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THE KINEMATIC EQUATIONS OF TWO ROBOTS OF THE TYPE 3TR-2R IN COOPERATION MOVEMENTS

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Abstract: *In this paper, the kinematic equations for tow robots working in cooperative movements are presented. To establish the equations of direct kinematics, the algorithm of exponential matrices in kinematics will be applique. The goal of the algorithm is to determine the Jacobian matrix, together with its first-order derivative with respect to time. These robots, working in cooperative movements, can be used to make complex parts.*
Keywords*: robot cooperation, kinematic equation, advanced mechanics*

1. INTRODUCTION

In general, cooperation robots correspond to redundant frames, in which the effective degrees of freedom are higher than the exact ones required to achieve a given processing.

This capacity growth the handiness of the mechanism and can be used to eschew the restriction of joints, singularity, and barrier of the workspace, as well as to diminish energy consumption or to improving their performance.

In kinematics, the main objective is the study of the mechanical movement of the material frame, without considering the mass and the force acting on them, the geometric aspect of the movement is pursued. [1]

In the geometric modelling, there are significant drawback like as: nonlinearity of the equations that imposes restrictions for the robot to be reversional, absence of control over speed and acceleration on the trajectory of motion. These snags are removed by using kinematic modelling to the robot control. The static hypothesis is removed, the column vectors of the operational and operational coordinates become a function of time.

In this paper, we will apply the matrix exponential algorithm. The algorithm of exponential matrices owed to its computational benefit and independency from reference frame, can be use to any robot structure [1].

2. KINEMATICS MODELING

To establish the direct kinematic model equations, the algorithm of matrices is use. According to this algorithm [1], the kinematic scheme is first presented in the zero configuration, $\overline{\theta}^{(0)}$ (see Fig. 1).

Conformable to the matrix exponential algorithm in kinematic modeling, iterations are applied in order to get the kinematic parameters. Using the symbolic computation in MATLAB, the absolute linear velocities and angular velocities of the reference frames are get, located in each driving kinematic joints, expressed in the mobile frames and in the fixed frame:

$$ {}^{1}v_1 = \begin{bmatrix} 0 \\ \dot{q}_1 \\ 0 \end{bmatrix};\ {}^{0}v_1 = \begin{bmatrix} 0 \\ \dot{q}_1 \\ 0 \end{bmatrix};\ {}^{1}\overline{\omega}_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix};\ {}^{0}\overline{\omega}_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad (1) $$

$$ {}^{2}v_2 = \begin{bmatrix} 0 \\ \dot{q}_1 \\ -\dot{q}_2 \end{bmatrix};\ {}^{0}v_2 = \begin{bmatrix} 0 \\ \dot{q}_1 \\ -\dot{q}_2 \end{bmatrix};\ {}^{1}\overline{\omega}_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}; $$

$$ {}^{0}\overline{\omega}_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad (2) $$

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$$ {}^{3}\bar{v}_3 = \begin{bmatrix} \dot{q}_3 \\ \dot{q}_1 \\ -\dot{q}_2 \end{bmatrix};\ {}^{0}\bar{v}_3 = \begin{bmatrix} \dot{q}_3 \\ \dot{q}_1 \\ -\dot{q}_2 \end{bmatrix};\ {}^{1}\bar{\omega}_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}; $$

$$ {}^{0}\bar{\omega}_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}. \tag{3} $$

The linear velocity of the origin of the frame $\{T\}$ in report to the fixed frame $\{0\}$, expressed in the mobile frame $\{T\}$ and in the fixed frame $\{0\}$, is equal to the linear velocity of the origin of the frame $\{4\}$ and is it characterized by the expressions:

$$ {}^{4}\bar{v}_4 \equiv {}^{T}\bar{v}_T = \begin{bmatrix} {}^{T}v_{xT} \\ {}^{T}v_{yT} \\ {}^{T}v_{zT} \end{bmatrix} = \begin{bmatrix} sq_4 \cdot \dot{q}_1 + cq_4 \cdot \dot{q}_3 \\ cq_4 \cdot \dot{q}_1 - sq_4 \cdot \dot{q}_3 \\ -\dot{q}_2 \end{bmatrix} \tag{4} $$

$$ {}^{0}\bar{v}_4 \equiv {}^{0}\bar{v}_T = \begin{bmatrix} \dot{q}_3 \\ \dot{q}_1 \\ -\dot{q}_2 \end{bmatrix}. \tag{5} $$

The last expression of the angular velocity of rotation of the frame $\{T\}$ in relation to the fixed frame $\{0\}$, expressed in the mobile frame $\{T\}$ and in the fixed frame $\{0\}$ is the same as the expression of the angular velocity of rotation of the frame $\{4\}$ and it is as follows:

$$ {}^{4}\bar{\omega}_4 \equiv {}^{T}\bar{\omega}_T = \begin{bmatrix} {}^{T}\omega_{xT} \\ {}^{T}\omega_{yT} \\ {}^{T}\omega_{zT} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \dot{q}_4 \end{bmatrix}. \tag{6} $$

$$ {}^{0}\bar{\omega}_4 \equiv {}^{0}\bar{\omega}_T = \begin{bmatrix} {}^{0}\omega_{xT} \\ {}^{0}\omega_{yT} \\ {}^{0}\omega_{zT} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \dot{q}_4 \end{bmatrix}. \tag{7} $$

Jacobian matrices were also determined, expressed in the frames $\{0\}$ and $\{T\}$:

$$ {}^{0}\underset{(6\times4)}{J}\left(\bar{\theta}\right) = \left[\begin{array}{ccc:c} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ \hdashline 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right] \tag{8} $$

$$ {}^{4}\underset{(6\times4)}{J}\left(\bar{\theta}\right) = {}^{T}\underset{(6\times4)}{J}\left(\bar{\theta}\right) = \left[\begin{array}{ccc:c} sq_4 & cq_4 & 1 & 0 \\ cq_4 & -sq_4 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ \hdashline 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]. \tag{9} $$

The expression Jacobian matrix is used to define the equations of direct kinematics (linear and angular velocities). The direct kinematics equations for the 3TR structure, represented by the column vector of the generalized velocities, expressed in the fixed frame $\{0\}$ and in the mobile frame $\{T\}$, are the following:

$$ {}^{0}\dot{\bar{X}} = \begin{bmatrix} (\dot{q}_3 & \dot{q}_1 & -\dot{q}_2)^T \\ (0 & 0 & \dot{q}_4)^T \end{bmatrix} \tag{10} $$

$$ {}^{4}\dot{\bar{X}} = {}^{4}J(\bar{\theta}) \cdot \dot{\bar{\theta}} = \begin{bmatrix} {}^{4}\bar{v}_4 \equiv {}^{T}\bar{v}_{T_T} \\ {}^{4}\bar{\omega}_4 \equiv {}^{T}\bar{\omega}_T \end{bmatrix}. \tag{11} $$

The equation (11) expresses the movement of the tool $\{T\}$ in the Cartesian space, it is relative to the fixed frame $\{0\}$ attached to the fixed base of the type 3TR robot.

The absolute linear accelerations and angular accelerations of the reference frame $\{i\}$, located in each driving kinematic joint, expressed in the mobile frame $\{i\}$ and in the fixed frame $\{0\}$ are obtained from the symbolic computation program in MATLAB:

$$ {}^{1}\dot{\bar{v}}_1 = \begin{bmatrix} 0 \\ \ddot{q}_1 \\ 0 \end{bmatrix};\ {}^{0}\dot{\bar{v}}_1 = \begin{bmatrix} 0 \\ \ddot{q}_1 \\ 0 \end{bmatrix};\ {}^{1}\dot{\bar{\omega}}_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}; $$

$$ {}^{0}\dot{\bar{\omega}}_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{12} $$

$$ {}^{2}\dot{v}_2 = \begin{bmatrix} 0 \\ \ddot{q}_1 \\ -\ddot{q}_2 \end{bmatrix}; \qquad {}^{0}\dot{v}_2 = \begin{bmatrix} 0 \\ \ddot{q}_1 \\ -\ddot{q}_2 \end{bmatrix}; $$

$$ {}^{2}\dot{\omega}_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}; \qquad {}^{0}\dot{\omega}_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}; \tag{13} $$

$$ {}^{3}\dot{v}_3 = \begin{bmatrix} \ddot{q}_3 \\ \ddot{q}_1 \\ -\ddot{q}_2 \end{bmatrix}; \qquad {}^{0}\dot{v}_3 = \begin{bmatrix} \ddot{q}_3 \\ \ddot{q}_1 \\ -\ddot{q}_2 \end{bmatrix}; $$

$$ {}^{3}\dot{\omega}_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}; \qquad {}^{0}\dot{\omega}_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}. \tag{14} $$

The linear acceleration of the origin of the frame $\{T\}$ in relation to the fixed frame $\{0\}$, expressed in the mobile frame $\{T\}$ and in the fixed frame $\{0\}$, is equal to the linear acceleration of the origin of the frame $\{4\}$ and is characterized by the expressions:

$$ {}^{4}\dot{v}_4 \equiv {}^{T}\dot{v}_T = \begin{bmatrix} {}^{T}\dot{v}_{xT} \\ {}^{T}\dot{v}_{yT} \\ {}^{T}\dot{v}_{zT} \end{bmatrix} = \begin{bmatrix} sq_4 \cdot \ddot{q}_1 \\ cq_4 \cdot \ddot{q}_1 - \\ -\ddot{q} \end{bmatrix} \tag{15} $$

$$ {}^{0}\dot{v}_4 \equiv {}^{0}\dot{v}_T - \begin{bmatrix} {}^{0}\dot{v}_{xT} \\ {}^{0}\dot{v}_{yT} \\ {}^{0}\dot{v}_{zT} \end{bmatrix} - \begin{bmatrix} \ddot{q}_3 \\ \ddot{q}_1 \\ -\ddot{q}_2 \end{bmatrix}. \tag{16} $$

The final expression of the angular acceleration of rotation of the frame $\{T\}$ in relation to the fixed frame $\{0\}$, expressed in the mobile frame $\{T\}$ and in the fixed frame $\{0\}$ is the same as the expression of the angular acceleration of rotation of the frame $\{4\}$ and it is as follows:

$$ {}^{4}\dot{\omega}_4 \equiv {}^{T}\dot{\omega}_T = \begin{bmatrix} {}^{T}\dot{\omega}_{xT} \\ {}^{T}\dot{\omega}_{yT} \\ {}^{T}\dot{\omega}_{zT} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \ddot{q}_4 \end{bmatrix}. \tag{17} $$

$$ {}^{0}\dot{\omega}_4 \equiv {}^{0}\dot{\omega}_T = \begin{bmatrix} {}^{0}\dot{\omega}_{xT} \\ {}^{0}\dot{\omega}_{yT} \\ {}^{0}\dot{\omega}_{zT} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \ddot{q}_4 \end{bmatrix}. \tag{18} $$

The Jacobian matrix derivatives expressed in the frames $\{0\}$ and $\{T\}$ have the following form:

$$ \underset{(6\times4)}{{}^{0}J}\left(\bar{\theta}\right) = \left[\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]. \tag{19} $$

The expression of the time derivative of the Jacobian matrix is used to define the equations of direct kinematics (linear and angular accelerations). The equations of direct kinematics, represented by the column vector of generalized accelerations, for the structure 3TR, expressed in the fixed frame $\{0\}$ and in the mobile frame $\{T\}$, are the following:

$$ {}^{0}\ddot{X} = \begin{bmatrix} (\ddot{q}_3 \quad \ddot{q}_1 \quad -\ddot{q}_2)^T \\ (0 \quad 0 \quad \ddot{q}_4)^T \end{bmatrix} \tag{20} $$

$$ {}^{4}\ddot{X} = \begin{bmatrix} (sq_4 \cdot \ddot{q}_1 + cq_4 \cdot \ddot{q}_3 \quad cq_4 \cdot \ddot{q}_1 - sq_4 \cdot \ddot{q}_3 \quad -\ddot{q}_2)^T \\ (0 \quad 0 \quad \ddot{q}_4)^T \end{bmatrix} \tag{21} $$

The expressions (20) and (21) represent the column vectors of the operational accelerations of the 3TR robot.

2.1 Direct Kinematics Equations for 2R structure

To determine the equations of direct kinematics, the algorithm of exponential matrices in kinematics will also be applied. For this purpose, the equations of direct geometry are used. [2]

Using the symbolic computation in MATLAB, the absolute linear and angular velocities of the reference frame $\{5\}$, expressed in the mobile frame $\{5\}$ and in the fixed frame, are obtained:

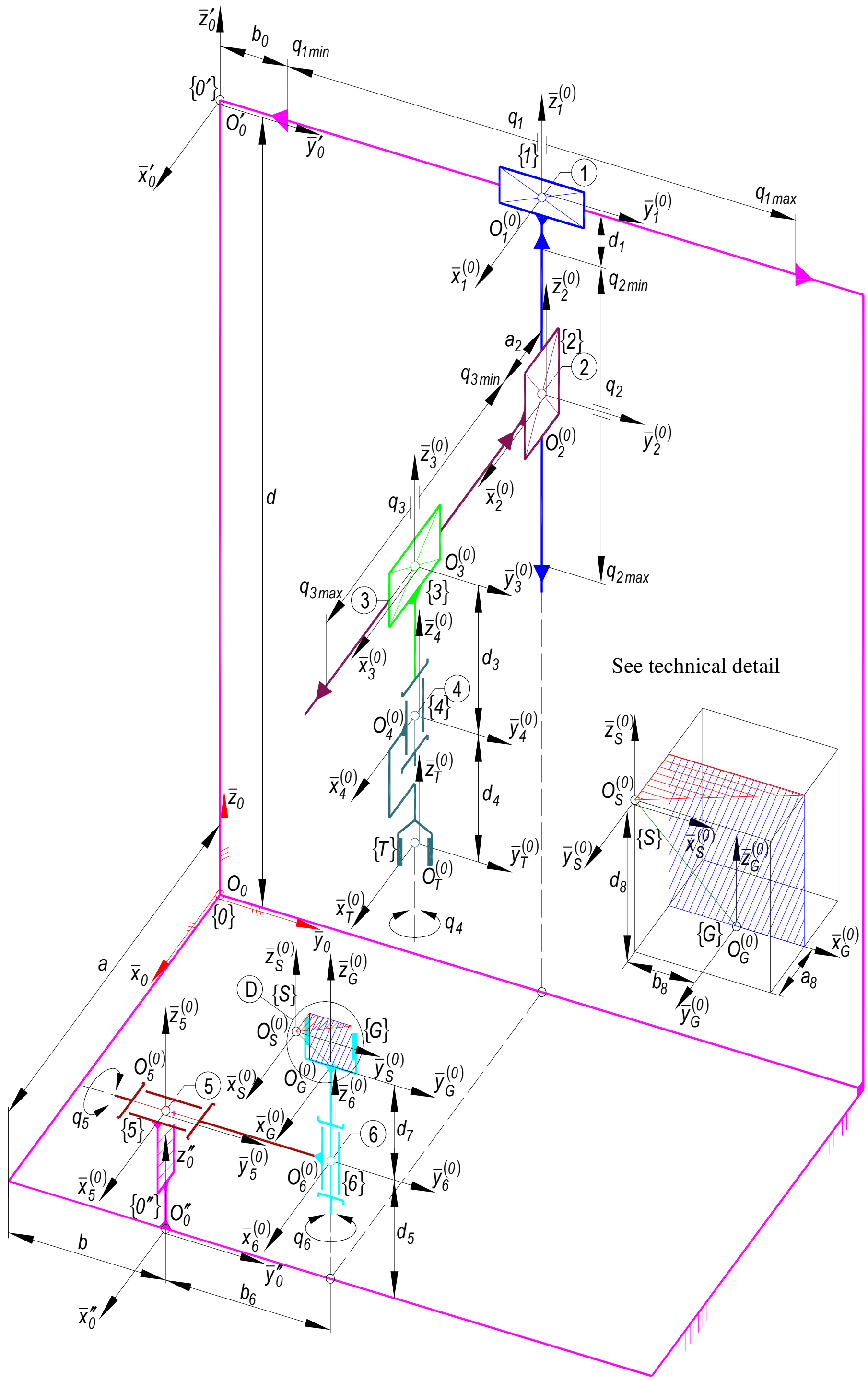

Fig.1 Kinematic scheme for tow robots working in cooperative movements

$$
{}^{5}\bar{v}_{5}=\begin{bmatrix}0\\0\\0\end{bmatrix};\ {}^{0}\bar{v}_{5}=\begin{bmatrix}0\\0\\0\end{bmatrix};
$$

$$
{}^{5}\bar{\omega}_{5}=\begin{bmatrix}0\\\dot{q}_{5}\\0\end{bmatrix};\ {}^{0}\bar{\omega}_{5}=\begin{bmatrix}0\\\dot{q}_{5}\\0\end{bmatrix}. \tag{22}
$$

The linear velocity of the origin of the frame $\{6\}$ in relation to the fixed frame $\{0\}$, expressed in the mobile frame $\{6\}$ and in the fixed frame $\{0\}$ is characterized by the expressions:

$$
{}^{6}\bar{v}_{6}=\begin{bmatrix}0\\0\\0\end{bmatrix};\qquad {}^{0}\bar{v}_{6}=\begin{bmatrix}0\\0\\0\end{bmatrix}. \tag{23}
$$

The final expression of the angular velocity of rotation of the frame $\{6\}$ relative to the fixed frame $\{0\}$, expressed in the mobile frame $\{6\}$ and in the fixed frame $\{0\}$, is the same as the expression of the angular acceleration of rotation of the frame $\{G\}$ and $\{S\}$ and it is as follows:

$$
{}^{6}\bar{\omega}_{6}\equiv{}^{G}\bar{\omega}_{G}\equiv{}^{S}\bar{\omega}_{S}=\begin{bmatrix}{}^{S}\omega_{xS}\\{}^{S}\omega_{yS}\\{}^{S}\omega_{zS}\end{bmatrix}=\begin{bmatrix}\dot{q}_{5}\cdot s\\\dot{q}_{5}\cdot c\\\dot{q}_{6}\end{bmatrix} \tag{24}
$$

;

$$
{}^{0}\bar{\omega}_{6}\equiv{}^{0}\bar{\omega}_{G}\equiv{}^{0}\bar{\omega}_{S}=\begin{bmatrix}{}^{0}\omega_{xS}\\{}^{0}\omega_{yS}\\{}^{0}\omega_{zS}\end{bmatrix}=\begin{bmatrix}\dot{q}_{6}\cdot s\\\dot{q}_{5}\\\dot{q}_{6}\cdot c\end{bmatrix} \tag{25}
$$

.

The linear velocity of the origin of the frame $\{G\}$ in relation to the fixed frame [0], expressed in the mobile frame $\{G\}$ and in the fixed frame $\{0\}$ is characterized by the expressions:

$$
{}^{G}\bar{v}_{G}=\begin{bmatrix}d_{7}\cdot\dot{q}_{5}\cdot cq_{6}\\-d_{7}\cdot\dot{q}_{5}\cdot sq_{6}\\0\end{bmatrix}; \tag{26}
$$

$$
{}^{0}\bar{v}_{S}=\dot{p}_{G}=\begin{bmatrix}\dot{p}_{xG}\\\dot{p}_{yG}\\\dot{p}_{zG}\end{bmatrix}\begin{bmatrix}d_{7}\cdot\dot{q}_{5}\cdot cq_{5}\\0\\-d_{7}\cdot\dot{q}_{5}\cdot sq_{5}\end{bmatrix}. \tag{27}
$$

The linear velocity of the origin of the frame $\{S\}$ in relation to the fixed frame $\{0\}$, expressed in the mobile frame $\{S\}$ and in the fixed frame $\{0\}$ is characterized by the expressions:

$$
{}^{S}\bar{v}_{S}=\begin{bmatrix}{}^{S}v_{xS}\\{}^{S}v_{yS}\\{}^{S}v_{zS}\end{bmatrix}=\begin{bmatrix}b_{8}\cdot\dot{q}_{6}+(d_{7}+d_{8})\cdot\dot{q}\\a_{8}\cdot\dot{q}_{6}-(d_{7}+d_{8})\cdot\dot{q}\\-\dot{q}_{5}\cdot(a_{8}\cdot cq_{6}+b_{8}\cdot\end{bmatrix} \tag{28}
$$

$$
{}^{0}\bar{v}_{S}=\begin{bmatrix}(d_{7}+d_{8})\cdot\dot{q}_{5}\cdot cq_{5}-\\-(a_{8}\cdot cq_{6}+b_{8}\cdot sq_{6})\cdot\dot{q}_{5}\cdot sq_{5}\cdot\\+(-a_{8}\cdot sq_{6}+b_{3}\cdot cq_{6})\cdot\dot{q}_{6}\cdot cq\\(a_{8}\cdot cq_{6}+b_{8}\cdot sq_{6})\cdot\dot{q}_{6}\\-(d_{7}+d_{8})\cdot\dot{q}_{5}\cdot sq_{5}-\\-(a_{8}\cdot cq_{6}+b_{8}\cdot sq_{6})\cdot\dot{q}_{5}\cdot cq_{5}\cdot\\+(a_{8}\cdot sq_{6}-b_{8}\cdot cq_{6})\cdot\dot{q}_{6}\cdot sq_{5}\end{bmatrix} \tag{29}
$$

The Jacobian matrices expressed in the frame $\{0\}$ and $\{S\}$ were also determined:

$$
\underset{(6\times2)}{{}^{0}J\left(\bar{\theta}\right)}=\begin{bmatrix}\chi_{1} & \left(-a_{8}\cdot sq_{6}+b_{8}\cdot cq_{6}\right)\cdot cq_{5}\\0 & a_{8}\cdot cq_{6}+b_{8}\cdot sq_{6}\\\chi_{2} & \left(a_{8}\cdot sq_{6}-b_{8}\cdot cq_{6}\right)\cdot sq_{5}\\0 & sq_{5}\\1 & 0\\0 & cq_{5}\end{bmatrix}
$$

$$
\chi_{1}=\begin{bmatrix}-\left(a_{8}\cdot cq_{6}+b_{8}\cdot sq_{6}\right)\cdot sq_{5}+\\+\left(d_{7}+d_{8}\right)\cdot cq_{5}\end{bmatrix} \tag{30}
$$

$$
\chi_{2}=\begin{bmatrix}-\left(a_{8}\cdot cq_{6}+b_{8}\cdot sq_{6}\right)\cdot cq_{5}-\\-\left(d_{7}+d_{8}\right)\cdot sq_{5}\end{bmatrix}
$$

$$
{}^{6}\underset{(6\times 2)}{J}\left(\bar{\theta}\right)=\begin{bmatrix} (d_7+d_8)\cdot cq_6 & b_8 \\ -(d_7+d_8)\cdot sq_6 & a_8 \\ -a_8\cdot cq_6-b_8\cdot sq_6 & 0 \\ sq_6 & 0 \\ cq_6 & 0 \\ 0 & 1 \end{bmatrix} \tag{31}
$$

$$
\dot{\bar{X}} = {}^{0}J(\bar{\theta})\cdot\dot{\bar{\theta}} = \begin{bmatrix} (d_7+d_8)\cdot\dot{q}_5\cdot cq_5 - \\ -(a_8\cdot cq_6+b_8\cdot sq_6)\cdot\dot{q}_5\cdot sq_5 + \\ +(-a_8\cdot sq_6+b_8\cdot cq_6)\cdot\dot{q}_6\cdot cq_5 \\ (a_8\cdot cq_6+b_8\cdot sq_6)\cdot\dot{q}_6 \\ -(d_7+d_8)\cdot\dot{q}_5\cdot sq_5 - \\ -(a_8\cdot cq_6+b_8\cdot sq_6)\cdot\dot{q}_5\cdot cq_5 + \\ +(a_8\cdot sq_6-b_8\cdot cq_6)\cdot\dot{q}_6\cdot sq_5 \end{bmatrix}
$$

$$
\begin{bmatrix} {}^{0}\bar{v}_S \\ \cdots\cdots\cdots \\ {}^{0}\bar{\omega}_6 \equiv {}^{0}\bar{\omega}_S \end{bmatrix} = \begin{bmatrix} \dot{q}_6\cdot sq_5 \\ \dot{q}_5 \\ \dot{q}_6\cdot cq_5 \end{bmatrix} \tag{32}
$$

$$
{}^{6}\dot{\bar{X}} = \begin{bmatrix} \begin{bmatrix} b_8\cdot\dot{q}_5+(d_7+d_8)\cdot\dot{q}_5\cdot cq_6 \\ a_8\cdot\dot{q}_6-(d_7+d_8)\cdot\dot{q}_5\cdot sq_6 \\ -\dot{q}_5\cdot(a_8\cdot cq_6+b_8\cdot sq_6) \end{bmatrix} \\ \cdots\cdots\cdots\cdots\cdots\cdots \\ \begin{bmatrix} \dot{q}_5\cdot sq_6 \\ \dot{q}_5\cdot cq_6 \\ \dot{q}_6 \end{bmatrix} \end{bmatrix}. \tag{33}
$$

The equation (32) expresses the motion of the part $\{S\}$ in the Cartesian space, is relative to the fixed frame $\{0\}$ attached to the fixed base of the type 2R robot.

The absolute linear accelerations and angular accelerations of the reference frame [S], expressed in the mobile frame $\{5\}$ and in the fixed frame $\{0\}$, are obtained from the symbolic computation in MATLAB:

$$
{}^{5}\dot{\bar{v}}_5 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix};\ {}^{0}\dot{\bar{v}}_5 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix};
$$
$$
{}^{5}\dot{\bar{\omega}}_5 = \begin{bmatrix} 0 \\ \ddot{q}_5 \\ 0 \end{bmatrix};\ {}^{0}\dot{\bar{\omega}}_5 = \begin{bmatrix} 0 \\ \ddot{q}_5 \\ 0 \end{bmatrix}. \tag{34}
$$

The linear acceleration of the origin of the frame $\{6\}$ in relation to the fixed frame $\{0\}$, expressed in the mobile frame and in the fixed frame $\{0\}$ is characterized by the expressions:

$$
{}^{6}\dot{\bar{v}}_6 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix};\quad {}^{0}\dot{\bar{v}}_6 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}. \tag{35}
$$

The final expression of the angular acceleration of rotation of the frame $\{6\}$ in relation to the fixed frame $\{0\}$, expressed in the mobile frame $\{6\}$ and in the fixed frame [0] is the same as the expression of the angular acceleration of rotation of the frames $\{G\}$ and $\{S\}$ and it is as follows:

$$
{}^{6}\dot{\bar{\omega}}_6 \equiv {}^{G}\dot{\bar{\omega}}_G \equiv {}^{S}\dot{\bar{\omega}}_S = \begin{bmatrix} \ddot{q}_5\cdot sq_6+\dot{q}_5\cdot\dot{q}_6\cdot cq_6 \\ \ddot{q}_5\cdot cq_6-\dot{q}_5\cdot\dot{q}_6\cdot sq_6 \\ \ddot{q}_6 \end{bmatrix}. \tag{36}
$$

$$
{}^{0}\dot{\bar{\omega}}_6 \equiv {}^{0}\dot{\bar{\omega}}_G \equiv {}^{0}\dot{\bar{\omega}}_S = \begin{bmatrix} \ddot{q}_6\cdot sq_5+\dot{q}_5\cdot\dot{q}_6\cdot cq_5 \\ \ddot{q}_5 \\ \ddot{q}_6\cdot cq_5-\dot{q}_5\cdot\dot{q}_6\cdot sq_5 \end{bmatrix}. \tag{37}
$$

The linear acceleration of the origin of the frame $\{G\}$ in relation to the fixed frame $\{0\}$, expressed in the mobile frame $\{G\}$ and in the fixed frame $\{0\}$ is:

$$
{}^{G}\dot{\bar{v}}_G = \begin{bmatrix} {}^{G}v_{xG} \\ {}^{G}v_{yG} \\ {}^{G}v_{zG} \end{bmatrix} = \begin{bmatrix} d_7\cdot\ddot{q}_5\cdot cq_6 \\ -d_7\cdot\ddot{q}_5\cdot sq_6 \\ -d_7\cdot\dot{q}_5^2 \end{bmatrix}; \tag{38}
$$

$$ {}^{\dot{0}}v_S = \begin{bmatrix} -d_7 \cdot (\dot{q}_5^2 \cdot sq_5 - \ddot{q}_5 \cdot cq_5) \\ 0 \\ -d_7 \cdot (\dot{q}_5^2 \cdot cq_5 + \ddot{q}_5 \cdot sq_5) \end{bmatrix}. \quad (39) $$

The linear acceleration of the origin of the frame $\{S\}$ in relation to the fixed frame $\{0\}$, expressed in the mobile frame $\{S\}$ and in the fixed frame $\{0\}$ is characterized by the expressions:

$$ {}^{\dot{s}}v_S = \begin{bmatrix} b_8 \cdot \ddot{q}_6 + (d_7 + d_8) \cdot \ddot{q}_5 \cdot cq_6 - \\ -\dot{q}_5^2 \cdot cq_6 \cdot (a_8 \cdot cq_6 + b_8 \cdot sq_6) - \\ -\dot{q}_6^2 \cdot a_8 \\ a_8 \cdot \ddot{q}_6 - (d_7 + d_8) \cdot \ddot{q}_5 \cdot sq_6 + \\ +\dot{q}_5^2 \cdot sq_6 \cdot (a_8 \cdot cq_6 + b_8 \cdot sq_6) + \\ +\dot{q}_6^2 \cdot b_8 \\ -\ddot{q}_5 \cdot (a_8 \cdot cq_6 + b_8 \cdot sq_6) + \\ +2 \cdot \dot{q}_5^2 \cdot \dot{q}_6^2 \cdot (a_8 \cdot sq_6 - b_8 \cdot cq_6) - \\ -\dot{q}_5^2 \cdot (d_7 + d_8) \end{bmatrix}; (40) $$

$$ {}^{\dot{0}}v_S = \begin{bmatrix} \left\{ \begin{matrix} (d_7 + d_8) \cdot (\ddot{q}_5 \cdot cq_5 - \dot{q}_5^2 \cdot sq_5) + \\ +(a_8 \cdot sq_6 - b_8 \cdot cq_6) \cdot \\ \cdot (2 \cdot \dot{q}_5 \cdot \dot{q}_6 \cdot sq_5 - \ddot{q}_6 \cdot cq_5) - \\ (a_8 \cdot cq_6 + b_8 \cdot sq_6) \cdot \\ \cdot [(\dot{q}_5^2 + \dot{q}_6^2) \cdot cq_5 + \ddot{q}_5 \cdot sq_5] \end{matrix} \right\} \\ (a_8 \cdot cq_6 + b_8 \cdot sq_6) \cdot \ddot{q}_6 - \\ -(a_8 \cdot sq_6 - b_8 \cdot cq_6) \cdot \dot{q}_5^2 \\ \left\{ \begin{matrix} -(d_7 + d_8) \cdot (\ddot{q}_5 \cdot sq_5 - \dot{q}_5^2 \cdot cq_5) + \\ +(a_8 \cdot sq_6 - b_8 \cdot cq_6) \cdot \\ \cdot (2 \cdot \dot{q}_5 \cdot \dot{q}_6 \cdot cq_5 + \ddot{q}_6 \cdot sq_5) + \\ +(a_8 \cdot cq_6 + b_8 \cdot sq_6) \cdot \\ \cdot [(\dot{q}_5^2 + \dot{q}_6^2) \cdot sq_5 - \ddot{q}_5 \cdot cq_5] \end{matrix} \right\} \end{bmatrix}. \quad (41) $$

The Jacobian matrix derivatives expressed in frames $\{0\}$ and $\{S\}$ have the following form:

$$ \underset{(6\times2)}{{}^{\dot{0}}J(\bar{\theta})} = \left\{ \begin{matrix} \begin{bmatrix} {}^{\dot{0}}J_1 & {}^{\dot{0}}J_2 \end{bmatrix} \\ respectively \\ \begin{bmatrix} \dot{J}_{1v} & \dot{J}_{2v} \\ \cdots & \cdots \\ \dot{J}_{1\omega} & \dot{J}_{2\omega} \end{bmatrix} \end{matrix} \right\} = \begin{bmatrix} \varepsilon_1 & \begin{matrix} \dot{q}_5 \cdot sq_5 \cdot (a_8 \cdot sq_6 - b_8 \cdot cq_6) - \\ -\dot{q}_6 \cdot cq_5 \cdot (a_8 \cdot cq_6 + b_8 \cdot sq_6) \end{matrix} \\ 0 & -\dot{q}_6 \cdot (a_8 \cdot sq_6 - b_8 \cdot cq_6) \\ \varepsilon_2 & \begin{matrix} \dot{q}_5 \cdot cq_5 \cdot (a_8 \cdot sq_6 - b_8 \cdot cq_6) + \\ +\dot{q}_6 \cdot sq_5 \cdot (a_8 \cdot cq_6 + b_8 \cdot sq_6) \end{matrix} \\ 0 & \dot{q}_5 \cdot cq_5 \\ 0 & 0 \\ 0 & -\dot{q}_5 \cdot sq_5 \end{bmatrix}. \quad (42) $$

$$ \varepsilon_1 = \begin{bmatrix} -sq_5 \cdot \begin{bmatrix} \dot{q}_6 \cdot (a_8 \cdot sq_6 - b_8 \cdot cq_6) + \\ +\dot{q}_5 \cdot (d_7 + d_8) \end{bmatrix} \\ -\dot{q}_5 \cdot cq_5 (a_8 \cdot cq_6 + b_8 \cdot sq_6) \end{bmatrix} $$

$$ \varepsilon_2 = \begin{bmatrix} \dot{q}_5 \cdot sq_5 (a_8 \cdot cq_6 + b_8 \cdot sq_6) + \\ +cq_5 \cdot \begin{bmatrix} \dot{q}_6 \cdot (a_8 \cdot sq_6 - b_8 \cdot cq_6) - \\ -\dot{q}_5 \cdot (d_7 + d_8) \end{bmatrix} \end{bmatrix} $$

$$ \underset{(6\times2)}{{}^{\dot{s}}J(\bar{\theta})} \equiv \underset{(6\times2)}{{}^{\dot{6}}J(\bar{\theta})} \equiv \underset{(6\times2)}{{}^{\dot{s}}J(\bar{\theta})} = \begin{bmatrix} -\dot{q}_5 \cdot cq_6 \cdot \begin{pmatrix} a_8 \cdot cq_6 + \\ +b_8 \cdot sq_6 \end{pmatrix} & -\dot{q}_6 \cdot a_8 \\ \dot{q}_6 \cdot \begin{pmatrix} a_8 \cdot sq_6 - \\ -b_8 \cdot cq_6 \end{pmatrix} - & \dot{q}_6 \cdot b_8 \\ -\dot{q}_5 \cdot (d_7 + d_8) & \dot{q}_5 \cdot \begin{pmatrix} a_8 \cdot sq_5 - \\ -b_8 \cdot cq_6 \end{pmatrix} \\ 0 & \dot{q}_5 \cdot cq_6 \\ 0 & -\dot{q}_5 \cdot sq_5 \\ 0 & 0 \end{bmatrix}. \quad (43) $$

The expression of the time derivative of the Jacobian matrix is used to define the equations of direct kinematics (linear and angular accelerations). The equations of direct kinematics, represented by the column vector of generalized accelerations, for the structure

2R, expressed in the mobile frame $\{S\}$ and in the fixed frame $\{0\}$, are the following:

$$ {}^0\ddot{X} = \begin{bmatrix} \left\{ \begin{matrix} (d_7+d_8)\cdot(\ddot{q}_5\cdot cq_5-\dot{q}_5^2\cdot sq_5 \\ +(a_8\cdot sq_6-b_8\cdot cq_6)\cdot \\ \cdot(2\cdot\dot{q}_5\cdot\dot{q}_6\cdot sq_5-\ddot{q}_6\cdot cq_5) \\ -(a_8\cdot cq_6+b_8\cdot sq_6)\cdot \\ \cdot[(\dot{q}_5^2+\dot{q}_6^2)\cdot cq_5+\ddot{q}_5\cdot sq_5 \end{matrix} \right. \\ (a_8\cdot cq_6+b_8\cdot sq_6)\cdot\ddot{q}_6 \\ -(a_3\cdot sq_6-b_8\cdot cq_6)\cdot\dot{q}_6^2 \\ \left\{ \begin{matrix} -(d_7+d_8)\cdot(\ddot{q}_5\cdot sq_5-\dot{q}_5^2\cdot cq \\ +(a_8\cdot sq_6-b_8\cdot cq_6)\cdot \\ \cdot(2\cdot\dot{q}_5\cdot\dot{q}_6\cdot cq_5+\ddot{q}_6\cdot sq_5) \\ +(a_8\cdot cq_6+b_8\cdot sq_6)\cdot \\ \cdot[(\dot{q}_5^2+\dot{q}_6^2)\cdot sq_5-\ddot{q}_5\cdot cq_5 \end{matrix} \right. \end{bmatrix} \quad (44)$$

$$\begin{bmatrix} {}^S\dot{v}_S \\ {}^6\dot{\omega}_6 = {}^S\dot{\omega}_S \end{bmatrix} = \begin{bmatrix} \ddot{q}_6\cdot sq_5+\dot{q}_5\cdot\dot{q}_6\cdot cq_5 \\ \ddot{q}_5 \\ \ddot{q}_6\cdot cq_5-\dot{q}_5\cdot\dot{q}_6\cdot sq_5 \end{bmatrix}. \quad (45)$$

$$ {}^6\ddot{X} = {}^S\ddot{X} = \begin{bmatrix} {}^S\dot{v}_S \\ {}^6\dot{\omega}_6 \equiv {}^S\dot{\omega}_S \end{bmatrix} = $$
$$ = \begin{bmatrix} {}^SJ(\bar{\theta}) & {}^S\dot{J}(\bar{\theta}) \end{bmatrix} \cdot \begin{bmatrix} \ddot{\theta} \\ \dot{\theta} \end{bmatrix}. \quad (46)$$

$$ {}^S\dot{v}_S = \begin{bmatrix} b_8\cdot\ddot{q}_6+(d_7+d_8)\cdot\ddot{q}_5\cdot cq_6- \\ -\dot{q}_5^2\cdot cq_6\cdot(a_8\cdot cq_6+b_8\cdot sq_6)-\dot{q}_6^2\cdot a_8 \\ a_8\cdot\ddot{q}_6-(d_7+d_8)\cdot\ddot{q}_5\cdot sq_6+ \\ +\dot{q}_5^2\cdot sq_6\cdot(a_8\cdot cq_6+b_8\cdot sq_6)+\dot{q}_6^2\cdot b_8 \\ -\ddot{q}_5\cdot(a_8\cdot cq_6+b_8\cdot sq_6)+ \\ +2\cdot\dot{q}_5^2\cdot\dot{q}_6^2\cdot(a_8\cdot sq_6-b_8\cdot cq_6)- \\ -\dot{q}_5^2\cdot(d_7+d_8) \end{bmatrix}. \quad (47)$$

$$ {}^6\dot{\omega}_6 = \begin{bmatrix} \ddot{q}_5\cdot sq_6+\dot{q}_5\cdot\dot{q}_6\cdot cq_6 \\ \ddot{q}_5\cdot cq_6-\dot{q}_5\cdot\dot{q}_6\cdot sq_6 \\ \ddot{q}_6 \end{bmatrix}. \quad (48)$$

Expressions (44), (45), (47), (48) represent the column vectors of the operational accelerations of the 2R robot expressed in the mobile frame $\{S\}$ and in the fixed frame $\{0\}$.

2.3 Equations of the direct kinematics of the cooperation between structure 3TR and structure 2R

The linear velocity and the angular velocity of rotation of the tool in relation to the frame of the workpiece are obtained from the equations presented in [2].

To determine the linear velocity of the origin of the frame $\{T\}$ in relation to $\{S\}$, the expressions are written:

$$ {}^S\bar{p}_{TS} = {}^0_S[R]^{-1}\cdot(\bar{p}_T-\bar{p}_S) = {}^0_S[R]^{-1}\cdot\bar{p}_{TS}. \quad (49)$$

$$ {}^S\dot{\bar{p}}_{TS} = {}^0_S[R]^{-1}\cdot\dot{\bar{p}}_{TS}. \quad (50)$$

$$ {}^S\dot{p}_{TS} = \begin{bmatrix} \dot{q}_1\cdot sq_6+(\dot{q}_2\cdot sq_5+\dot{q}_3\cdot cq_5)\cdot cq_6- \\ -(d_7+d_8)\cdot\dot{q}_5\cdot cq_6-\dot{q}_6\cdot b_8 \\ \dot{q}_1\cdot cq_6-(\dot{q}_2\cdot sq_5+\dot{q}_3\cdot cq_5)\cdot sq_6+ \\ +(d_7+d_8)\cdot\dot{q}_5\cdot sq_6-\dot{q}_6\cdot a_8 \\ \dot{q}_5\cdot(a_8\cdot cq_6+b_8\cdot sq_6)- \\ -\dot{q}_2\cdot cq_5+\dot{q}_3\cdot sq_5 \end{bmatrix}. \quad (51)$$

$$ \left\{ {}^S\omega_S \times \right\} = \begin{bmatrix} 0 & -\dot{q}_6 & \dot{q}_5\cdot cq_6 \\ \dot{q}_6 & 0 & -\dot{q}_5\cdot sq_6 \\ -\dot{q}_5\cdot cq_6 & \dot{q}_5\cdot sq_6 & 0 \end{bmatrix}. \quad (52)$$

The final expression for the angular velocity of the frame $\{T\}$ relative to $\{S\}$ is:

$$ {}^S\omega_T = \begin{bmatrix} -\dot{q}_4\cdot sq_5\cdot cq_6-\dot{q}_5\cdot sq_6 \\ \dot{q}_4\cdot sq_5\cdot sq_6-\dot{q}_5\cdot cq_6 \\ \dot{q}_4\cdot cq_5-\dot{q}_6 \end{bmatrix}. \quad (53)$$

Equations (51), (53) characterize the linear velocity and the angular velocity of rotation of the frame attached to the tool, $\{T\}$, relative to the frame attached to the part, $\{S\}$.

To determine the linear velocity of the origin of the frame $\{S\}$ in relation to $\{T\}$, the following expressions are written:

$$ {}^{T}\overline{p}_{ST} = {}^{0}_{T}[R]^{-1} \cdot (\overline{p}_S - \overline{p}_T) = {}^{0}_{T}[R]^{-1} \cdot \overline{p}_{ST} . \quad (54) $$

$$ {}^{T}\dot{\overline{p}}_{ST} = {}^{0}_{T}[R]^{-1} \cdot \dot{\overline{p}}_{ST} . \quad (55) $$

$$ \frac{\partial}{\partial t}\left({}^{T}\overline{p}_{ST} \right) = {}^{0}_{T}\left[\dot{R}\right]^{-1} \cdot \overline{p}_{ST} + {}^{T}\dot{\overline{p}}_{ST} . \quad (56) $$

$$ {}^{T}\dot{\overline{p}}_{ST} = \{ {}^{T}\overline{\omega}_T \times \} \cdot {}^{T}\overline{p}_{ST} + \frac{\partial}{\partial t}\left({}^{T}\overline{p}_{ST} \right). \quad (57) $$

$$ \{ {}^{T}\overline{\omega}_T \times \} = \begin{bmatrix} 0 & -\dot{q}_4 & 0 \\ \dot{q}_4 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \quad (58) $$

$$ {}^{T}\overline{\omega}_T = \begin{bmatrix} 0 \\ 0 \\ \dot{q}_4 \end{bmatrix}. \quad (59) $$

The linear velocity of the origin of the frame $\{S\}$ in relation to the frame of the tool $\{T\}$ is characterized by the expressions:

$$ {}^{T}\dot{\overline{p}}_{ST} = \{ {}^{T}\overline{\omega}_T \times \} \cdot {}^{T}\overline{p}_{ST} + \frac{\partial}{\partial t}\left({}^{T}\overline{p}_{ST} \right) \quad (60) $$

$$ {}^{T}\dot{\overline{p}}_{ST} = \begin{bmatrix} \left\{ \begin{matrix} \begin{bmatrix} a_6 \cdot \dot{q}_5 \cdot sq_5 + \\ +(d_7 - d_6) \cdot \dot{q}_5 \cdot cq_5 \end{bmatrix} \cdot c(q_4 \cdot \Delta_4) - \\ -\dot{q}_1 \cdot c(q_4 \cdot \Delta_4) - \dot{q}_2 \cdot s(q_4 \cdot \Delta_4) \end{matrix} \right\} \\ \left\{ \begin{matrix} -\begin{bmatrix} a_6 \cdot \dot{q}_5 \cdot sq_5 + \\ +(d_7 - d_6) \cdot \dot{q}_5 \cdot cq_5 \end{bmatrix} \cdot s(q_4 \cdot \Delta_4) + \\ +\dot{q}_1 \cdot s(q_4 \cdot \Delta_4) - \dot{q}_2 \cdot c(q_4 \cdot \Delta_4) \end{matrix} \right\} \\ \left\{ \begin{matrix} -\dot{q}_3 + a_6 \cdot \dot{q}_5 \cdot cq_5 - \\ -(d_7 - d_6) \cdot \dot{q}_5 \cdot sq_5 \end{matrix} \right\} \end{bmatrix} . \quad (61) $$

The final expression of the angular velocity of rotation of the workpiece frame $\{S\}$ relative to the tool frame $\{T\}$ is:

$$ {}^{T}\overline{\omega}_S = \begin{bmatrix} \dot{q}_5 \cdot sq_4 + \dot{q}_6 \cdot cq_4 \cdot sq_5 \\ \dot{q}_5 \cdot cq_4 - \dot{q}_6 \cdot sq_4 \cdot sq_5 \\ -\dot{q}_4 + \dot{q}_6 \cdot cq_5 \end{bmatrix}. \quad (62) $$

Equation (62) describe the linear velocity and the angular velocity of rotation of the frame attached to the part, $\{S\}$, in relation to the frame attached to the tool, $\{T\}$.

3. CONCLUSIONS

In this work we obtained: the direct kinematics equations for a robot that has four degrees of freedom three translations and one rotation; the direct kinematics equations for a robot that has two degrees of freedom two rotations; the direct kinematics equations for the cooperative movements between the two robots.

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ECUAȚIILE CINEMATICE A DOI ROBOȚI DE TIPUL 3TR-2R ÎN MIȘCĂRI DE COOPERARE

În această lucrare sunt prezentate ecuațiile cinematice pentru doi roboții care lucrează în mișcări de cooperare. Pentru a stabili ecuațiile cinematicii directe, se va aplica algoritmul matricelor exponențiale în cinematică. Scopul algoritmului este de a determina matricea Jacobiană, împreună cu derivata sa de ordinul întâi în raport cu timpul. Acești roboți, care lucrează în mișcări de cooperare, pot fi folosiți pentru a realiza piese complexe.

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