TECHNICAL UNIVERSITY OF CLUJ-NAPOCA

ACTA TECHNICA NAPOCENSIS

Series: Applied Mathematics, Mechanics, and Engineering
Vol. 65, Issue Special IV, December, 2022

ANALYSIS AND ALLOCATION OF DIMENSIONAL TOLERANCES USING GeoGebra SOFTWARE

Aurel TULCAN, Felicia Veronica BANCIU, Ion GROZAV

Abstract: *For mechanical products one of the features that define products quality is the existence of dimensional chains (DCs) whose closing dimensions meet the design requirements, respectively fall within the desired tolerance range. The paper presents a software program for calculating the tolerances for the 3D, 2D and 1D dimensional chains. This program helps to check the tolerances initially allocated and, in the event that they are unsatisfactory, the initial tolerances shall be modified using the proportional reduction method, or other tolerances shall be allocated in accordance with the coefficient of transfer of the component dimensions and a weighting factor on the costs regarding the tolerances achieving. The verification of the tolerances is done by the Monte-Carlo method.*

Key words: *dimensional chain, tolerance allocation, Monte–Carlo method, weighting coefficient, capability parameter*

1. INTRODUCTION

The manufacturing of parts/products with respect to the functional parameters imposed by the design engineers largely dictates the sequence of processing procedures and operations necessary to obtain the precision of the finished product, having direct implications on manufacturing costs.

The parts/components have indications of dimensional, geometric tolerances, surface roughness that must be strictly attained so that at the time of assembly/use they form the indicated types of fits and serve the functional role for which they were designed [1].

Thus, design engineers and technologists must know the methods of analysis, prescription and allocation of tolerances for dimensional chains closing elements, elements related to dimensional chains interchangeability, geometric and dimensional precision, the costs incurred by them, all constituting essential conditions for ensuring the products quality, their efficient and effective design, implementation and operation.

A dimensional chain (DC) can include any dimension of a part, dimension that could be the closing dimension (CD) of this dimensional chain. The tolerances of the closing dimensions are dependent on the tolerance of the dimensions that compose the dimensional chain. For the closing dimension it is necessary to obtain the admitted tolerance having tolerances as large as possible at dimensional chain component dimensions, in order to obtain a lower cost.

Tolerance analysis and tolerance allocation are two different methods [2]. In tolerance analysis the dimensional tolerances of a dimensional chain are specified and known, and based on them the resulting tolerance variation is calculated. This could be solved using different mathematical models for the geometrical deviations and gaps (e.g., the torsors of the small displacements) [3], three-dimensional (3D) tolerance analysis models [4], geometric model for representing statistically-based tolerance regions [5].

Another analysis methods, such as worst-case (complete interchangeability of the parts) and statistical analysis methods are presented in specialized literature. Designing to worst-case can require extremely tight tolerances that in turn produce and increases the cost of the parts. The Root Squares Sum (RSS) method and

Monte-Carlo (MC) method [3,6-8] are often used in statistical tolerance.

If the assembly tolerance is well known from design specifications (the closing element of dimensional chain specifications) and the size of component tolerances that have to meet the assembly requirements are unknown – this case is about tolerance allocation.

Among the methods for tolerance allocation is the Proportional Scaling Method for tolerance allocation [9,10], which by proportional scaling allows bringing the tolerance zone of the closing dimension to the desired value, the allocation using weighting factor. In [11] the allocation of tolerances using weighting factors is presented. In this way can be controlled which tolerance fields are bigger and smaller. These scalars apply to the tolerance rather than to the dimensions in dimensional chain as in applied MC simulation.

Least-cost tolerance allocation using optimization techniques is complex because it uses various information regarding tolerance, manufacturing costs, optimization, modelling and programming. Research on the allocation of tolerances by optimizing their costs, based on Lagrange multipliers combined with discrete search techniques to find the optimum processes from a set of alternatives and nonlinear programming techniques [12,13] and optimization based on genetic algorithms [14] are presented in specialized literature. In order to avoid part deviations and to achieve the desired tolerances, in dimensional chain closing dimension the designers will specify and allocate tolerances to the dimensions of a dimensional chain.

A software program that analyses tolerances is SolidWorks/TolAnalyst [15]. The tolerances are analyzed using statistical method Worst Case and RSS (Root Square Sum) without taking into account the tolerances costs. Our paper subject is to allocate tolerance using supplementary the cost of tolerances which are taken into account by means of weighting coefficient wk. SolidWorks solution offers good tolerances but satisfying only the transfer coefficients the results are good but not at the minimum price. For obtaining minimum price, a factor must be used that is a function of the tolerances costs, the weighting coefficient wk.

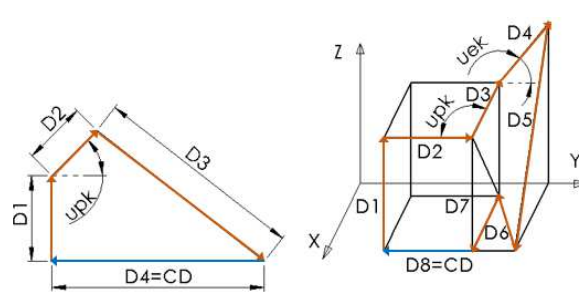
The development of the currently offered software program ensures the possibility of creating useful and efficient calculation programs for the relatively easy analysis and allocation of the tolerances of dimensional chains.

Thus, the existence of the GeoGebra software, [16-19], allows an effective analysis of tolerances, as well as their allocation in accordance with quality requirements.

The paper presents such a software program, which, using GeoGebra software, analyses some existing tolerances, checking whether the closing dimension (CD) of the dimensional chain (DC) falls within the tolerances allowed for it. In the next step the created software program proportionally reduces the existing tolerances, so that the closing dimension (CD) falls within the allowed tolerance field.

2. TOLERANCES ANALYSIS

The software program is dedicated for planar (2D) and spatial (3D) dimensional chains show in figure 1.



Planar DC Spatial DC

Fig. 1. Dimensional chains.

The closing dimension (CD) of a dimensional chain can be calculate using the mathematical equation (1) and the closing dimension's angles in XY plane, (up) and (ue), can be calculate using the equation (2) [1], where:

- D_k is the size of the dimensions of the dimensional chain;
- up_k is the angle in plane of each dimension D_k of DC;

- Cx, Cy, Cz are the closing dimension’s projections onto X, Y, Z axis;
- ue_k is the elevation angle, in degrees, of each dimension D_k of the DC.

In order to present the software operation mode, it was set up the initial data for spatial (3D) dimensional chain with 4 dimensions (Fig. 2a.) and the initial data for this dimensional chain was introduced in the matrix (Fig. 2b.).

$$\begin{cases} C_x = \sum_{k=1}^{nD} mD_k * \cos(up_k) * \cos(ue_k) \\ C_y = \sum_{k=1}^{nD} mD_k * \sin(up_k) * \cos(ue_k) \\ C_z = \sum_{k=1}^{nD} mD_k * \sin(ue_k) \\ C = \sqrt{C_x{}^2 + C_y{}^2 + C_z{}^2} \end{cases} \quad (1)$$

$$\begin{cases} up = \tan^{-1}\left(\frac{Cy}{Cx}\right) \\ ue = \tan^{-1}\left(\frac{C_z}{\sqrt{C_x{}^2 + C_y{}^2}}\right) \end{cases} \quad (2)$$

Figure 2a shows all 4 dimensions with their vectors located in origin of the reference system (light blue color) and the 3D dimension chain, created with the previously drawn vectors (dark blue color).

The number of columns of the D_{at} matrix represents the number of n_D component dimensions of the DC. The allowed (desired) tolerances for the closing dimension (CD) of the dimensional chain (DC) is Td=0.05 mm.

The D_{at} matrix has in its lines:
- The size of dimensions D_k of dimensional chain;
- The angle in plane u_{pk} for each dimension D_k of DC;
- The closing dimension’s projections Cx, Cy, Cz in X, Y, Z axis;
- The elevation angle ue_k, in degrees, of each dimension D_k of the DC;
- The initially assigned tolerance fields T_k of each dimension D_k of the DC;
- The weighting factor of tolerance, w_k=1,...,5, dependent on costs: small values for low costs and large values for higher costs.

Since the verification of tolerances is done by the Monte-Carlo method, the number of random dimensions of D_k that are generated is nc=1000, and the capability parameter considered in the calculations is Cp=1.33.

The GeoGebra software has an important facility, it can calculate the angles g_k between component dimension vectors D_k and the closing dimension vector C (see Fig. 3.). In figure 3a are presented the vectors with their origin at the origin of the reference system, in order to observe the angles formed between them (D_k) and the closing vector (C). These angles were calculated with GeoGebra and the value of these angles are presented in the figure 3b.

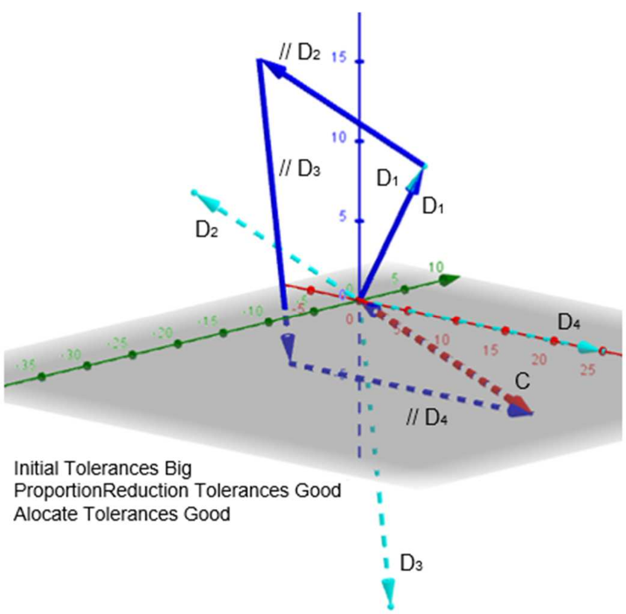


Fig. 2a. Spatial (3D) dimensional chain.

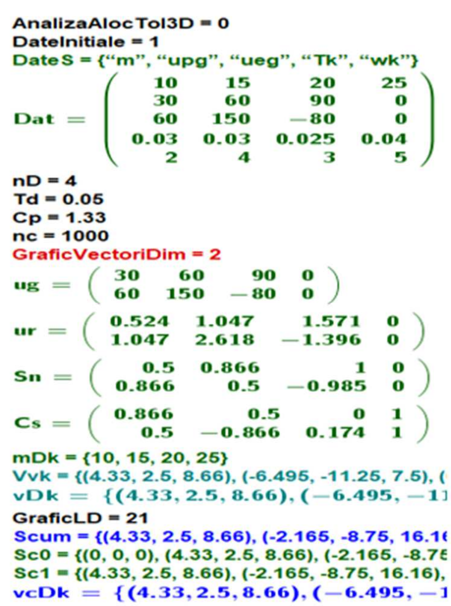


Fig. 2b. Initial data for spatial (3D) dimensional chain.

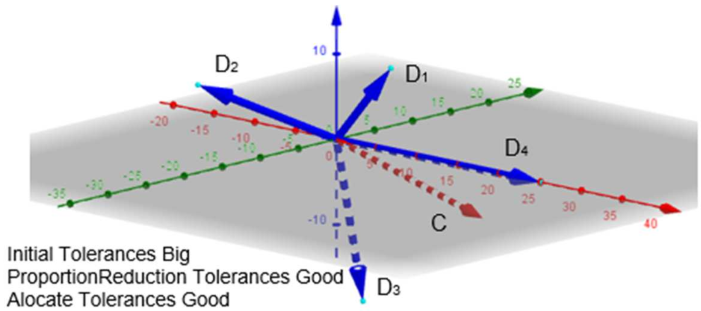


Fig. 3a. D_k and C vectors.

UnghiurilrDkSiC = 21
gk = {76.566°, 108.952°, 83.785°, 15.545°}
ct = {0.232, -0.325, 0.108, 0.963}

Fig. 3b. g_k angle values.

These angles facilitate the verification of the initial tolerance fields [12] and allow the calculation of the projections of the component dimensions D_k in the direction of the closing dimension vector C, and as a result the closing dimension of DC will be calculated more easily. To check the initial tolerances, nc random value of the component dimensions will be generated the initial tolerances for them T_k, using the uniform distribution, the most deficient in this case.

The GeoGebra software allows the creation of lists with these projections of the random component dimensions D_k on the direction of the closing dimension vector C. By summing these nc random value a list of nc value of the closing dimension IC_k will be obtained, for which some statistical value will be calculated, that is shown in figure 4.

AnalizaTolINITIALE = 3
ProiectiiDkPeDirC = 31
pmDkc = {2.323, -4.872, 2.165, 24.085}
Tic = {0.007, -0.01, 0.003, 0.039}
LIpk = {2.32, -4.867, 2.164, 24.066}
LSpk = {2.327, -4.877, 2.166, 24.105}

$$IpDkc = \begin{pmatrix} 2.322 & 2.321 & 2.324 & 2 \\ -4.875 & -4.868 & -4.875 & -4 \\ 2.164 & 2.165 & 2.166 & 2 \\ 24.071 & 24.078 & 24.077 & 24 \end{pmatrix}$$

ICk = {23.683, 23.696, 23.692, 23.724, 23.692, 23.6
StatisticaCkINITIAL = 32
Ckimin = 23.678
Ckimax = 23.728
Rci = 0.05
sci = 0.012
Dci = 0.07
MedCki = 23.702
VerificRebuturi = 33
LTi = 23.677
LTs = 23.727
VerRbIntInf = {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
VerRbIntSup = {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0
RbIntInf = 0
RbIntSup = 1
RebInt = 1
Cpi = 0.711
VerTolINITIALE = "Initial Tolerances BIG "

Fig. 4. Initial tolerances verification.

The average of the closing dimensions from random DCs is MedCk=23.702 mm, and coincides with the theoretical average of the closing dimension C (mC=23.702 mm)

The range in the initial case is R_{ci}=0.05 mm, equal to T_d=0.05 mm. The standard deviation for the original case is s_{ci}=0.012 mm, which indicates a possible variance of closing dimension $d_c=6*s_{ci}$=0.07 mm. This dispersion field is greater than the allowed tolerance T_d=0.05. It follows that in the case of the initial tolerances we will obtain parts out of tolerances. Moreover, the production capability parameter $C_{pi}=Td/(6*s_{ci})$=0.711, is less than 1, so it is inadequate.

The GeoGebra software has a built-in probability calculator. Introducing the value of the desired mean mC=23.702, the standard deviation obtained from previous statistical calculations s_{ci}=0.012 and the limits of the closing dimensions, taking into account the allowed tolerances T_d=0.05, LTi=mC-Td/2 =23.677 mm and LTs=mC+Td/2=23.727 mm, we get the probability to obtain parts conform to tolerances, that is represented in figure 5.

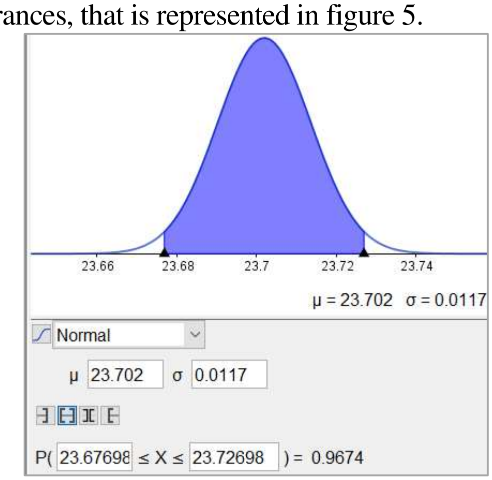


Fig. 5. Probability of obtaining parts within tolerance.

It can be seen that initially we have about 96.74% parts conform to tolerances. So, 3.26% is waste. Therefore, the initial tolerances are not appropriate. It is desired to change the tolerances to have 100% the percentage of parts within tolerances.

To reduce the tolerances, there are two possibilities, namely:

- Proportional reduction of initial tolerances;
- Allocation of new tolerances in accordance with the transfer coefficient of the component dimensions and the costs of obtaining the tolerances.

3. PROPORTIONAL REDUCTION OF (EXISTENT) INITIAL TOLERANCE

In the case of using the proportional reduction of the initial tolerances, the initial tolerances are proportionally adjusted (reducing or increasing them proportionally) in order to obtain the desired tolerances at the closing dimension of dimensional chain. A capability parameter will be ensured, preferably Cp ≥1.33.

A proportional reduction factor rr is calculated depending on the desired proportionality parameter Cp and the one obtained in the initial case:

$$rr=\frac{Cp_i}{Cp}=\frac{0{,}738}{1{,}33}=0{,}535 \quad (3)$$

With this correction factor rr, the initial tolerances are multiplied and the reduced tolerances Tr are obtained:

$$Tr = Tk * rr = \{0.016, 0.016, 0.01, 0.021\} \quad (4)$$

In the following, these tolerances will be verified by the Monte-Carlo method, similar to the case of the initial tolerances. The sequence in the GeoGebra software, which performs these calculations, is shown in figure 6.

The closing dimensions C_{kr} statistically calculated value is represented in figure 6. They are resulted from Monte-Carlo method applied in this case and the results show that they have a range $R_{cr} = 0.029 < T_d$ and a dispersion field of C_{kr}, $D_{cr}=6*s_{cr}= 0.038 < T_d$.

It also results the capability parameter C_{pr}=1.37, that satisfy. So proportionally reduced

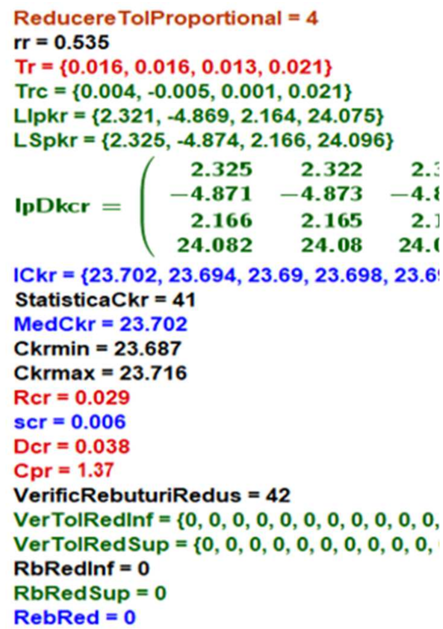


Fig. 6. The GeoGebra sequence – verify by MC method.

tolerance ranges satisfy our requirements. The probability calculator of the GeoGebra software, applied in the case of proportional reduction is shown in figure 7. It is observed that 99.99% of parts are conform to tolerances. So basically, all the parts.

4. TOLERANCES ALLOCATION

If it is desired to change the tolerances for the component dimensions of the dimensional chains, taking into account the transfer coefficients of the component dimensions and with some approximation, the costs necessary to

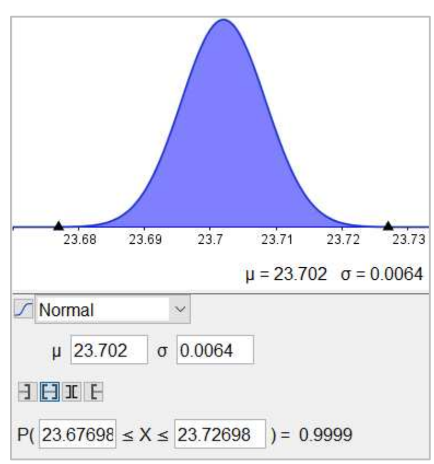


Fig. 7. Accepted parts – case of proportionally reduced tolerances.

achieve those tolerances, the software will allocate these new tolerances, which ensure the inclusion of the field dispersion of the closing dimension to fall within the allowed tolerances for the closing dimension (CD).

In some cases, it is desired to allocate other tolerances for the component dimensions D_k, which are consistent with the transfer coefficients of each dimension D_k and dependent on the costs of obtaining the tolerances.

The transfer coefficients are known from the previously performed calculations ct = {0.232, -0.325, 0.108, 0.963}, and the costs of the tolerances are taken into account by means of weighting coefficients w_k = {2, 4, 3, 5}. The weighting coefficient value is taken in the range (1=very low costs, and 5=very high costs).

Using the transfer coefficients ct and the weighting factors w_k, the assigned tolerances T_{ka} were calculated and this GeoGebra sequence is presented in figure 8.

The statistical calculations for the multiple values of the closing dimension C_{ka}, obtained by the Monte-Carlo method, reveal a range $R_{ca}=0.031 < T_d$, respectively a field of dispersion

AlocareaTolerantelor = 5
wk = {2, 4, 3, 5}
ngk0 = {76.566, 108.952, 83.785, 15.545
ngk1 = {76.566, 71.048, 83.785, 15.545}
ngk2 = {76.566, 71.048, 83.785, 15.545}
ngk3 = {76.566, 71.048, 83.785, 15.545}
ngk = {3.828, 3.552, 4.189, 1}
Sngk = 12.57
wPt = {7.657, 14.21, 12.568, 5}
SwPt = 39.434
rTd = 0.001
Tkac = {0.007, 0.014, 0.012, 0.005}
Tka = {0.031, 0.042, 0.111, 0.005}
LIpka = {2.32, -4.865, 2.159, 24.083}
LSpka = {2.327, -4.879, 2.171, 24.088}

IpDkca =	2.323	2.32	2.32
	−4.876	−4.875	−4.874
	2.164	2.161	2.171
	24.083	24.087	24.088

lCka = {23.694, 23.694, 23.704, 23.71, 23.696, 23.6
StatisticaCka = 51
Ckamin = 23.686
Ckamax = 23.718
Rca = 0.031
sca = 0.006
Dca = 0.035
Cpa = 1.43
VerifRebAlocare = 52
VerRbAlocInf = {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
VerRbAlocSup = {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0
RbAlocInf = 0
RbAlocSup = 0
RebAloc = 0
VerTolALOCATE = “Alocate Tolerances Good”

Fig. 8. The GeoGebra sequence – verified by MC method of allocated tolerances.

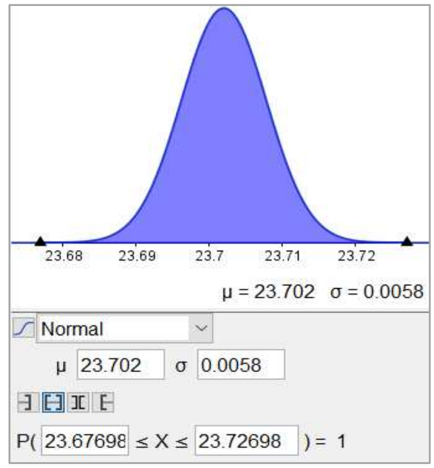


Fig. 9. Accepted parts – case of allocated tolerances.

of the closing dimensions C_{ka}, in the case of allocation $D_{ca}=0.035 < T_d$.

The production capacity parameter, (capability parameter) is $C_{pa}=1.43$, which is higher than recommended ($C_p=1.33$).

When applying the Probability Calculator, existing in the GeoGebra software, in this allocation case, the value get is shown in figure 9. The percentage of 100% parts conform to tolerances will be obtained when assigning tolerances. So, all the parts will be accepted.

When applying the Probability Calculator, existing in the GeoGebra software, in this allocation case, the value get is shown in figure 9. The percentage of 100% parts conform to tolerances will be obtained when assigning tolerances. So, all the parts will be accepted.

5. CASE OF PLANAR DIMENSIONAL CHAINS CALCULATIONS

The calculation program was designed for spatial (3D) dimensional chain (DC), but it can also be used for planar dimensional chain (Fig. 10.) or even linear dimensional chain (Fig. 11.).

For 2D dimensional chains, the elevation angle ue=0 will be introduced for all component dimensions, respectively line 3 of the D_{at} matrix (see Fig. 10.) will have all zero value.

For linear dimensional chain, the plane angle up=0 and the elevation angle ue=0 will be introduced for all component dimensions, respectively lines 2 and 3 of the D_{at} matrix (see Fig. 11.) will have all zero value.

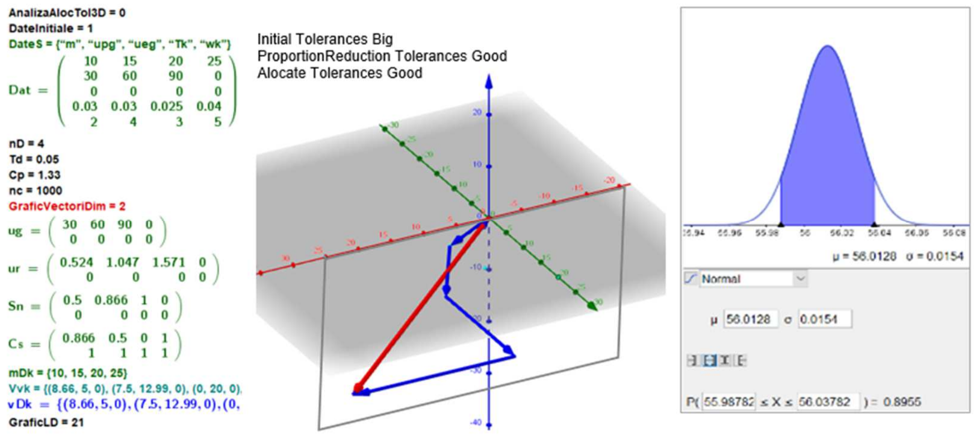


Fig. 10. The GeoGebra sequence – verify by MC method 2D dimensional chain.

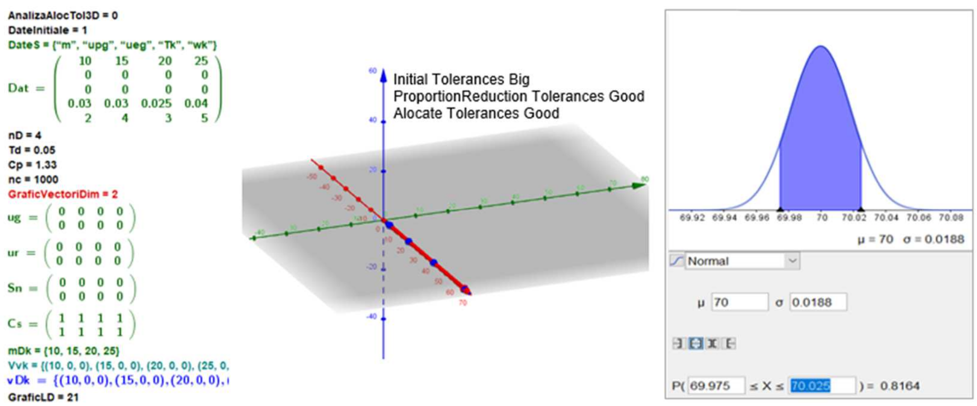


Fig. 11. The GeoGebra sequence – verify by MC method 1D dimensional chain.

6. CONCLUSION

The existence of high-performance software, such as GeoGebra, which can be obtained for free, allows the development of programs to calculate and verify the dimensional tolerances for dimensional chains dimensions.

Thus, 3D, 2D and 1D dimensional chains can be solved with the same calculation program.

This greatly facilitates the design work of engineers. They can quickly estimate the conditions for the manufacturing processes to generate parts conform to specifications, so in the end quality products can be obtained.

7. REFERENCES

[1] Grozav, I. *Workholding devices used in machine building technology*. Publishing House Politehnica, ISBN (10) 973-625-378-3, Timişoara, in Romanian, 2006.

[2] Chase, K.W. *Tolerance Allocation Methods for Designers*. in ADCATS Report No. 99-6, Brigham Young University, available at http://adcats.et.byu.edu/Publication/99-6/ToleranceAlloc.pdf , 1999.

[3] Barkallah, M., Louati, J., Haddar, M. *Evaluating of manufacturing tolerance using a statistical method and experimentation*. Int J Simul Model, 11(1), 5-16, 2012, https://doi.org/10.2507/IJSIMM11(1)1.194

[4] Chen, H., Jin, S., Li, Z., Lai, X. *A comprehensive study of three dimensional tolerance analysis methods*. Comput-Aided Des, 53, 1-13, 2014, https://doi.org/10.1016/j.cad.2014.02.014

[5] Xu, S., Keyser, J. *Statistical geometric computation on tolerances for dimensioning*. Comput-Aided Des, 70, 193-201, 2016, https://doi.org/10.1016/j.cad.2015.06.012

[6] Dantan, J.Y., Qureshi, A.J. *Worst-case and statistical tolerance analysis based on quantified constraint satisfaction problems and Monte Carlo simulation*. Comput-Aided Des, 41(1), 1-12, 2009, https://doi.org/10.1016/j.cad.2008.11.003

[7] Qureshi, A.J., Dantan, J.Y., Sabri, V., Beaucaire, P., Gayton, N. *A statistical tolerance analysis approach for over-constrained mechanism based on optimization and Monte Carlo simulation*. Comput-Aided Des, 44(2), 132-42, 2012, https://doi.org/10.1016/j.cad.2011.10.004

[8] Askri, R., Bois, C., Wargnier, H., Gayton, N. *Tolerance synthesis of fastened metal-composite joints based on probabilistic and worst-case approaches*. Comput-Aided Des, 100, 39-51, 2018, https://doi.org/10.1016/j.cad.2018.02.008

[9] Chase, K.W., Greenwood, W.H. *Design Issues in Mechanical Tolerance Analyses*, Winter Annual Meeting of the ASME, Boston, Massachusetts, 1987.

[10] Kenneth, W.C. *Tolerance Allocation Methods for Designer*, ADCATS Report No. 99-6, 1999.

[11] Abazar, S. *An Improved Differential Evolution Optimization Algorithm*. IJRRAS, 15(2), 132-145. 2013, https://www.arpapress.com/Volumes/Vol15Issue2/IJRRAS_15_2_01.pdf

[12] Chase, K.W., Greenwood, W.H., Loosli, B.G., Hauglund, L.F. *Least Cost Tolerance Allocation for Mechanical Assemblies with Automated Process Selection*, Manufacturing Review, ASME, 3(1), 49–59, 1990.

[13] Cheng, K.M., Tsai, J.C. *Optimal Statistical Tolerance Allocation of Assemblies for Minimum Manufacturing Cost*. Appl Mech Mater, 52-54, 1818-23, 2011, https://doi.org/10.4028/www.scientific.net/AMM.52-54.1818

[14] Prabhaharan, G., Asokan, P., Ramesh, P. Rajendran S. *Genetic-algorithm-based optimal tolerance allocation using a least-cost model.* Int J Adv Manuf Technol, 24, 647–60, 2004, https://doi.org/10.1007/s00170-003-1606-1

[15] https://help.solidworks.com/2020/English/SolidWorks/tolanalyst/c_TolAnalyst_Overview.htm, (accessed 10 October 2022).

[16] http://mdmetric.com/tech/DINISO1302extract.htm (accessed 15 May 2022).

[17] GeoGebra Classic, https://www.geogebra.org/classic (accessed 15 May 2022).

[18] www.geogebra.org (accessed on 15 May 2022).

[19] GeoGebra - Wikipedia, https://en.wikipedia.org/wiki/GeoGebra (accessed 16 May 2022).

ANALIZA ŞI ALOCAREA TOLERANŢELOR DIMENSIONALE FOLOSIND PROGRAMUL DE CALCUL GEOGEBRA

O problemă curentă în ingineria mecanică este aceea a indicării a unor toleranţe pentru dimensiunile unui lanţ de dimensiuni cât şi pentru dimensiunea sa de închidere astfel încât fabricarea pieselor la dimensiunile indicate să se realizeze cu costuri minime. Lucrarea prezintă un program de calcul al toleranțelor pentru lanțurile de dimensiuni 3D, 2D şi 1D. Acest program verifică toleranțele inițial acordate, iar în cazul în care acestea sunt nesatisfăcătoare vor fi modificate toleranţele iniţiate utilizând metoda reducerii proporţionale sau se vor aloca alte valori toleranțelor folosind coeficienți de transfer şi factori de ponderare. Verificarea toleranțelor se realizează folosind metoda Monte-Carlo.

TULCAN Aurel, PhD, Associate Professor, Politehnica University of Timişoara, Materials and Manufacturing Engineering Department, Romania, aurel.tulcan@upt.ro

BANCIU Felicia Veronica, PhD, Lecturer, Politehnica University of Timişoara, Materials and Manufacturing Engineering Department, Romania, felicia.banciu@upt.ro

GROZAV Ion, PhD, Associate Professor, Politehnica University of Timişoara, Materials and Manufacturing Engineering Department, Romania, ion.grozav@upt.ro