TECHNICAL UNIVERSITY OF CLUJ-NAPOCA

ACTA TECHNICA NAPOCENSIS

Series: Applied Mathematics, Mechanics, and Engineering
Vol. 66, Issue I, March, 2023

NEW ASPECTS CONCERNING VIBRATION SUPPRESSION IN MULTI DEGREE OF FREEDOM MECHANICAL SYSTEMS

Maria Luminita SCUTARU, Mircea MIHĂLCICĂ, Marin MARIN, Omar Abdulah SHRRAT OMAR

Abstract: *Mechanical systems with several degrees of freedom present vibration properties that allow such a system to be used as an absorber for certain exciting frequencies. The paper aims to expand an analysis previously made by the authors, regarding the dynamic absorption capacity of some mechanical systems in a more general case, which can be encountered in engineering practice. An example shows how to apply these properties in the calculation and the resulting advantages.*
Key words: *Vibration Suppression; Multi Degree of Freedom; mechanical system; forced vibration; eigenpulsations, eigenmodes*

1. INTRODUCTION

At the time of their appearance, dynamic absorbers represented an ingenious and inexpensive method of vibration reduction, which proved to have many uses in various fields of engineering. The first patent was obtained in 1909 [1] and a theoretical explanation of their operation was made in 1928 [2]. Over the years, of course, many improvements have been made and the field of use has always been expanded. The classic dynamic absorber is a passive element and currently dynamic absorbers with an active role have been developed. The numerous opportunities for use and the importance of this vibration reduction system made the continuous study of this device necessary.

Obviously, in practical applications it may be necessary to suppress more than one harmful frequency. For this, a device was imagined that can be tuned to attenuate three resonance frequencies simultaneously [3]. Obviously, to solve such a problem, three different absorbers can be attached to the system. But this approach leads to additional costs and to the creation of a more complicated assembly. For this reason, a single device that can achieve the attenuation of three different frequencies is preferable. A numerical experiment validates the proposed solution. A complex system in which a dynamic vibration absorber (DVA) is proposed and analyzed is presented in [4]. The absorber has a non-linear behavior and variable damping. Such an absorber proves to be useful in the case of systems that may have different behaviors in operation, depending on the external conditions in which they operate. Seismic protection is an extremely researched subject in which numerous solutions have been proposed to mitigate the shocks induced by earthquakes. A complex solution for solving this problem is proposed in [5] and consists of a DVA built in the form of an elastic continuum. The material used has several natural frequencies that are found in the frequency range of seismic effects. A pendulum damper for solving such problems is studied in [6].

The main use of DVA is found in vibration damping where we have a harmful harmonic excitation that is eliminated. An unconventional DVA system that uses magnetic force to eliminate unwanted vibration is proposed in [7]. Using a known expression for the magnetic force, the classical analysis theory of such a system provides the optimal parameters for the

Received: 30.01.23; **Similarities**: 09.02.23: **Reviewed**: 05.03./07.02.23: **Accepted**:21.03.23.

system to ensure the desired absorption. Experimental results have validated the model and the proposed solution.

Another problem involving the use of DVAs is represented by devices operating at supercritical speeds. These, in order to reach the operating speed, must pass continuously, through all the speeds, including the critical ones, after the moment of starting. Passing through resonance zones leads to unwanted additional demands on the devices and possible malfunctions/damages, if the amplitudes, when passing through the resonance zone, become too high. To solve this problem, DVAs are used, which will act when passing through the resonance zone. Such a situation is described and studied in [8]. The finite element method is used to model the system. It is thus possible to reach a supercritical speed, through a smooth acceleration, without observing a significant increase in the amplitude in the resonance area.

Innovative methods such as the use of magneto-rheological elastomer can be used to achieve vibration absorption [9,10]. Active or semi-active systems have started to be important in recent years, as a result of the development of technology. Such a solution in two versions (with magneto-rheological damper and a combination of a magneto-rheological damper and a classic absorber) for a semi-active suspension is presented in [11]. Genetic algorithms are used to determine the optimal passive parameters. A series of results regarding the analysis of different aspects of the design and realization of DVAs are presented in [12-17]. An application regarding the reduction of vibrations in the case of drilling processes is presented in [18]. The construction solution for DVA is a thin steel tube filled with natural rubber. The parameter adjustment procedure is carried out by varying the number of inserted rubber sleeves. A system that provides damping adjustment depending on road excitation for a car is presented in [19]. In this way, an increase in comfort and stability of maneuverability is achieved while driving. Other methods of calculating the dynamic absorber are studied and analyzed in [20-23].

Mechanical systems with elastic elements from engineering applications and where vibration absorption is necessary work in most cases in stationary regimes, in which energy is introduced into the system. This requires the study of the forced vibrations of the analyzed systems.

In aerospace engineering applications, thin-walled elements are widely used, which, due to their low rigidity, can lead to forced vibrations. In order to solve this problem, a vibration suppression method for this type of vibrations is proposed in [24]. The proposed solution is verified experimentally.

The phenomenon of anti-resonance leads to the stabilization and even cancellation of self-excited vibrations. In [25], another possibility of applying the anti-resonance phenomenon is presented, namely the damping of externally excited resonant vibrations. Amplitude suppression due to parametric antiresonance is characterized by several parameters that define the system: a depth of parametric excitation, mass ratio, damping coefficient and small frequency deviations from the parametric anti-resonance.

In the operation of articulated aerial robots, the elastic vibration of the chain-shaped structure appears, which has a negative influence on stable flight. For the control of such a structure, a control method with an innovative design is presented in [26]. The feasibility of the proposed method is demonstrated through experiments on a multilink physical model.

In [27], dynamic modeling, finite-time trajectory tracking control, and vibration suppression of a flexible two-link space robot are studied. A dynamic model of a flexible space robot based on Lagrange's method is used to suppress the vibrations of this system. Thus, a new vibration suppression method is offered which is used in a simulation using a hybrid control scheme. In a study [28] that addresses the dynamic modeling and suppression of the disturbing force of some rotating flywheels, the finite element method is used for the analysis. An annular dynamic absorber is proposed to suppress the radial and axial excitations at different rotation speeds of the shaft. It is shown that the proposed absorber is able to suppress certain high frequencies that may appear in the system. In [29,30] are presented some practical solution of absorbers used in the engineering applications.

In [31] it is analysed if a judicious dimensioning of a system offers it the ability to function as a dynamic absorber for some frequency. So it is possible to reduce the vibration for certain frequency, without using a DVA. If it is possible that, the design and construction of the elastic system is much simplified, not needing a supplementary absorber. The design and manufacturing costs decrease. Following the analysis made in [31], it was concluded that it is possible for a system to act as an absorber, through the topology, masses and stiffnesses of the elements that make up the system, without the need to add additional elements for vibration absorption. Compared to the research previously presented, the problem develops the case when one part of the system is excited and another part is not. It is studied, in this situation, if there is absorption of vibration for an element of the system and what are the conditions under which this exists.

2. MATERIALS AND METHODS

In the following we will formulate the problem we want to study. Let's consider a mechanical system, made up of masses linked together by elastic elements. For ease of presentation, let's consider a system without damping, excited by a system of forces with harmonic variation of pulsation p.The equations of motion for this system are given by the system of differential equations [32-38]:

$$[M]\{\ddot{X}\} + [K]\{X\} = \{F\}\, sin\, p\, t \tag{1}$$

where $[M]$ represents the mass matrix, $[K]$ represents the stifness matrix and $\{F\}$ the excitation forces vector.

We divided the masses considered in the system into three categories. The first represents concentrated masses, in number s, which are not required by any force. The second category is represented by concentrated masses, in number r, which are subject to external excitations. And the last category is represented by a mass that is excited with force F_3 but will absorb the vibration of the two sets of forces $\{F_2\}$ and $\{F_3\}$ and will remain at rest. In this case, relation (1) can be written:

$$\begin{bmatrix} [M_{11}] & [M_{12}] & [M_{13}] \\ [M_{21}] & [M_{22}] & [M_{23}] \\ [M_{31}] & [M_{32}] & [M_{33}] \end{bmatrix} \begin{Bmatrix} \{\ddot{X}_1\} \\ \{\ddot{X}_2\} \\ \{\ddot{X}_3\} \end{Bmatrix} + \\ + \begin{bmatrix} [K_{11}] & [K_{12}] & [K_{13}] \\ [K_{21}] & [K_{22}] & [K_{23}] \\ [K_{31}] & [K_{32}] & [K_{33}] \end{bmatrix} \begin{Bmatrix} \{X_1\} \\ \{X_2\} \\ \{X_3\} \end{Bmatrix} = \begin{Bmatrix} 0 \\ \{F_2\} \\ \{F_3\} \end{Bmatrix} sin\, p\, t \tag{2}$$

In the relation above, the vector $\{X_1\}=[x_1\ x_2\ \mathrm{K}\ \Lambda\ x_s]^T$ represents the displacements of unexcited masses, $\{X_2\}=[x_{s+1}\ x_{s+2}\ \mathrm{K}\ \Lambda\ x_{s+r}]^T$ represent the displacements of the excited masses and $\{X_3\}=[x_{s+r+1}]$ the mass that will remain at rest despite the excitations that act on it and on the other r excited masses. The matrices $[M_{11}]$, $[K_{11}]$ have dimension ($s \times s$), matrices matricele $[M_{12}]$, $[K_{12}]$ have dimension ($s \times r$), the matrices $[M_{21}]$, $[K_{21}]$ have the dimension (r × s), $[M_{13}]$, $[K_{13}]$ have the dimension ($s \times 1$), $[M_{31}]$, $[K_{31}]$ dimension ($1 \times s$), $[M_{22}]$, $[K_{22}]$ the dimension r × r, $[M_{23}]$, $[K_{23}]$ dimension $r \times 1$, $[M_{32}]$, $[K_{32}]$ dimension ($1 \times r$) and $[M_{33}]$, $[K_{33}]$ dimension (1×1). Using the clasical metohd to solve this system we chose the solution under the form:

$$\begin{Bmatrix} \{X_1\} \\ \{X_2\} \\ \{X_3\} \end{Bmatrix} = \begin{Bmatrix} \{A_1\} \\ \{A_2\} \\ \{A_3\} \end{Bmatrix} \sin pt \quad ; \tag{3}$$

One obtains:

$$\begin{Bmatrix} \{\ddot{X}_1\} \\ \{\ddot{X}_2\} \\ \{\ddot{X}_3\} \end{Bmatrix} = -p^2 \begin{Bmatrix} \{A_1\} \\ \{A_2\} \\ \{A_3\} \end{Bmatrix} sin\, p\, t \tag{4}$$

Putting the condition that the solution (3) verifies Equation (2), it obtains:

$$\begin{bmatrix} [K_{11}]-p^2[M_{11}] & [K_{12}]-p^2[M_{12}] & [K_{13}]-p^2[M_{13}] \\ [K_{21}]-p^2[M_{21}] & [K_{22}]-p^2[M_{22}] & [K_{23}]-p^2[M_{23}] \\ [K_{31}]-p^2[M_{31}] & [K_{32}]-p^2[M_{32}] & [K_{33}]-p^2[M_{33}] \end{bmatrix} \begin{Bmatrix} \{A_1\} \\ \{A_2\} \\ \{A_3\} \end{Bmatrix} = \begin{Bmatrix} \{0\} \\ \{F_2\} \\ \{F_3\} \end{Bmatrix} \tag{5}$$

In the conditions in which $\{F_1\}=0$ (there is no excitation on the first part of the system) it is impose that $\{A_3\}=0$. Let's look for the conditions for this to happen. By entering this conditions in Equation (5), we get:

$$([K_{11}]-p^2[M_{11}])\{A_1\}+([K_{12}]-p^2[M_{12}])\{A_2\}=\{0\}, \quad (6)$$

$$([K_{21}]-p^2[M_{21}])\{A_1\}+([K_{22}]-p^2[M_{22}])\{A_2\}=\{F_2\}, \quad (7)$$

$$([K_{31}]-p^2[M_{31}])\{A_1\}+([K_{32}]-p^2[M_{32}])\{A_2\}=\{F_3\}, \quad (8)$$

For the linear system (6),(7) and (8) to be compatible and determined it is necessar to have the condition:

$$\det\left(\begin{bmatrix}[K_{11}]-p^2[M_{11}] & [K_{12}]-p^2[M_{12}] & 0\\ [K_{21}]-p^2[M_{21}] & [K_{22}]-p^2[M_{22}] & \{F_2\}\\ [K_{31}]-p^2[M_{31}] & [K_{32}]-p^2[M_{32}] & \{F_3\}\end{bmatrix}\right)=0 \quad (9)$$

Equations (9) offer us the conditions in which the amplitude A_3 can become equal with zero. If all the geometric, masic and stifness are defined, the Eq.(9) offers us the pulsations p for which the wheel 7 rest.

In the engineering applications encountered in practice, there is obviously depreciation. In any mechanical system there are frictions and different processes through which the energy of the system is dissipated in the external environment. The most common damping in such systems is viscous damping. Of course, the calculation can also be done for these cases, but the number of operations increases significantly. If we analyze the results, the previous considerations made for the case where there is no depreciation remain valid from a qualitative point of view, of the system's behavior. What appears in addition in the case of viscous damping is that the phases of the oscillations of the different masses differ from each other. If we also introduce friction, the number of parameters that define the system increases, but this can represent an advantage for designers, the number of parameters that can be manipulated increases and thus the possibilities of tuning the system to the frequencies we want to suppress increase.

3. RESULTES

An example is presented in the following. Consider a system having 7 flywheels linked together by elastic shafts.

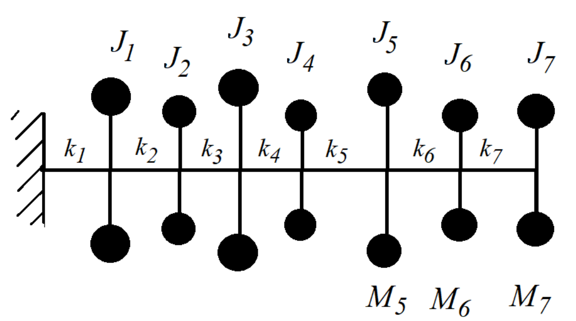

Fig.1. The system of wheels

$$\begin{aligned}
&J_1\ddot{\phi}_1 + k_1\phi_1 - k_2(\phi_2-\phi_1) = 0 \quad ;\\
&J_2\ddot{\phi}_2 + k_2(\phi_2-\phi_1) - k_3(\phi_3-\phi_2) = 0 \quad ;\\
&J_3\ddot{\phi}_3 + k_3(\phi_3-\phi_2) - k_4(\phi_4-\phi_3) = 0 \quad ;\\
&J_4\ddot{\phi}_4 + k_4(\phi_4-\phi_3) - k_5(\phi_5-\phi_4) = 0 \quad ;\\
&J_5\ddot{\phi}_5 + k_5(\phi_5-\phi_4) - k_6(\phi_6-\phi_5) = M_5 \sin p\,t \quad ;\\
&J_6\ddot{\phi}_6 + k_6(\phi_6-\phi_5) - k_7(\phi_7-\phi_6) = M_6 \sin p\,t \quad ;\\
&J_7\ddot{\phi}_7 + k_7(\phi_7-\phi_6) = M_7 \sin p\,t \ .
\end{aligned} \quad (10)$$

$$\begin{bmatrix} J_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & J_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & J_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & J_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & J_5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & J_6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & J_7 \end{bmatrix} \begin{Bmatrix} \ddot{\phi}_1 \\ \ddot{\phi}_2 \\ \ddot{\phi}_3 \\ \ddot{\phi}_4 \\ \ddot{\phi}_5 \\ \ddot{\phi}_6 \\ \ddot{\phi}_7 \end{Bmatrix} +$$

$$+ \begin{bmatrix} k_1+k_2 & -k_2 & 0 & 0 & 0 & 0 & 0 \\ -k_2 & k_2+k_3 & -k_3 & 0 & 0 & 0 & 0 \\ 0 & -k_3 & k_3+k_4 & -k_4 & 0 & 0 & 0 \\ 0 & 0 & -k_4 & k_4+k_5 & -k_5 & 0 & 0 \\ 0 & 0 & 0 & -k_5 & k_5+k_6 & -k_6 & 0 \\ 0 & 0 & 0 & 0 & -k_6 & k_6+k_7 & -k_7 \\ 0 & 0 & 0 & 0 & 0 & -k_7 & k_7 \end{bmatrix} \begin{Bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \\ \phi_5 \\ \phi_6 \\ \phi_7 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ M_5 \\ M_6 \\ M_7 \end{Bmatrix} sin\, p\, t \qquad (11)$$

$$\begin{bmatrix} [J_{11}] & [J_{12}] & [J_{13}] \\ [J_{21}] & [J_{22}] & [J_{23}] \\ [J_{31}] & [J_{32}] & [J_{33}] \end{bmatrix} \begin{Bmatrix} \{\ddot{\Phi}_1\} \\ \{\ddot{\Phi}_2\} \\ \{\ddot{\Phi}_3\} \end{Bmatrix} +$$

$$+ \begin{bmatrix} [K_{11}] & [K_{12}] & [K_{13}] \\ [K_{21}] & [K_{22}] & [K_{23}] \\ [K_{31}] & [K_{32}] & [[K_{33}]] \end{bmatrix} \begin{Bmatrix} \{\Phi_1\} \\ \{\Phi_2\} \\ \{\Phi_3\} \end{Bmatrix} = \begin{Bmatrix} 0 \\ \{Q_2\} \\ \{Q_3\} \end{Bmatrix} sin\, p\, t \qquad (12)$$

where:

$$[J_{11}] = \begin{bmatrix} J_1 & 0 & 0 & 0 \\ 0 & J_2 & 0 & 0 \\ 0 & 0 & J_3 & 0 \\ 0 & 0 & 0 & J_4 \end{bmatrix} \quad [J_{22}] = \begin{bmatrix} J_5 & 0 \\ 0 & J_6 \end{bmatrix}$$

$$[J_{33}] = [J_7] \;;\; [J_{12}] = [O_{4\times 2}] \;;\; [J_{21}] = [J_{12}]^T \;;$$

$$[J_{13}] = [O_{4\times 1}] \;;\; [J_{31}] = [J_{13}]^T \;;\; [J_{23}] = [O_{2\times 1}] \;;$$

$$[J_{32}] = [J_{23}]^T \qquad (13)$$

$$[K_{11}] = \begin{bmatrix} k_1+k_2 & -k_2 & 0 & 0 \\ -k_2 & k_2+k_3 & -k_3 & 0 \\ 0 & -k_3 & k_3+k_4 & -k_4 \\ 0 & 0 & -k_4 & k_4+k_5 \end{bmatrix}$$

$$[K_{22}] = \begin{bmatrix} k_5+k_6 & -k_6 \\ -k_6 & k_6+k_7 \end{bmatrix} ; [K_{33}] = [k_7] \;;$$

$$[K_{12}] = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ -k_5 & 0 \end{bmatrix} ;$$

$$[K_{21}] = [K_{12}]^T \;;\; [K_{13}] = [O_{4\times 1}] \;;\; [K_{31}] = [K_{13}]^T \;;$$

$$[K_{23}] = \begin{bmatrix} 0 \\ -k_7 \end{bmatrix} \;;\; [K_{32}] = [K_{23}]^T \qquad (14)$$

$$\{\Phi_1\} = \begin{Bmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \\ \varphi_4 \end{Bmatrix} \;;\; \{\Phi_2\} = \begin{Bmatrix} \varphi_5 \\ \varphi_6 \end{Bmatrix} \;;\; \{\Phi_3\} = \{\varphi_7\} . \qquad (15)$$

The solution for the system (12) must be under the form:

$$\begin{Bmatrix} \{\Phi_1\} \\ \{\Phi_2\} \\ \{\Phi_3\} \end{Bmatrix} = \begin{Bmatrix} \{A_1\} \\ \{A_2\} \\ \{A_3\} \end{Bmatrix} \sin pt \;;$$

It results:

[illegible]

Introducing in (12) it obtains:

$$\begin{bmatrix} [K_{11}]-p^2[J_{11}] & [K_{12}]-p^2[J_{12}] & [K_{13}]-p^2[J_{13}] \\ [K_{21}]-p^2[J_{21}] & [K_{22}]-p^2[J_{22}] & [K_{23}]-p^2[J_{23}] \\ [K_{31}]-p^2[J_{31}] & [K_{32}]-p^2[J_{32}] & [K_{33}]-p^2[J_{33}] \end{bmatrix} \begin{Bmatrix} \{A_1\} \\ \{A_2\} \\ \{A_3\} \end{Bmatrix} = \begin{Bmatrix} 0 \\ \{Q_2\} \\ \{Q_3\} \end{Bmatrix} \qquad (16)$$

We want: $A_3 = 0$. The system (13) becomes:

$$\begin{aligned} &([K_{11}] - p^2[J_{11}])\{A_1\} + ([K_{12}] - p^2[J_{12}])\{A_2\} = \{0\} \\ &([K_{21}] - p^2[J_{21}])\{A_1\} + ([K_{22}] - p^2[J_{22}])\{A_2\} = \{Q_2\} \\ &([K_{31}] - p^2[J_{31}])\{A_1\} + ([K_{32}] - p^2[J_{32}])\{A_2\} = \{Q_3\} \end{aligned} \qquad (17)$$

The condition to have a determined solution of this system is:

$$\begin{vmatrix} [K_{11}]-p^2[J_{11}] & [K_{12}]-p^2[J_{12}] & \{0\} \\ [K_{21}]-p^2[J_{21}] & [K_{22}]-p^2[J_{22}] & \{Q_2\} \\ [K_{31}]-p^2[J_{31}] & [K_{32}]-p^2[J_{32}] & \{Q_3\} \end{vmatrix} = 0 \qquad (18)$$

Solving this polynomial equation, is possible to obtain the values *p* for which the amplitude $A_3 = 0$.

We can represent this determinant and we can obtain a first approximation for the values p for which the determinant is zero.

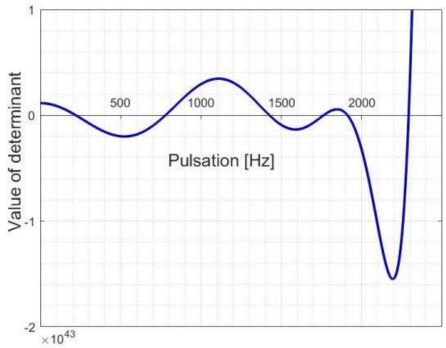

Fig.2. Value of determinant. The representation of the value of determinant for different pulsations p

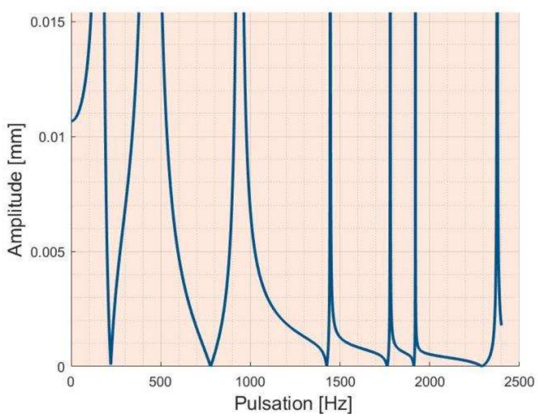

Fig.3. The amplitude of the vibration of wheel 7. There are six frequencies that can be suppressed.

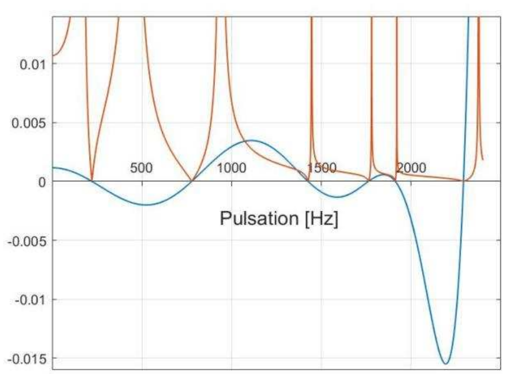

Fig.4. The superposition of the values of determinant and the values of amplitude of wheel 7

If we superpose the graph of the determinant with the points where it cancels with the graph of the amplitudes, it is immediately noticed that the amplitudes are zero exactly at the points where the determinant is equal to zero, as it should happen (see Figure 4).

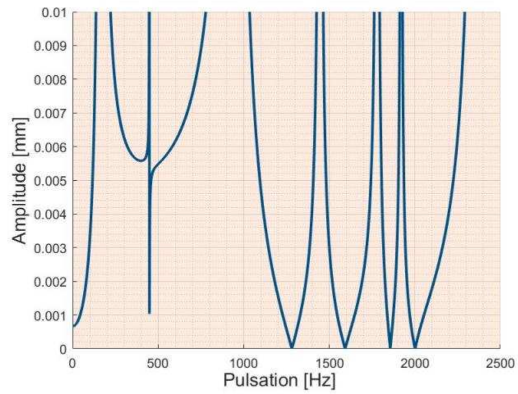

Fig.5. The amplitude of the vibration of wheel 6.

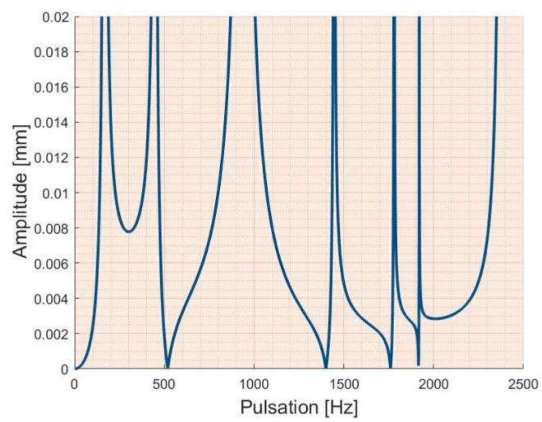

Fig.6. The amplitude of the vibration of wheel 5. There are four frequencies that can be suppressed.

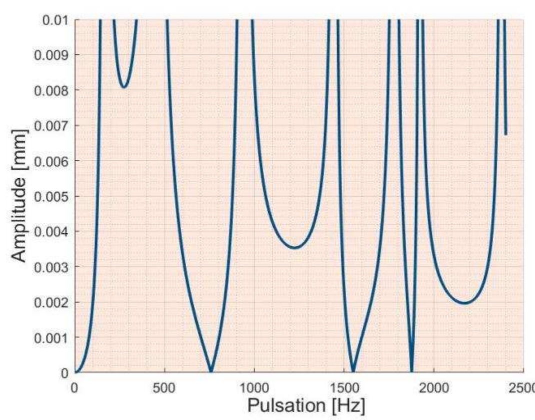

Fig.7. The amplitude of the vibration of wheel 4. There are three frequencies that can be suppressed.

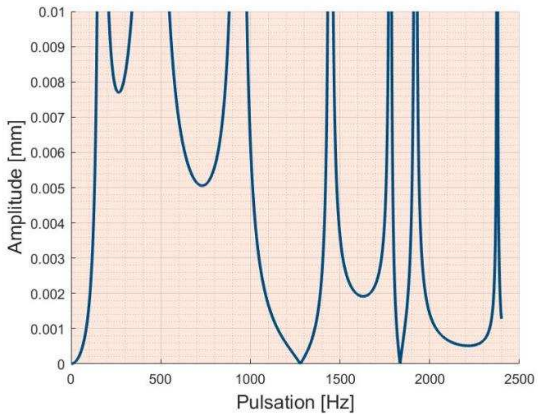

Fig.8. The amplitude of the vibration of wheel 3. There are two frequencies that can be suppressed.

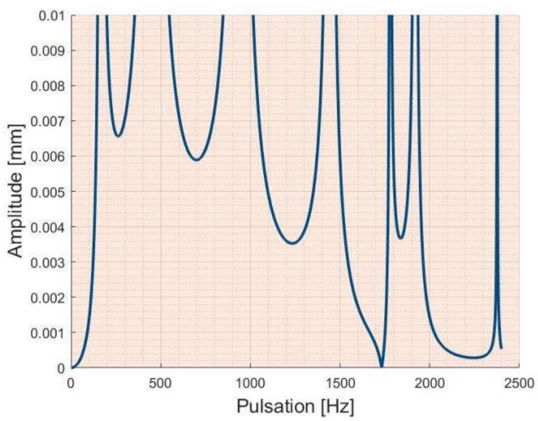

Fig.9. The amplitude of the vibration of wheel 2. There is one frequency that can be suppressed

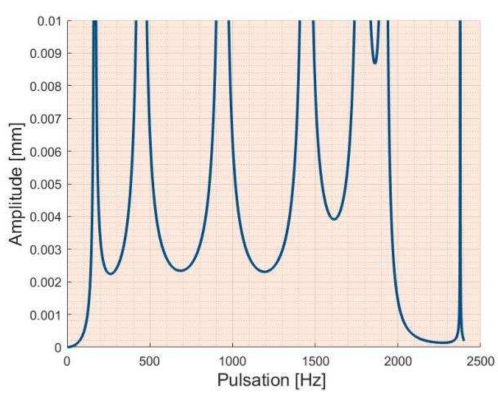

Fig.10. The amplitude of the vibration of wheel 1. There is no suppression of the vibration for this element

4. DISCUSSION AND CONCLUSIONS

The important result that we demonstrated in the paper is that a mechanical system with several degrees of freedom can ensure the suppression of the vibrations of one of the elements, for certain frequencies. Moreover, for a system with 7 degrees of freedom, as we considered in the paper, for the last element we can obtain 6 different frequency values for which the complete suppression of vibrations can be ensured, even if, as in our example, there are three elements which supports a harmonic excitation. Also, for the other elements, vibration suppression can be ensured for certain frequencies.

It follows from this that through a careful design of a mechanical structure with elastic elements, it is possible to ensure the suppression of vibrations for one of the elements of the structure, for certain frequencies. Obviously, the requirement of a certain frequency through the design theme impose a careful study and he determination of the stiffness and inertias of the system elements so as to ensure the suppression of unwanted vibrations.

5. REFERENCES

[1]Frahm, H. Device for Damping Vibrations of Bodies. U.S. Patent US989958A, 30 October 1909.

[2]Ormondroyd, J.; Den Hartog, J.P. Theory of the dynamic vibration absorber. *Trans. ASME* **1928**, *50*, 9–22.

[3]Yoon, G.H.; Choi, H.; So, H.Y. Development and optimization of a resonance-based mechanical dynamic absorber structure for multiple frequencies. *J. Low Freq. Noise Vib. Act. Control* **2021**, *40*, 880–897.

[4]Wang, T.; Tian, R.L.; Yang, X.W.; Zhang, Z.W. A Novel Dynamic Absorber with Variable Frequency and Damping. *Shock Vib.* **2021**, *2021*, 8833089.

[5]Makarov, S.B.; Pankova, N.V. On the Possibility of Applying a Multi-frequency Dynamic Absorber (MDA) to Seismic Protection Tasks. *Adv. Intell. Syst. Comput.* **2020**, *1127*, 395–403.

[6]Di Egidio, A.; Alaggio, R.; Aloisio, A.; de Leo, A.M.; Contento, A.; Tursini, M.

Analytical and experimental investigation into the effectiveness of a pendulum dynamic absorber to protect rigid blocks from overturning. *Int. J. Non-Linear Mech.* **2019**, *115*, 1–10.
[7]Heidari, H.; Monjezi, B. Vibration control of imbalanced Jeffcott rotor by virtual passive dynamic absorber with optimal parameter values. *Proc. Inst. Mech. Eng. Part C J. Mech. Eng. Sci.* **2018**, *232*, 4278–4288.
[8]Fontes, Y.C.; Nicoletti, R. Rotating dynamic absorber with viscoelastic element. *J. Braz. Soc. Mech. Sci. Eng.* **2016**, *38*, 377–383.
[9]Komatsuzaki, T.; Inoue, T.; Terashima, O. A broadband frequency-tunable dynamic absorber for the vibration control of structures. In Proceedings of the 13th International Conference on Motion and Vibration Control (MOVIC), Southampton, UK, 3–6 July 2016; Volume 744, p. 012167.
[10]Komatsuzaki, T.; Inoue, T.; Iwata, Y. MRE-based adaptive-tuned dynamic absorber with self-sensing function for vibration control of structures. In Proceedings of the 7th Annual ASME Conference on Smart Materials, Adaptive Structures and Intelligent Systems (SMASIS), Newport, RI, USA, 8–10 September 2014; Volume 1, p. V001T03A011.
[11]Orecny, M.; Segl'a, S.; Hunady, R.; Ferkova, Z. Application of a magneto-rheological damper and a dynamic absorber for a suspension of a working machine seat. *Procedia Eng.* **2014**, *96*, 338–344.
[12]Nicoara, D.D. The Damped Dynamic Vibration Absorber—A Numerical Optimization Method. In Proceedings of the International Conference COMAT 2018, Brasov, Romania, 25–26 October 2018.
[13]Pennestri, E. An application of Chebyshev's min-max criterion to the optimum design of a damped dynamic vibration absorber. *J. Sound Vib.* **1998**, *217*, 757–765.
[14]Song, J.; Si, P.; Hua, H.; Li, Z. A DVA-Beam Element for Dynamic Simulation of DVA-Beam System: Modeling, Validation and Application. *Symmetry* **2022**, *14*, 1608.
[15]Byrnes, P.W.G.; Lacy, G. Modal vibration testing of the DVA-1 radio telescope. In *Ground-Based and Airborne Telescopes VI, Proceedings of the SPIE Astronomical Telescopes + Instrumentation, Edinburgh, UK, 26 June–1 July 2016*; Book Series; SPIE: Paris, France, 2016; Volume 9906, Part 1, p. 99063P.
[16]Sharma, S.K.; Sharma, R.C.; Lee, J.; Jang, H.L. Numerical and Experimental Analysis of DVA on the Flexible-Rigid Rail Veicle Carbody Resonant Vibration. *Sensors* **2022**, *22*, 1922.
[17]Dong, G.; Xiaojie, C.; Jing, L.; Peiben, W.; Zhengwei, Y.; Xingjian, J. Theoretical modeling and optimal matching on the damping property of mechatronic shock absorber with low speed and heavy load capacity. *J. Sound Vib.* **2022**, *535*, 117113.
[18]Hendrowati, W.; Merdekawan, N. Modeling and analysis of boring bar vibration response in internal turning due to variation of the amount of DVA rubber in finish boring cut. *J. Mech. Sci. Technol.* **2021**, *35*, 4353–4362.
[19]Gu, C.; Zhu, J.; Chen, X. A Novel E-DVA Module Synthesis Featuring of Synergy between Driving and Vibration Attenuation. *Shock Vib.* **2016**, *2016*, 8464317.
[20]Abouelregal, A.E.; Marin, M. Thesize-dependent thermoelastic vibrations of nanobeams subjected to harmonic excitation and rectified sine wave heating. *Mathematics* **2020**, *8*, 1128.
[21]Mocanu, S.; Rece, L.; Burlacu, A.; Florescu, V.; Rontescu, C.; Modrea, A. Novel Procedures for Sustainable Design in Structural Rehabilitation on Oversized Metal Structures. *Metals* **2022**, *12*, 1107.
[22]Diveyev, B.; Dorosh, I.; Cherchyk, H.; Burtak, V.; Ostashuk, M.; Hlobchak,

M.; Kotiv, M. DVA for the High-Rise Object. In Proceedings of the 16th Intenational Conference on Experience of Designing and Application of CAD Systems in Microelectronics—CADSM, Lviv, Ukraine, 22–26 February 2021.

[23]Vlase, S.; Marin, M.; Scutaru, M.L.; Munteanu, R. New analytical method based on dynamic response of planar mechanical elastic systems. *AIP Adv.* **2017**, *7*, 1–15.

[24]Yuan, X.; Wang, S.T.; Mao, X.Y.; Liu, H.Q.; Liang, Z.S.; Guo, Q.S.; Yan, R. Forced vibration mechanism and suppression method for thin-walled workpiece milling. INTERNATIONAL JOURNAL OF MECHANICAL SCIENCES, **2022**, Vol. 230, Article Number 107553, DOI 10.1016/j.ijmecsci.2022.107553.

[25]Pesek, L.; Sulc, P.; Pust, L. Numerical Study of Forced Vibration Suppression by Parametric Anti-Resonance. ARCHIVES OF ACOUSTICS, **2016,** Vol. 41, Issue 3, Page 527-533, DOI 10.1515/aoa-2016-0051.

[26]Maki, T.; Zhao, M.J.; Okada, K.; Inaba, M. Elastic Vibration Suppression Contro l for Multilinked Aerial Robot Using Redundant Degrees-of-Freedom of Thrust Force. IEEE ROBOTICS AND AUTOMATION LETTERS, **2022,** Vol.7, Issue 2, Page 2859-2866, DOI 10.1109/LRA.2022.3145060

[27]Lei, R.H.; Chen, L. Finite-time tracking control and vibration suppression based on the concept of virtual control force for flexible two-link space robot. Defence Technology **2021,** Vol.17, Issue 3, Page 874-883, DOI 10.1016/j.dt.2020.04.013.

[28]Huang, X.C.; Su, Z.W.; Wang, S.; Wei, X.S.; Wang, Y.; Hua, H.X. High-frequency disturbance force suppression mechanism of a flywheel equipped with a flexible dynamic vibration absorber. Journal of Vibration and Control **2020,** Vol.26, Issue 23-24, Page 2113-2124, DOI 10.1177/1077546320915340

[29]Balcau, MC and Arghir, M. Case Study of a Mechanics System Composed of Four reduced Masses and the Dynamic Absorber Placed to One of the Extremities of the Mechanics System and Subjected to Four Harmoinic. 20th International Danube-Adria-Association for Automation and Manufacturing Symposium. 2009, Annals of DAAAM for 2009 & Proceedings of the 20th International DAAAM Symposium 20, pp.1421-1422.

[30]Balcau, M,; Cristea, A.F. Theoretical Considerations regardding the Dynamic Absorber. *Acta Technica Napocensis. Series-Applied Mathematics, Mechanics and Engineering* 2019, 62 (3) , pp.417-422z.

[31]Scutaru, M.L.; Marin, M.; Vlase, S. Dynamic Absorption of Vibration in a Multi Degree of Freedom Elastic System. *Mathematics* **2022**, *10*, 4045. https://doi.org/10.3390/math10214045.

[32]Vlase, S.; Nastac, C.; Marin, M.; Mihalcica, M. A Method for the Study of the Vibration of Mechanical Bars Systems with Symmetries. *Acta Tech. Napoc. Ser. Appl. Math. Mech. Eng.* **2017**, *60*, 539–544.

[33]Vlase, S.; Marin, M.; Iuliu, N. Finite Element Method-Based Elastic Analysis of Multibody Systems: A Review. *Mathematics* **2022**, *10*, 257.

[34]Bencze, A.; Scutaru, M.L.; Marin, M.; Vlase, S.; Toderiță, A. Adder Box Used in the Heavy Trucks Transmission Noise Reduction. *Symmetry* **2021**, *13*, 2165.

[35]Scutaru, M.L.; Vlase, S.; Marin, M.; Modrea, A. New analytical method based on dynamic response of planar mechanical elastic systems. *Bound. Value Probl.* **2020**, *2020*, 104.

[36]Potirniche, A.M., Vasile, O. and Capatana, G.F. 2022. Modal Analysis of a Mechanical System Modeled as a 6 Degrees-of-Freedom Solid Body with Elastic Bearings and Structural Symmetries. *Romanian Journal of Acoustics and Vibration*. 19, 1 (Mar. 2022), 36-40.
[37]Tufisi, C., Minda, A.A., Burtea, D.-G. and Gillich, G.-R. 2022. Frequency Estimation using Spectral Techniques with the Support of a Deep Learning Method. *Romanian Journal of Acoustics and Vibration*. 19, 1 (Jun. 2022), 49-55.
[38]Bratu, P., Nicolae, G.L., Iacovescu, S., Ion,D. and Cotoban, A. 2022. The Effect of The Symmetrical Elastic Nonlinearity on Structural Vibrations Transmitted by Dynamic Equipment on The Construction Envelope. *Romanian Journal of Acoustics and Vibration*. 19, 1 (Mar. 2022),25-28

Aspecte noi privind suprimarea vibrațiilor în sistemele mecanice cu mai multe grade de libertate

Rezumat

Sistemele mecanice cu mai multe grade de libertate prezintă proprietăți la vibratii care permit ca un astfel de system sa poate fi utilizat ca absorbitor pentru anumite frecvente excitatoare. Lucrarea își propune să lărgească o analiză făcută anterior de autori, privind capacitatea dinamică de absorbție a unor sisteme mecanice într-un caz mai general, care poate fi întâlnită în practica inginerească. Pe un exemplu sunt prezentate modul de aplicare a acestor proprietăți în calcul și avantajele rezultate.

Maria Luminița SCUTARU, Professor,Dr.hab., Transylvania University of Brașov, Department of Mechanical Engineering, lscutaru@unitbv.ro, Office phone: +40-268-418992, 29, B-dul Eroilor, 500036,Braşov, home phone: +40-723-242735.
Mircea MIHĂLCICĂ, Associate Prof. Transylvania University of Brașov, Department of Mechanical Engineering, mihalcica.mircea@unitbv.ro, Office phone: +40-268-418992, 29, B-dul Eroilor, 500036, Brașov.
Marin MARIN, Prof. Dr.hab., Transylvania University of Brașov, Department of Mathematics, m.marin@unitbv.ro, Office phone: +40-268-418992, 29, B-dul Eroilor, 500036, Brașov.
Omar Abdulah SHRRAT OMAR, PhD Student, Transylvania University of Brașov, Department of Mechanical Engineering, omar.shrrat@unitbv.ro, Office phone: +40-268-418992, 29, B-dul Eroilor, 500036, Brașov.