar{r}_{12} = [x_{11}\ y_{11}\ z_{11}]^T$ and wheel - worm $\bar{r}_{21} = [x_{22}\ y_{22}\ z_{22}]^T$ and these are given by relations (8) and (9):

$$\begin{cases} x_{11} = x_2 cos\varphi_1 - (z_2 cos\varphi_2 - y_2 sin\varphi_2) \cdot sin\varphi_1 \\ y_{11} = z_2 cos\varphi_2 + y_2 sin\varphi_2 \\ z_{11} = x_2 sin\varphi_1 + (z_2 cos\varphi_2 - y_2 sin\varphi_2) \cdot cos\varphi_1 \end{cases} \quad (8)$$

$$\begin{cases} x_{22} = x_1 cos\varphi_1 + z_1 sin\varphi_1 \\ y_{22} = y_1 cos\varphi_2 - (z_2 cos\varphi_1 - x_1 sin\varphi_1) \cdot sin\varphi_2 \\ z_{22} = y_1 cos\varphi_2 - (z_1 cos\varphi_1 - x_1 sin\varphi_1) \cdot cos\varphi_2 \end{cases} \quad (9)$$

To continue the study, the following simplifying assumptions will be taken into account:

- It was considered that the point M belonging to the worm falls on the same side of the flank of the tooth also at the second tooth of the nonconventional gear;

- Consider s_0 - as the generating helical profile, having as parameter h the distance between two flanks of the worm teeth (Fig.3);
- The following geometric elements of the worm are taken into account, in the axial plane:
  - α_1, α_2 – the angles on which make them the vertical line of the point M with the left flank, respective right of the worm;
  - r_e – worm evolvent radius;
  - p_c –independent parameter;
  - $\bar{r}_c$ position vector of some point M_c which appertain the surface of the worm.

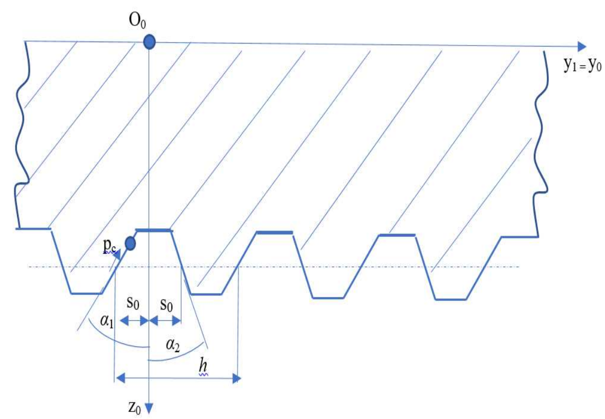

Fig. 3 Axial profile of the worm.

From a point *M* belonging to the flank of the snail, the normal to that surface can be determined as follows:

$$\bar{n} = \nabla f = \frac{\partial f}{\partial x}\bar{\imath} + \frac{\partial f}{\partial y}\bar{\jmath} + \frac{\partial f}{\partial z}\bar{k} \quad (10)$$

where: $\bar{\imath}, \bar{\jmath}, \bar{k}$ – are axes versors of Ox_1, Oy_1, Oz_1, but *f(x,y,z)* is a flank surface function to a tooth.

The normal to the flank surface is given by:

$$\bar{N} = \lambda \cdot \bar{n} \quad (11)$$

where: λ – scalar parameter

And the normal, in the general form, on the surface, depending on the $x_0y_0z_0$ system and the given geometric elements becomes:

$$\bar{N}_c = \begin{bmatrix} N_{cx} \\ N_{cy} \\ N_{cz} \end{bmatrix} = \begin{bmatrix} -(2c-3)\cdot sin\alpha_c \cdot cos\varphi_1 - \lambda \cdot cos\alpha_c \cdot sin\varphi_1 \\ cos\alpha_c \\ (2c-3)\cdot cos\alpha_c \cdot cos\varphi_1 + \lambda \cdot sin\alpha_c \cdot sin\varphi_1 \end{bmatrix}$$

respective, $N_{cx} \cdot v_{12}^x + y \cdot v_{12}^y + N_{cz} \cdot v_{12}^z = \varphi_1$

(12)

c – flank variation coefficient.

$$\lambda = \frac{h}{r_c + p_c \cdot cos\alpha_c} \quad (13)$$

The position vector of a certain point of the flank of the worm vis a vis relative to the fixed system $x_0y_0z_0$ is:

$$\bar{r}_c = \begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix} = \begin{bmatrix} 0 \\ (s_0 + p_c \cdot sin\alpha_c)\cdot(2c-3) \\ r_c + p_c \cdot cos\alpha_c \end{bmatrix} \quad (14)$$

The relation (14) reporting of the reference system $x_1y_1z_1$ will be:

$$\bar{r}_{1c} = \begin{bmatrix} x_{1c} \\ y_{1c} \\ z_{1c} \end{bmatrix} = \begin{bmatrix} -(r_c + p_c \cdot sin\alpha_c)\cdot cos\varphi_1 \\ (s_0 + p_c \cdot cos\alpha_c)\cdot(2c-3) + h \\ (r_c + p_c \cdot sin\alpha_c)\cdot sin\varphi_1 \end{bmatrix} \quad (15)$$

The relations (12) and (15) completely define, of the view point of geometrical study, the elicoidal surface of the worm.

2.2. The kinematical study of the gear

The study of nonconventional gearing from the point of view of kinematics [12], [13], refers to the velocity with which the flank of the worm tooth moves vis-a-vis the driven wheel.

In the kinematic study, the parameters and axis systems shown in figure 2 and the position vectors will be taken into account $\bar{r}_{12}$ and $\bar{r}_{21}$ given by relations (8) and (9).

Also, it note with:

$\omega_1 = \dot{\varphi}_1$ and $\omega_2 = \dot{\varphi}_2$ – the angular velocities of the worm and driven wheel.

also, $\bar{v}_{12} = \begin{bmatrix} v_{12}^x & v_{12}^y & v_{12}^z \end{bmatrix}^T$ is the relative velocity of the worm regarding the driven wheel, and it will be:

$$\bar{v}_{12} = \dot{\overline{r_{12}}} = \frac{\partial \bar{r}_{12}}{\partial t} + \bar{\omega}_1 \times \bar{r}_{12} \quad (16)$$

$$\frac{\partial \bar{r}_{12}}{\partial t} = \begin{bmatrix} -x_2 sin\varphi_1 - (z_2 cos\varphi_2 - y_2 sin\varphi_2)\cdot cos\varphi_1 \\ z_2 cos\varphi_2 + y_2 sin\varphi_2 \\ z_{11} = x_2 cos\varphi_1 - (z_2 cos\varphi_2 - y_2 sin\varphi_2)\cdot sin\varphi_1 \end{bmatrix} \cdot \omega_1 \quad (17)$$

$$\bar{\omega}_1 \times \bar{r}_{12} = \begin{vmatrix} \bar{\imath} & \bar{\jmath} & \bar{k} \\ 0 & \omega_1 & 0 \\ x_{11} & y_{11} & z_{11} \end{vmatrix} = \begin{bmatrix} \omega_1 z_{11} \\ 0 \\ -\omega_1 x_{11} \end{bmatrix} \quad (18)$$

Substituting relation (8) into (18) and taking into account the relation (16), we will have the relations of the relative velocity of the worm with respect to the gear wheel:

$$\bar{v}_{12} = \bar{v}_{12}(\varphi_1, \varphi_2) \quad (19)$$

The gearing condition of the worm, respectively the surface of the tooth flanks at the driven wheel teeth must satisfy the condition:

$$\bar{N}_c \cdot \bar{v}_{12} = 0 \quad (20)$$

and the definition of the normal, in the vector form, to the flank of the tooth of the driven wheel becomes:

$$\bar{N}_{2c} = \lambda \cdot \bar{n}_{2c} = \bar{N}_{2c}(p_c, \varphi_2) \quad (21)$$

So, relations (9) and (21) completely define the flank surface of the tooth of the driven wheel of the gear.

3. CONCLUSIONS

The paper brings as a novelty element simplified mathematical modeling, in the sense that some factors that vary will be considered fixed and the movement of the gear wheel relative to the worm and vice versa is analyzed, both from the point of view of the trajectory and the relative velocity between them.

Regardless of the type of gear, an important factor is its performance. For the performance of the gear, the bearing capacity, the vibrations in the mechanism, the noise it produces, the working temperature, and the heating of the gear are analyzed. These represent energy losses leading to a decrease in the efficiency of the mechanism.

This was also, one of the goals of this study. The worm gear had a number of problems related to geometry, sizing, and machining technology. Over the years, these problems have been clarified through the mathematical interpretation of the gearing processes, but, also of the experimental results and the relationships between the geometrical elements and the processing conditions of the worm gears. They also resulted in unitary conceptions of their calculation and construction.

If the application fields of worm gears are taken into account when high precision is desired, they can be replaced by worm gears with bearings (nonconventional), which come with several advantages such as: the worm gear with bearings has eliminate the backlash into the gear and replace the sliding friction into the teeth, between the worm and the driven wheel of the gear with the rolling friction between the worm spiral and the bearings.

All this leads to the need for a lower actuation force, increasing in this way the efficiency and also the life of the gear.

4. REFERENCES

[1]Adler, O. R. – Angrenaje I, Editura Tehnică, București, 1968.

[2] Bălcău, M., Arghir, M., *The use of dynamical absorbers*, 13th International Conference „Automation in Production Planning and Manufacturing“, Zilina ISBN 978-80-89276-35-6, pag. 9-14, 2012.

[3]Monica Bălcău, Andrei Ripianu, *Case study of a mechanic system composed of for reduced masses and the dynamic absorber is placed to one of the extremities of the mechanic system and subjected to two harmonics X*, în Acta Technica Napocensis, seria Applied Mathematics and Mechanics, nr. 51, vol. IV , ISSN 1221-5872, pag.107-112, 2008.

[4]Boloş, V. – *Angrenaje melcate spiroide. Danturarea roților plane.* Ed. Univ. "Petru Maior", Tg. Mureş.

[5]Deng, X., Wang, J., Wang, S., Liu, Y. – *Investigation on the Backlash of Roller Enveloping Hourglass Worm Gear: Theoretical Analysis and Experiment*, Mississippi State University, 2018.

[6]Haragâș, S. – *Organe de mașini*, Editura Napoca Star, Cluj-Napoca, 2014.

[7]Haragâș, S. – *Reductoare cu o treaptă. Calcul și proiectare*, Editura Risoprint, Cluj-Napoca, 2014.

[8]Maros, D. – Angrenaje melcate, Editura Tehnică, București, 1966.

[9]Ninacs, R. - *Angrenaje melcate neconvenționale*, raport de cercetare, IOSUD-UTCN, 2022.

[10] Schonstein, C. – *Considerations about matrix exponentials in geometrical modeling of the robots*, Acta Technica Napocensis, Series: Applied Mathematics, Mechanics, and Engineering, Issue 2, No. 62, June, Cluj-Napoca, ISSN 1221-5872, 2019.

[11] Schonstein, C. – *Kinematic control functions for a serial robot structure based on the time derivative Jacobian matrix*, Acta Technica Napocensis, Series: Applied Mathematics and Mechanics, No. 61, Issue II, Cluj-Napoca, ISSN 1221-5872, Romania, 2018.

[12] Sedgwick, R. – *Recirculating ball worm drive*, Wisconsin, 1968.

[13] Vyatkin, A. – *Analysis of the geometry and contact density of globoid gearing*, MATEC Web of Conferences 329, 03008 (2020) https://doi.org/10.1051/matecconf/20203290 3008, ICMTMTE, 2020.

[14]https://www.nikken-kosakusho.co.jp/en/product/index.php?seq=42
[15]https://www.semanticscholar.org/paper/Innovative-Design-for-A-Ball-Worm-Gear-Mechanism-Kocak/687de07f9e3ad543171689a4686e94d5f514bc94/figure/7

Studii privind cinematica și funcționarea angrenajului melcat neconvențional

Rezumat: Lucrarea are ca scop analiza cinematică a unui angrenaj melcat neconvențional, respectiv acela la care dinții roții melcate sunt de fapt rulmenți. Analiza unui astfel de angrenaj neconvențional e relativ dificil de realizat din cauza parametrilor variabili necunoscuți care pot apărea. Astfel, în lucrare se va analiza cinematic acest angrenaj fixând anumiți parametrii. Studiul realizat este o etapă importantă și obligatorie în stabilirea unui model matematic al acestui tip de angrenaj, model care să țină cont și de aspecte de dinamică.

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