## TECHNICAL UNIVERSITY OF CLUJ-NAPOCA

 ACTA TECHNICA NAPOCENSISSeries: Applied Mathematics, Mechanics, and Engineering Vol. 66, Issue I, March, 2023

# POSSIBILITIES OF ASSESSING THE VEHICLE STABILITY PARAMETERS IN THE CASE OF ROLLOVER AND ROLLING ACCIDENTS 

Adrian TODORUŢ, Nicolae CORDOŞ, Irina DUMA, István BARABÁS, Nicolae BURNETE Radu-Mihai EFRIM, Marian-Daniel DRAGOSTE


#### Abstract

The paper evaluates, from a physical-mathematical point of view, the stability parameters of the motor vehicles in road accidents with rollover and rolling. Such accidents occur frequently throughout the world, and current standards in this area do not provide an acceptable method of assessing the vehicle rollover stability. The stability of the vehicle during the cornering is a risky stage of the journey due to the additional factors acting on it. The main stability factor is the centrifugal force, which depends on the curvature radius of the road and is very sensitive to the vehicle speed, usually controlled by the driver. The counterforce is produced at the wheel-road interaction, where different types and conditions of the road cause a large variation of the reactions between the wheels and the roadway. Respectively, the stability and manoeuvrability of the vehicle is highly influenced. The paper mainly focuses on the reconstruction of these types of accidents, starting from the final to the initial stage. The initial values can be included in certain stability criteria which could contribute to the analysis of possible errors of the driver, the possibilities to avoid the accident and other technical conditions that can justify the evolution of the road event, respectively the possible conditions for maintaining the transverse stability in the event of overturning and the possibilities to avoid the rolling accidents. Thus, the condition of the vehicle's wheels detaching from the inside of the curve and the condition of reaching the neutral stability position are identified, after which the accident reconstruction is proceeded. It is carried out in the reverse order of the events, starting from the rolling phase, continuing with the skidding phase and the pre-slip phase.


Key words: vehicle, traffic accident, numerical modelling, pre-skid, skid, overturn, roll

## 1. INTRODUCTION

Rollovers and rolling of the vehicle are chaotic events, often involving multiple impacts. While all other collision types follow the laws of physics, overturns and rollovers of the vehicle are so unrepeatable as a whole that they can be considered, up to some extent, as having a random nature [17].

Vehicle overturning is a major type of accident that greatly endangers the safety of the occupants [5, 12].

A road accident with the vehicle overturning can be staged according to the particularities of its kinematic and dynamic parameters [1, 2, 9 , 15]: the appearance of the cause that can produce the overturning (pre-skid phase); the
manifestation and amplification of the forces that cause the overturning (skid phase); overturning and rollover (rollover phase).

To describe and analyse the dynamics of vehicle rollovers, in [12] different numerical models are studied that capture certain situations of rollover accidents.

The overturning condition is given by the situation in which a vehicle travels in a cornering or is struck from the side, and the overturning force is large enough to cause the displacement of the centre of mass to a certain point, having the vertical projection outside the gauge and thus the vehicle will overturn [16].

Methods for estimating the vehicle travel speed are suggested in [10], by using quarter turns and performing a repetitive simulation in
established typical scenarios. Also here, simple physical models for overturning are developed, each analysed to identify the overturning criteria, and verified by simulating various of overturning situations.

In [11] the effects of tire characteristics on vehicle rollover and lateral stability are investigated, showing that their grip has an opposite effect on vehicle lateral stability and rollover tendency, while both suspension and road parameters significantly influence the rollover of the vehicle and lateral stability.

Lateral rollover produced around the longitudinal axis of the vehicle is one of the most frequently encountered categories of overturning [4]. Often, it is difficult to specify the value of the total roll angle of the vehicle (the rotation can be around the longitudinal axis of the vehicle, around an axis parallel to it, or in a combination of them - called twisting) [4]. If the lateral rotation is not complete, there are the situations of rollover on one side (a quarter of a rotation), rollover on the ceiling (half of a rotation), rollover on the other side (three quarters of a rotation) [4].

The average duration of a side rollover or screw-up is approx. 2.3 s [4]. If two complete rotations occur, the average duration of one rotation is about 1.5 s , and if three (or more than three) complete rotations occur, the average duration of one rotation is approx. 1.1 s [4]. Some authors recommend using a single value, 1.7 s , for a complete rotation (regardless the number of rotations) [4].

Lateral overturning (which is not generated by the impact with another vehicle, or with an element on the road side) can occur when the vehicle moves on a curvilinear trajectory, when an overturning moment is generated by a centrifugal force that is considered to be concentrated and applied in the vehicle's centre of gravity [4]. Lateral overturning due to centrifugal force requires the simultaneous fulfilment of two conditions [4]: the overturning moment generated by this force must be greater than the equilibrium moment given by the weight of the vehicle and its arm (half of the gauge); the transverse grip of the wheels on the side that constitutes the overturning axis must be
high enough to prevent the vehicle from sliding sideways (skidding), respectively the sliding tendency must be prevented by the presence of some elements on the roadway (longitudinal micro ditches, sidewalks).

Generally, the overturning is preceded by the skidding phenomenon. The skid can later bring the vehicle into overturning conditions, either by increasing the transverse grip coefficient, or by the appearance of obstacles that prevent the skid. The adhesion force is manifested at equilibrium as a force equal and of the opposite direction to the lateral force from the contact surface of the wheels with the roadway. At the limit, this force is the product between the weight of the vehicle and the grip coefficient in the transverse direction [4].

The grip coefficient in the transverse direction is considered to have higher values, corresponding to the roadway in good conditions, $\varphi_{y}=0.40 \ldots 0.65$ [4]. In [14] there are recommended the values $\varphi_{y}=0.36 \ldots 0.61$, and as an average value, in [1] is found $\varphi_{y}=0.48$ and in [3] $\varphi_{y}=0.50$.

In 2001, the NHTSA (National Highway Traffic Safety Administration) developed the rating system (with 5 stars) for vehicle rolling resistance, based on the static stability factor, which is given by the ratio of the half- wheelbase to the height of the vehicle's centre of gravity [ 9 , 12, 13]. The rolling tendency of a vehicle increases with the decreasing value of the static stability factor $[9,13]$.

In general, at vehicles, the static stability factor is between the values $0.85 \ldots 1.40(\geq 1$ in passenger cars) [9], and the duration corresponding to reaching the Neutral Stability Position of the vehicle (NSP - the position of the vehicle when its centre of mass is on the vertical axis that passes through the point of contact of the wheels with the road $[9,10,15]$ ), is under $0.80 \mathrm{~s}[2,9]$. This depends on the nature of the road (for example, for calloused road it is between the values $0.40 \ldots 0.60 \mathrm{~s}$, and for asphalt, between $0.20 \ldots 0.30 \mathrm{~s})[2,9]$.

In passenger cars, the neutral stability angle has an average value of $50^{\circ}$, and the estimated duration of the skid of approximately 0.50 s , so it can be considered that this phase ends when
the angular velocity of the vehicle in relation to the longitudinal axis upon reaching the vehicle's neutral stability position is $(2 \cdot 50) / 0.50 \%$, respectively $200 \%$ [9].

When the skidding distance cannot be established, the skidding duration can be imposed (for skidding on grassy ground, this is approximately 0.50 s , and for asphalt road, 0.20 s ) [9].

As the static stability factor increases, so does the impulse required for rollover [9]. Rollover stability is improved on hard-surfaced roads. For example, given the static stability factors considered, rolling on asphalt requires nearly double the impulses compared to rolling over gravel [9].

Along the rolling distance, the vehicle has a uniformly decelerated movement, with a forward resistance coefficient between the limits of $0.40 \ldots 0.50$ [ 9,17$]$, the usual values being between the limits of $0.43 \ldots 0.47$ [9]. During the rolling phase, there is a progressive decrease in the vehicle's kinetic energy, it being consumed by friction-rolling and deformation of the body [9].

During skidding, the resistance coefficient has significantly higher values (1.20...1.70, the usual values being around 1.50) than the grip coefficient, and this because [9]: during the skid there is an additional energy consumption necessary for the rotation of the vehicle and the elevation of its centre of mass until reaching the NSP; the friction with the road is more intense, usually accompanied by getting stuck in the roughness of the road and pulling out the particles from the tires and rims along this path; there are compressions and expansions of the suspension elements whose returns occur after reaching the NSP. All these manifestations occur in the skidding phase, with an energy consumption equivalent to the increase in resistance to the forward movement.

Unlike the coefficient of driving resistance in the skidding phase, which takes into account the energy losses related to the rotation of the vehicle, its elevations up to the NSP and the friction of the tires with the ground, the sliding coefficient related to the skidding distance refers only to the frictional losses and has a value
between 0.80...1.10 [9], lower than the coefficient of resistance to progress in the skidding phase, but higher than a grip coefficient, since in the skidding phase the wheels tend to grab the soil and break pieces of it.

Thus, the reconstruction of the rolling and skidding phases is based on the adoption of forward resistance coefficients. The quantities at the border between the phases are of interest, as they can lead to assessments of the correspondence with reality of both reconstruction stages. One interest parameter is represented by the angular velocity of the vehicle between the skidding phase and the rolling phase [9].

The global resistance coefficient for both phases, skidding and rolling, can take values between $0.65 \ldots 0.70$, for asphalted or stonepaved roads and between $0.70 \ldots 0.80$, for grassy land [2, 9]. Many technical experts use such a working methodology to establish the minimum speed at which it is possible to reach the NSP, since the variation limits of the overall vehicle driving resistance coefficient for both skid and roll phases are narrower than those of the vehicle driving resistance coefficient in the skid phase or the vehicle driving resistance coefficient in the rolling phase [9]. In such situations, the subsequent detailed reconstruction of the phases by imposing the duration of the slip is not recommended, because the precision of the results is not acceptable [9].

In the present paper, the condition of the vehicle's wheels detaching from the inside of the curve and the condition of reaching the neutral stability position are identified. Afterwards, the accident reconstruction is carried out, in the reverse order of the events, starting from the rolling phase, continuing with the skidding and the pre-slip phase.

## 2. CONDITIONS OF STABILITY IN ROLLOVER ACCIDENTS

### 2.1. The condition for the wheels detaching inside the curve

In order to establish the transverse stability criteria, the vehicle is considered while
cornering on a road with the transverse slope $\beta$. The transverse overturning of the vehicle occurs in relation to the point $S$ (Fig. 1) [18, 19, 20, 21, 22, 23, 24].


Fig. 1. The forces and moments that act on the vehicle in cornering on a road with a transverse slope $\beta$.

In the case of a turn with a superelevation (transverse slope of the $\operatorname{road} \beta>0$ ), the equation of moments with respect to the overturning point $S$ (see Fig. 1), by the condition of maintaining the transverse stability when overturning at the limit (the normal reaction of the road with respect to the wheels on the right side, $Z_{d}=0$ ), is $[2,8,9,22,23$, 24]:

$$
\begin{gather*}
(\Sigma M)_{S}:\left(F_{i y} \cdot \cos \beta-G_{a} \cdot \sin \beta\right) \cdot h_{g}- \\
-\left(G_{a} \cdot \cos \beta+F_{i y} \cdot \sin \beta\right) \cdot \frac{E}{2}=0, \tag{1}
\end{gather*}
$$

where: E is the gauge; $\mathrm{h}_{\mathrm{g}}$ - the height of the vehicle's centre of gravity; $G_{a}$ - vehicle weight.

By processing the relation (1) and taking into account that $\mathrm{F}_{\mathrm{iy}}=\mathrm{m}_{\mathrm{a}} \cdot \mathrm{a}_{\mathrm{y}}$, the static stability factor can be obtained according to the relation [9]:

$$
\begin{equation*}
\frac{\mathrm{a}_{\mathrm{y}}}{\mathrm{~g}}=\frac{\mathrm{m}_{\mathrm{a}} \cdot\left(\frac{\mathrm{E}}{2} \cdot \cos \beta+\mathrm{h}_{\mathrm{g}} \cdot \sin \beta\right)}{\mathrm{m}_{\mathrm{a}} \cdot h_{\mathrm{g}} \cdot \cos \beta-\frac{\mathrm{m}_{\mathrm{a}} \mathrm{E}}{2} \cdot \sin \beta} . \tag{2}
\end{equation*}
$$

in which: $\mathrm{a}_{\mathrm{y}}$ is the acceleration in the direction of the speed vector; $\mathrm{m}_{\mathrm{a}}$ - the mass of the vehicle.

If the road has no superelevation $(\beta=0)$, then [2, 9, 12]:

$$
\begin{equation*}
\frac{\mathrm{a}_{\mathrm{y}}}{\mathrm{~g}}=\frac{\mathrm{E}}{2 \cdot \mathrm{hg}} . \tag{3}
\end{equation*}
$$

The ratio $\frac{\mathrm{a}_{\mathrm{y}}}{\mathrm{g}}$ has the meaning of a grip coefficient in the direction of the velocity vector,
$\varphi_{y}^{\prime}$, and the ratio $\frac{\mathrm{E}}{2 \cdot \mathrm{hg}_{\mathrm{g}}}$ is called static stability factor.

Relation (3) establishes only the static condition for the start of rolling/ For example, if the road is with a lower grip, the vehicle can still skid without overturning. The fulfilment of the condition (3) depends on the drift angle (the angle between the longitudinal axis and the direction at that moment of the movement of the centre of gravity). This dependence is expressed through the relation [9]:

$$
\begin{equation*}
\varphi_{\mathrm{y}}^{\prime}=\varphi_{\mathrm{y}} \cdot \sin \delta, \tag{4}
\end{equation*}
$$

in which $\varphi_{y}$ is the of lateral grip coefficient of the vehicle tires with the road.

The vehicle continues its skidding until its kinetic energy is consumed, regardless of the drift angle's $\delta$ value, if [9]:

$$
\begin{equation*}
\varphi_{\mathrm{y}}^{\prime}>\varphi_{\mathrm{y}} \tag{5}
\end{equation*}
$$

In this situation, the vehicle can only overturn if the wheels encounter an obstacle that locks them.

The condition of static stability is worse in reality, and this is because the stiffness coefficients of the suspension and the wheel tires are taken into consideration. Because of them, the centre of mass (Fig. 2) [9] moves to the outside of the curve, and the weight of the suspended mass $\mathrm{G}_{\mathrm{e}}$ additionally generates a destabilizing moment.

In this case, for $\beta=0$, the moment equation with respect to the overturning point S (see Fig. 2), by the condition of maintaining overturning transverse stability at the limit (the normal reaction of the road on the wheels on the right side, $\mathrm{Z}_{\mathrm{d}}=0$ ), is $[8,9]$ :

$$
\begin{gather*}
(\Sigma M)_{S}: F_{i y_{e}} \cdot h_{e}+F_{i y_{r}} \cdot h_{r}-G_{r} \cdot \frac{E}{2}-G_{e} .  \tag{6}\\
\cdot\left[\frac{E}{2}-\left(h_{e}-h_{o}\right) \cdot \Theta_{v}\right]=0 \\
\Theta_{v}=\xi+v \tag{7}
\end{gather*}
$$

in which: $\xi$ is the angle [rad] at which the suspended mass is tilted in relation to the wheels axis; $u$ - angle inclination [rad] of the wheel axis in relation to the road; $\mathrm{m}_{\mathrm{r}}$ - mass of the rigid assembly; $\mathrm{m}_{\mathrm{e}}-$ suspended mass; $\mathrm{m}_{\mathrm{a}}=\mathrm{m}_{\mathrm{e}}+\mathrm{m}_{\mathrm{r}}$; $h_{e}$ and $h_{r}$ - height of the centres of gravity of the suspended and rigid mass; $\mathrm{O}_{\mathrm{e}}$ - the rotational centre of the suspended mass; $\mathrm{O}_{\mathrm{r}}$ - centre of gravity of the rigid mass; $\mathrm{h}_{\mathrm{o}}$ - oscillation point of
height $O ; \quad F_{i y_{e}}=m_{e} \cdot a_{y} ; \quad F_{i y_{r}}=m_{r} \cdot a_{y}$; $\mathrm{a}_{\mathrm{y}}$ - acceleration in the direction of the speed vector (lateral acceleration).


Fig. 2. Scheme for determining the static stability factor when taking into account the stiffness of the suspension.

From relation (6) the static stability factor at which the rollover can begin [9] is obtained:

$$
\begin{equation*}
\frac{a_{y}}{g}=\frac{m_{a} \cdot \frac{E}{2}-m_{e} \cdot\left(h_{e}-h_{o}\right) \cdot \Theta_{v}}{m_{e} \cdot h_{e}+m_{r} \cdot h_{r}} . \tag{8}
\end{equation*}
$$

From the comparison of the relations (3) and (8), it can be inferred that a rigid suspension provides a higher static stability factor, so the danger of rollover is lower.

The static stability factor is important in the reconstruction of the accident because it sets a limit that, once overtaken, may be accompanied by the overturning of the vehicle. Therefore, it is possible for the vehicle to overturn only if condition [9] is met:

$$
\begin{equation*}
\varphi_{y}^{\prime}=\frac{a_{y}}{g} \geq \varphi_{y} \tag{9}
\end{equation*}
$$

Overturning can therefore occur before the vehicle reaches the skid with the wheelbase perpendicular to the direction of the momentary movement of its centre of gravity. So, the rollover can begin when the drift angle $\delta$ has reached the value [9]:

$$
\begin{equation*}
\delta \geq \arcsin \frac{\varphi_{\mathrm{y}}^{\prime}}{\varphi_{\mathrm{y}}}, \text { in }^{\circ} \tag{10}
\end{equation*}
$$

### 2.2. Condition of reaching the neutral stability position

In order for the vehicle to reach the NSP (Fig. 3) [9, 10], the condition in relation (9) must be met, but it is not sufficient. In addition, the
vehicle must have an energy enabling the centre of gravity to be raised from $h_{g}$ to $h_{p}\left(h_{p}\right.$ being the height of the centre of gravity in the NSP, $h_{p}=\sqrt{\left.(E / 2)^{2}+h_{g}^{2}\right)}$ and, at the same time, to overcome the inertia of its mass.

The energy corresponding to the deformation of the tires and the elastic elements of the suspension may be considered to be low, with negligible values. In this case, the minimum kinetic energy of the vehicle that can generate the displacement of its centre of mass to the NSP must be equal to the potential energy equivalent to the translation of its weight on the height ( $h_{p}-h_{g}$ ). Thus, for $\beta=0$ (Fig. 3), the outlining of the mathematical model can begin with the relation [9]:

$$
\begin{equation*}
\frac{\mathrm{m}_{\mathrm{a}} \cdot \mathrm{v}_{\mathrm{d}}^{2}}{2}=\mathrm{m}_{\mathrm{a}} \cdot \mathrm{~g} \cdot\left(\mathrm{~h}_{\mathrm{p}}-\mathrm{h}_{\mathrm{g}}\right) \tag{11}
\end{equation*}
$$

from which the minimum speed required for reaching the NSP, $\mathrm{v}_{\mathrm{d}}$, may be deducted $[2,9]$ :
$\mathrm{v}_{\mathrm{d}}=\sqrt{2 \cdot \mathrm{~g} \cdot \mathrm{~h}_{\mathrm{g}} \cdot\left[\sqrt{\left(\frac{\mathrm{E}}{2 \cdot \mathrm{~h}_{\mathrm{g}}}\right)^{2}+1}-1\right]}$, in $\mathrm{m} / \mathrm{s}$.


Fig. 3. Scheme for determining the conditions for reaching the neutral stability position.

The minimum energy required to rotate the centre of mass until the angle of neutral stability $\Theta_{d}$ is reached may also be written according to the moment of mechanical inertia J of the vehicle in relation to the longitudinal axis
passing through the point of contact of the wheels with the road [2, 8, 9]:

$$
\begin{equation*}
\frac{\mathrm{J} \cdot \omega_{\mathrm{d}}^{2}}{2}=\mathrm{m}_{\mathrm{a}} \cdot \mathrm{~g} \cdot\left(\mathrm{~h}_{\mathrm{p}}-\mathrm{h}_{\mathrm{g}}\right) . \tag{13}
\end{equation*}
$$

The angular velocity of the vehicle in the NSP can be expressed as a function of the conservation of the momentum generated by the velocity $\mathrm{v}_{\mathrm{d}}$ relative to the contact point at the beginning of the rollover, which is equal to the momentum in the NSP [2, 8, 9]:

$$
\begin{equation*}
\mathrm{J} \cdot \omega_{\mathrm{d}}=\mathrm{m}_{\mathrm{a}} \cdot \mathrm{v}_{\mathrm{d}} \cdot \mathrm{~h}_{\mathrm{g}}, \text { in } \mathrm{kg} \cdot \mathrm{~m}^{2} / \mathrm{s} . \tag{14}
\end{equation*}
$$

Replacing $\omega_{\mathrm{d}}$ from (14) in (13) it is obtained [2, 9, 10]:

$$
\begin{equation*}
\mathrm{v}_{\mathrm{d}}=\sqrt{\frac{2 \cdot \mathrm{~g} \cdot \mathrm{~J}}{\mathrm{~m}_{\mathrm{a}} \cdot \mathrm{~h}_{\mathrm{g}}} \cdot\left[\sqrt{\left(\frac{\mathrm{E}}{2 \cdot \mathrm{~h}_{\mathrm{g}}}\right)^{2}+1}-1\right]} \text {, in } \mathrm{m} / \mathrm{s} \tag{15}
\end{equation*}
$$

The minimum speed, $\mathrm{v}_{\mathrm{d}_{0}}$, at which it is possible to reach the NSP, established with relation (15) is more precise than (12), since in addition to raising the centre of mass, it also takes into account the rotational inertia.

The application of relation (15) depends on knowing the momentum J; for this, Steiner's theorem $[8,9]$ is used:

$$
\begin{equation*}
\mathrm{J}=\mathrm{J}_{\mathrm{xx}}+\mathrm{m}_{\mathrm{a}} \cdot \mathrm{~h}_{\mathrm{p}}^{2}, \text { in } \mathrm{kg} \cdot \mathrm{~m}^{2}, \tag{16}
\end{equation*}
$$

where $J_{x x}$ is the moment of mechanical inertia of the vehicle in relation to its longitudinal axis (passing through its centre of gravity), specific to the rolling motion (lateral rolling). To estimate the moment $J_{\mathrm{xx}}$, the relation [7,9] is used:

$$
\begin{equation*}
\mathrm{J}_{\mathrm{xx}}=\frac{\mathrm{h}_{\mathrm{v}}+\mathrm{h}_{\mathrm{g}}}{\mathrm{~K}_{\mathrm{v}}} \cdot \mathrm{E} \cdot \mathrm{~m}_{\mathrm{a}} \text {, in } \mathrm{kg} \cdot \mathrm{~m}^{2} \tag{17}
\end{equation*}
$$

where: $h_{v}$ represents the height of the vehicle; $\mathrm{K}_{\mathrm{v}}$ - the coefficient of the mass moment of roll inertia, the values of which depend on the type of vehicle (for passenger cars, $\mathrm{K}_{\mathrm{v}}=7.9846$ [7, 9]).

By replacing (16) and (17) in (15), $\mathrm{v}_{\mathrm{d}}$ is obtained, in $\mathrm{m} / \mathrm{s}[8,9]$ :

$$
\begin{equation*}
\mathrm{v}_{\mathrm{d}}=\sqrt{\frac{2 \cdot \mathrm{~g}}{\mathrm{~h}_{\mathrm{g}}} \cdot\left\{\frac{\mathrm{~h}_{\mathrm{v}}+\mathrm{h}_{\mathrm{g}}}{\mathrm{~K}_{\mathrm{v}}} \cdot \mathrm{E}+\left[\left(\frac{\mathrm{E}}{2}\right)^{2}+\mathrm{h}_{\mathrm{g}}^{2}\right]\right\} \cdot\left[\sqrt{\left.\left(\frac{\mathrm{E}}{2 \cdot h_{\mathrm{B}}}\right)^{2}+1-1\right]}\right.} . \tag{18}
\end{equation*}
$$

The relations (12) and (18) establish the minimum translational speeds required, from an energy point of view, to achieve the NSP.

Experimentally, it was found [9] that after the wheels inside the curve detach from the road, the centre of mass of the vehicle evolves towards the

NSP at an upward angular velocity $\frac{d \Theta}{d t}$ linearly over time, i.e. practically with constant angular acceleration, as if it were driven by a tangential force $F_{t}$ constant (the force of the minimum impulse to reach the NSP). Consequently, it can be considered that reaching the NSP is conditioned by an impulse of a tangential force $F_{t}$ that is manifested for a duration $\Delta t$ corresponding to the NSP reaching. From this, a third way of determining the speed $v_{d_{0}}$ at which the rollover occurs can be inferred. For this purpose, from the introduction of the condition of compensating the translation energies integrated in the period $\Delta t$, in the equations of the mass centre movement in the NSP by translation and rotation, the equality $[2,9]$ is obtained:

$$
\begin{gather*}
\left(\frac{\mathrm{F}_{\mathrm{t}}}{\mathrm{~m}_{\mathrm{a}} \cdot \mathrm{~g}}-\frac{\mathrm{E}}{2 \cdot \mathrm{~h}_{\mathrm{g}}}\right)^{2} \cdot \Delta \mathrm{t}^{2}+\frac{\mathrm{E}}{2 \cdot \mathrm{~h}_{\mathrm{g}}} \cdot\left(\frac{\mathrm{~F}_{\mathrm{t}}}{\mathrm{~m}_{\mathrm{a}} \cdot \mathrm{~g}}-\frac{\mathrm{E}}{2 \cdot \mathrm{~h}_{\mathrm{g}}}\right) \cdot \Delta \mathrm{t}^{2}= \\
=\frac{2}{\mathrm{~m}_{\mathrm{a}} \cdot \mathrm{~g} \cdot \mathrm{~h}_{\mathrm{g}}} \cdot\left(\mathrm{~J}_{\mathrm{xx}}+\frac{\mathrm{m}_{\mathrm{a}} \cdot \mathrm{E}}{4}\right)^{2} \cdot\left[\sqrt{\left.\left(\frac{\mathrm{E}}{2 \cdot \mathrm{~h}_{\mathrm{g}}}\right)^{2}+1-1\right],}\right. \tag{19}
\end{gather*}
$$

which, after solving in relation to $\frac{F_{t}}{m_{a} \cdot g}$, offers the solution [2, 9]:

$$
\begin{gather*}
\frac{\mathrm{F}_{\mathrm{t}}}{\mathrm{~m}_{\mathrm{a}} \cdot \mathrm{~g}}=\frac{1}{2 \cdot \Delta \mathrm{t}} \cdot\left[\Delta \mathrm{t} \cdot\left(\frac{\mathrm{E}}{2 \cdot \mathrm{~h}_{\mathrm{g}}}\right)+\right. \\
+\sqrt{\left.\Delta \mathrm{t}^{2} \cdot\left(\frac{\mathrm{E}}{2 \cdot \mathrm{~h}_{\mathrm{g}}}\right)^{2}+\left(\frac{32 \cdot \mathrm{~h}_{\mathrm{g}} \cdot \mathrm{~J}_{\mathrm{xx}}}{\mathrm{~m}_{\mathrm{a}} \cdot g}+\frac{8 \cdot \mathrm{~h}_{\mathrm{g}} \cdot \mathrm{E}^{2}}{\mathrm{~g}}\right) \cdot\left(\sqrt{\left(\frac{\mathrm{E}}{2 \cdot \mathrm{~h}_{\mathrm{g}}}\right)^{2}+1}-1\right)\right]} . \tag{20}
\end{gather*}
$$

Equations (19) and (20) are valid only if this condition [9] is fulfilled:

$$
\begin{equation*}
\frac{\mathrm{F}_{\mathrm{t}}}{\mathrm{~m}_{\mathrm{a}} \cdot \mathrm{~g}}>\frac{\mathrm{E}}{2 \cdot \mathrm{~h}_{\mathrm{g}}} . \tag{21}
\end{equation*}
$$

Generally, in passenger cars $\frac{\mathrm{E}}{2 \cdot \mathrm{~h}_{\mathrm{g}}} \geq 1$ [9], hence the force that should be applied to the wheels inside the curve would be greater than the weight of the vehicle, when it could be overturned with a force slightly greater than half its weight. In this case, the angular motion up to the NSP would no longer be uniformly accelerated as in the real case, so compliance with the condition (21) becomes mandatory.

The relation (20) allows the assessment of the amount of momentum that can cause the wheels to roll over when the wheels pass over or through a bump.

In order to produce the rollover, the momentum generated by the bump crossing (the
product between the vertical force transmitted to the wheels and its time of manifestation) should be higher than that resulting from the relation (20). If the rollover is the result of not adapting the cornering speed, the angular velocity in the NSP can be determined from the conservation of angular momentum [9]:

$$
\begin{array}{r}
\mathrm{v}_{\mathrm{d}_{0}} \cdot \mathrm{~m}_{\mathrm{a}} \cdot \mathrm{~h}_{\mathrm{g}}=\omega_{\mathrm{d}} \cdot \mathrm{~m}_{\mathrm{a}} \cdot \mathrm{~h}_{\mathrm{p}}^{2} \\
\omega_{\mathrm{d}}=\frac{\mathrm{v}_{\mathrm{d}_{0}} \cdot \mathrm{~h}_{\mathrm{g}}}{\left(\frac{\mathrm{E}}{2}\right)^{2}+\mathrm{h}_{\mathrm{g}}^{2}}, \text { in rad} / \mathrm{s} \tag{23}
\end{array}
$$

Considering that up to the NSP the movement is uniformly accelerated, the result is the angular acceleration $\varepsilon$ [9]:

$$
\begin{equation*}
\varepsilon=\frac{\omega_{\mathrm{d}}}{\Delta \mathrm{t}} \text {, in } \mathrm{rad} / \mathrm{s}^{2} . \tag{24}
\end{equation*}
$$

Knowing that [9, 10],

$$
\Theta_{\mathrm{d}}=\frac{\pi}{180} \cdot\left(90-\operatorname{arctg} \frac{2 \cdot \mathrm{~h}_{\mathrm{g}}}{\mathrm{E}}\right), \text { in rad, (25) }
$$

and,

$$
\begin{equation*}
\Theta_{\mathrm{d}}=\varepsilon \cdot \frac{\Delta \mathrm{t}^{2}}{2}, \text { in } \mathrm{rad} \tag{26}
\end{equation*}
$$

it is obtained [9],

$$
\begin{equation*}
\Delta \mathrm{t}=\frac{2 \cdot \Theta_{\mathrm{d}}}{\omega_{\mathrm{d}}}, \text { in } \mathrm{s} \tag{27}
\end{equation*}
$$

The value of $\Delta t$ is entered in relation (20), from which the force $F_{t}$ is then obtained. The speed $v_{t}$ derived from this third methodology is determined by equating the initial kinetic energy with the mechanical work produced by $\mathrm{F}_{\mathrm{t}}$ during reaching the NSP [9]:

$$
\begin{align*}
& \frac{\mathrm{m}_{\mathrm{a}} \cdot \mathrm{v}_{\mathrm{d}}^{2}}{2}=\mathrm{F}_{\mathrm{t}} \cdot \Theta_{\mathrm{d}} \cdot \sqrt{\left(\frac{\mathrm{E}}{2 \cdot \mathrm{~h}_{\mathrm{g}}}\right)^{2}+\mathrm{h}_{\mathrm{g}}^{2}}  \tag{28}\\
& \mathrm{v}_{\mathrm{d}}=\sqrt{\frac{2 \mathrm{~F}_{\mathrm{t}} \cdot \Theta_{\mathrm{d}}}{\mathrm{~m}_{\mathrm{a}}} \cdot \sqrt{\left(\frac{\mathrm{E}}{2 \cdot \mathrm{~h}_{\mathrm{g}}}\right)^{2}+\mathrm{h}_{\mathrm{g}}^{2}}} \text {, in } \mathrm{m} / \mathrm{s} \tag{29}
\end{align*}
$$

in which $\Theta_{d}$ it is determined with the relation (26).
In order to estimate the various influences of some dimensional characteristics of the vehicle on its behaviour in curves, an example of an approach to the stability of two vehicles with the same mass, but differing in heights of the mass centres, overall heights and suspension stiffness, is presented. The results of the calculations, as a result of the development of the numerical calculation model, as well as the characteristics of the vehicles, are captured in Table 1. As it can be seen, the vehicle with the lower centre of mass and rigid suspension cannot overturn, as the static stability factor is superior to the lateral
grip: regardless of the curved turn, it will skid without overturning until its kinetic energy is consumed. However, if the same vehicle has an elastic suspension, its static stability factor will decrease and thus be able to overturn, as will the vehicle with the higher centre of mass, which can overturn even with a rigid suspension.

For the rollover to occur, the transverse speed of the vehicle with a lower height of the mass centre should be higher: for example, it appears from the case under consideration that when the centre of mass is raised by $36.36 \%$, the minimum speed at which it is possible to reach the NSP, which takes into account both the raising of the centre of mass and the rotational inertia, decreases by $23.93 \%$. These proportions shall be maintained regardless of the methodology for determining the minimum speed.

## 3. MODELS FOR THE RECONSTRUCTION OF ROLLOVER AND ROLLING ACCIDENTS

The development of the vehicle movement in the rolling phase depends on the determination of its speed and direction in the previous stages. Consequently, the reconstruction of such an accident must be performed in the reverse order of events [1, 2, 9, 15], starting from the rolling phase (Fig. 4).

### 3.1. Reconstruction of the rolling phase

The reconstruction of the rolling phase aims to establish two aspects [9]:

- determining the speed of the vehicle at the beginning of the phase and the time corresponding to the rolling movement according to the total rolling distance;
- determining the moment of rolling, the evolution over time of the angular and translational velocities, respectively the orientation of the vehicle during rolling.
The distance $S_{r_{0}}$ on which the vehicle rolls is apparent from the investigation of the scene. The end point of the trajectory corresponds to the position where the vehicle was found after the accident $\mathrm{S}_{\mathrm{r}_{0}}$. The place where the rollover began
(where the skidding phase ended) can be determined by the traces left by the tires and the rims of the wheels around which the rollover began. Thus, the rolling distance $S_{r_{0}}$ can be determined with sufficient precision [9].

The speed of the vehicle $\mathrm{v}_{\mathrm{r}_{0}}$ at the beginning of the rolling phase can be determined according to the relation [9]:

$$
\begin{equation*}
\mathrm{v}_{\mathrm{r}_{0}}=\sqrt{2 \cdot \mathrm{~g} \cdot \mathrm{c}_{\mathrm{r}} \cdot \mathrm{~S}_{\mathrm{r}_{0}}}, \text { in } \mathrm{m} / \mathrm{s} \tag{30}
\end{equation*}
$$

in which: $c_{r}$ is the coefficient of driving resistance; $\mathrm{S}_{\mathrm{r}_{0}}$ - the rolling distance of the vehicle.

Table 1
The results of the calculation of the quantities necessary to start the rollover of two vehicles

| $\begin{aligned} & \text { N } \\ & \text { N } \\ & \text { N } \\ & 0 \\ & 0 \end{aligned}$ | Characteristic size | Notation |  | Calculated or adopted values |  | Unit of measurement |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | Vehicle 1 | Vehicle 2 |  |
|  | $\square$ mass of the vehicle | $\mathrm{m}_{\mathrm{a}}$ | - | 1250 | 1250 | kg |
|  | $\square$ the suspended mass of the vehicle | $\mathrm{m}_{\text {e }}$ | - | 1020 | 1020 | kg |
|  | $\square$ mass of the rigid assembly | $\mathrm{m}_{\mathrm{r}}$ | - | 230 | 230 | kg |
|  | $\square$ gauge | E | - | 1.35 | 1.35 | m |
|  | overall height of the vehicle | $\mathrm{h}_{\mathrm{v}}$ | - | 1.25 | 1.60 | m |
|  | deight of the centre of mass | $\mathrm{h}_{\mathrm{g}}$ | - | 0.55 | 0.75 | m |
|  | height of the mass centre of the suspended part | $\mathrm{h}_{\mathrm{e}}$ | - | 0.75 | 0.85 | m |
|  | height of the mass centre of the rigid part | $\mathrm{h}_{\mathrm{r}}$ | - | 0.30 | 0.30 | m |
|  | - height of the oscillation centre of the suspended part | $\mathrm{h}_{0}$ | - | 0.35 | 0.35 | m |
|  | - the lateral inclination angle in turn of the suspended part | $\Theta_{\mathrm{v}}$ | - | 0.253 | 0.253 | rad |
|  | - coefficient of the mass moment of roll inertia | $\mathrm{k}_{\mathrm{v}}$ | - | 7.9846 | 7.9846 | - |
|  | grip coefficient in longitudinal direction | $\varphi$ | - | 1.10 | 1.10 | - |
|  | ] grip coefficient in the transverse direction | $\varphi_{y}$ | - | 0.90 | 0.90 | - |
| 幾 | $\square$ static stability factor in rigid suspension variant | $\varphi_{y}^{\prime}$ | 3 | 1.227 | 0.900 | - |
|  | - static stability factor in the elastic suspension | $\varphi_{y}^{\prime}$ | 8 | 0.888 | 0.764 | - |
|  | the drift angle at which the turning instability occurs in the elastic suspension | $\delta$ | 10 | 80.602 380.420 | 58.042 496.659 | kg. ${ }^{2}$ |
|  | - the moment of mass inertia of roll | $\mathrm{J}_{\mathrm{xx}}$ | $17$ | $380.420$ | $496.659$ | $\mathrm{kg} \cdot \mathrm{m}^{2}$ |
|  | - mass inertia moment in relation to the axis passing through the wheels' contact points with the road | J | $16$ | $1328$ | $1769$ | $\mathrm{kg} \cdot \mathrm{m}^{2}$ |
|  | - angle of rotation to the neutral stability position | $\Theta_{\text {d }}$ | 25 | 0.887 | 0.733 | rad |
|  | - the minimum speed required to reach the NSP by translation | $\mathrm{v}_{\mathrm{d}}$ | 12 | $\begin{gathered} \hline 2.508 \\ (9.030) \end{gathered}$ | $\begin{gathered} \hline 2.254 \\ (8.116) \end{gathered}$ | $\begin{gathered} \mathrm{m} / \mathrm{s} \\ (\mathrm{~km} / \mathrm{h}) \end{gathered}$ |
|  | - the minimum speed required to reach the NSP by rotation | $\mathrm{v}_{\mathrm{d}}$ | 15 | $\begin{gathered} 4.701 \\ (16.924) \end{gathered}$ | $\begin{gathered} 3.576 \\ (12.874) \end{gathered}$ | $\begin{gathered} \mathrm{m} / \mathrm{s} \\ (\mathrm{~km} / \mathrm{h}) \end{gathered}$ |
|  | - the minimum angular speed of the vehicle in relation to the longitudinal axis when the NSP is reached | $\omega_{\mathrm{d}}$ | 23 | $\begin{gathered} 3.410 \\ (195.407) \end{gathered}$ | $\begin{gathered} 2.634 \\ (150.934) \end{gathered}$ | $\begin{gathered} \mathrm{rad} / \mathrm{s} \\ (\% / \mathrm{s}) \end{gathered}$ |
|  | $\square$ the time of reaching the NSP | $\Delta \mathrm{t}$ | 27 | 0.520 | 0.556 | s |
|  | - the force of the minimum momentum to reach the NSP | $\mathrm{F}_{\mathrm{t}}$ | 20 | 19686 | 13846 | N |
|  | - the condition of overturning by the impulse of the force $F_{t}$ |  | 21 | 19686>15049 | $13846>11036$ | N |
|  | - the minimum speed required to reach the NSP by the force impulse $F_{t}$ | $\mathrm{v}_{\mathrm{d}}$ | 29 | $\begin{gathered} 6.130 \\ (22.068) \\ \hline \end{gathered}$ | $\begin{gathered} 4.361 \\ (15.700) \\ \hline \end{gathered}$ | $\begin{gathered} \mathrm{m} / \mathrm{s} \\ (\mathrm{~km} / \mathrm{h}) \end{gathered}$ |



Fig. 4. The scheme of vehicles road accidents reconstruction with rollover and rolling.

The duration $t_{r_{0}}$, corresponding to the movement with a uniformly decelerated movement over the distance $\mathrm{S}_{\mathrm{r}_{0}}$, can be established with the relation [9]:

$$
\begin{equation*}
\mathrm{t}_{\mathrm{r}_{0}}=\frac{\mathrm{v}_{\mathrm{r}_{0}}}{\mathrm{~g} \cdot \mathrm{c}_{\mathrm{r}}} \text {, in } \mathrm{s} . \tag{31}
\end{equation*}
$$

In order to estimate the maximum angular speed of the vehicle, when rotating it around its longitudinal axis, a relation of type [6, 9] may be used:

$$
\begin{equation*}
\omega_{\max }=\frac{3 \cdot \Theta_{\mathrm{r}_{0}}}{2 \cdot \mathrm{t}_{\mathrm{r}_{0}}}, \tag{32}
\end{equation*}
$$

in which $\Theta_{\mathrm{r}_{0}}$ represents the angle of rotation of the vehicle during $\mathrm{S}_{\mathrm{r}_{0}}$. In order to determine $\Theta_{\mathrm{r}_{0}}$, it is necessary to establish, from the scene of the deed, the number of rotations $n_{r}$ made during the rolling over the distance $\mathrm{S}_{\mathrm{r}_{0}}$.

Knowing $n_{r}$, the total angle of rotation $\Theta_{t}$ and the average angular speed $\omega_{\mathrm{m}}$ can be calculated [9]:

$$
\begin{equation*}
\Theta_{\mathrm{t}}=2 \pi \cdot \mathrm{n}_{\mathrm{r}}, \mathrm{rad}=360 \cdot \mathrm{n}_{\mathrm{r}},{ }^{\circ}, \tag{33}
\end{equation*}
$$

$$
\begin{equation*}
\omega_{\mathrm{m}}=\frac{\Theta_{\mathrm{r}_{0}}}{\mathrm{t}_{\mathrm{r}_{0}}}=\frac{\Theta_{\mathrm{t}}-\Theta_{\mathrm{d}}}{\mathrm{t}_{\mathrm{r}_{0}}} \tag{34}
\end{equation*}
$$

in which: $\Theta_{t}=\Theta_{r_{0}}+\Theta_{d}$.
In the case of rolling-over accidents caused by exceeding the stability factor in curves or by obstacles encountered on the road, when the vehicle rotates from the outset only around its longitudinal axis, in order to identify developments in time of angular velocity and angle during rolling, shape equations may be used [6, 9]:

$$
\begin{align*}
& \omega=4 \cdot \omega_{\max } \cdot\left(\sqrt{\frac{\mathrm{t}}{\mathrm{t}_{\mathrm{r}_{0}}}}-\frac{\mathrm{t}}{\mathrm{t}_{\mathrm{r}_{0}}}\right),  \tag{35}\\
& \Theta=4 \cdot \omega_{\max } \cdot\left(\frac{2 \cdot \mathrm{t}^{3 / 2}}{3 \cdot \sqrt{t_{\mathrm{r}_{0}}}}-\frac{\mathrm{t}^{2}}{2 \cdot \mathrm{t}_{\mathrm{r}_{0}}}\right), \tag{36}
\end{align*}
$$

where $t$ changes between values $\frac{2 \cdot \Theta_{\mathrm{d}}}{\Theta_{\text {max }}}$ and $\mathrm{t}_{\mathrm{r}_{0}}$, because the rolling phase begins with a delay corresponding to the completion of the angle $\Theta_{d}$ compared to the initial moment $(t=0)$ to which the relations (35) and (36) refer.

From the relation (35) it is inferred that the vehicle reaches its maximum angular speed after a time $\mathrm{t}=\frac{\mathrm{tr}_{\mathrm{r}_{0}}}{4}$ [9].

### 3.2. Reconstruction of the skid phase

The end of the skidding phase coincides with the beginning of the rolling phase. If the overturning is caused by the contact of the wheels with a rigid obstacle in the road or a bump on it, it is considered that the rolling starts from these places. After lifting the wheels from one side of the vehicle off the ground, it takes from the available kinetic energy a quantity necessary to reach the NSP and at the same time the wheels on the other side rub against the ground for a distance $S_{d_{0}}$, for this consuming another part of the initial energy. Thus, the vehicle enters the rolling phase, the distance travelled depending on the remaining energy and possibly also on the potential energy associated with some bumps [9]. It is admitted [9] that the rollover begins when the vehicle passes the NSP; at these moments the sliding friction is replaced by rolling, which reduces the contact forces and thus explains the disappearance of the crawling marks from the rolling phase.

The skid phase begins when the wheels on one side of the vehicle detach from the road. The reconstruction of the skid phase is based more on methodologies developed in accordance with the results of the experimental tests [9]. One reconstruction methodology implies the knowledge of the skid distance $S_{d_{0}}$ and consists of the imposion of the driving resitance coefficient $c_{d}$ for this stage, according to the results obtained from experimental tests; based on this, the speed at the beginning of the skid phase $\mathrm{v}_{\mathrm{d}_{0}}$ and the time $\mathrm{t}_{\mathrm{d}_{0}}$ are determined [9]:

$$
\begin{gather*}
\mathrm{v}_{\mathrm{d}_{0}}=\sqrt{\mathrm{v}_{\mathrm{r}_{0}}^{2}+2 \cdot \mathrm{~g} \cdot \mathrm{c}_{\mathrm{d}} \cdot \mathrm{~S}_{\mathrm{d}_{0}}}, \text { in } \mathrm{m} / \mathrm{s}  \tag{37}\\
\mathrm{t}_{\mathrm{d}_{0}}=\frac{\mathrm{v}_{\mathrm{d}_{0}}-\mathrm{v}_{\mathrm{r}_{0}}}{\mathrm{~g}_{\mathrm{d}} \cdot \mathrm{c}_{\mathrm{d}}}, \text { in } \mathrm{s} \tag{38}
\end{gather*}
$$

in which $v_{r_{0}}$ results from the relation (30).
At the same time, the angular speed $\omega_{\mathrm{d}_{0}}$ can be established at the end of the slippage, with a relation similar to (27) [9]:

$$
\begin{equation*}
\omega_{\mathrm{d}_{0}}=\frac{2 \cdot \Theta_{\mathrm{d}}}{\mathrm{t}_{\mathrm{d}_{0}}} \text {, in } \mathrm{rad} / \mathrm{s} \tag{39}
\end{equation*}
$$

in which $\Theta_{d}$ is calculated according to the relation (25). Angular velocity $\omega_{\mathrm{d}_{0}}$ can also be determined from the energy balance: kinetic energy $\frac{m_{a} \cdot v_{d_{0}}^{2}}{2}$ of the vehicle at the beginning of the skid phase is used for rotation around the wheel contact line until it is reached $\Theta_{\mathrm{d}}$, respectively $\frac{\mathrm{J} \cdot \omega_{\mathrm{d}_{\mathrm{d}}}^{2}}{2}$ (J being the mass moment of inertia in relation to the mentioned line), to defeat sliding friction over the distance $S_{d_{0}}$, respectively $\mathrm{m}_{\mathrm{a}} \cdot \mathrm{g} \cdot \mathrm{S}_{\mathrm{d}_{0}} \cdot \varphi^{\prime}\left(\varphi^{\prime}\right.$ being the related sliding coefficient), for raising the centre of mass in the position NSP to which an energy contribution
corresponds $\mathrm{m}_{\mathrm{a}} \cdot \mathrm{g} \cdot\left(\mathrm{h}_{\mathrm{p}}-\mathrm{h}_{\mathrm{g}}\right)$ and for rolling down to stop with the energy $\frac{\mathrm{m} \cdot \mathrm{v}_{\mathrm{c}_{0}}}{2}$. Thus, the equation of the energy balance has the form of [9]:

$$
\begin{align*}
& \frac{\mathrm{m}_{\mathrm{a}} \cdot \mathrm{v}_{\mathrm{d}_{0}}^{2}}{2}=\frac{\mathrm{J} \cdot \omega_{\mathrm{d}_{0}}^{2}}{2}+\mathrm{m}_{\mathrm{a}} \cdot \mathrm{~g} \cdot \mathrm{~S}_{\mathrm{d}_{0}} \cdot \varphi^{\prime}+ \\
& +\mathrm{m}_{\mathrm{a}} \cdot \mathrm{~g} \cdot\left[\sqrt{\left.\left(\frac{\mathrm{E}}{2}\right)^{2}+\mathrm{h}_{\mathrm{g}}^{2}-\mathrm{h}_{\mathrm{g}}\right]+\frac{\mathrm{m}_{\mathrm{a}} \cdot v_{\mathrm{r}_{0}}^{2}}{2}},\right. \tag{40}
\end{align*}
$$

from which results [9]:

$$
\begin{equation*}
\omega_{\mathrm{d}_{0}}=\sqrt{\frac{\mathrm{v}_{\mathrm{d}_{0}}^{2}-\mathrm{v}_{\mathrm{r}_{0}}^{2}-2 \cdot \mathrm{~g} \cdot \mathrm{~S}_{\mathrm{d}_{0}} \cdot \varphi \neq-2 \cdot \mathrm{~g} \cdot\left[\sqrt{\left(\frac{\mathrm{E}}{2}\right)^{2}+\mathrm{h}_{\mathrm{g}}^{2}}-\mathrm{h}_{\mathrm{g}}\right]}{\frac{\mathrm{h}_{\mathrm{v}}+\mathrm{h}_{\mathrm{g}}}{\mathrm{~K}_{\mathrm{v}}} \cdot \mathrm{E}+\left(\frac{\mathrm{E}}{2}\right)^{2}+\mathrm{h}_{\mathrm{g}}^{2}}} . \tag{41}
\end{equation*}
$$

It is possible that the skid and rolling phases will be treated uniformly, in which case the kinematic quantities related to the evolution of the angular speed of the vehicle's rotation cannot be determined. In such situations, the distance $\mathrm{S}_{\mathrm{dr}}$ (on which the skidding and rolling phases were carried out) must be known, and the speed $\mathrm{v}_{\mathrm{d}_{0}}$ is obtained with the relation [9]:

$$
\begin{equation*}
\mathrm{v}_{\mathrm{d}_{0}}=\sqrt{2 \cdot \mathrm{~g} \cdot \mathrm{c}_{\mathrm{g}} \cdot \mathrm{~S}_{\mathrm{dr}}}, \tag{42}
\end{equation*}
$$

in which $\mathrm{cg}_{\mathrm{g}}$ is a global resistance coefficient for both phases, skidding and rolling.

Thus, on this path, speed $v_{r_{0}}$ is obtained with the relation [9]:

$$
\begin{equation*}
\mathrm{v}_{\mathrm{r}_{0}}=\mathrm{v}_{\mathrm{d}_{0}}-\mathrm{t}_{\mathrm{d}_{0}} \cdot \mathrm{~g} \cdot \mathrm{c}_{\mathrm{g}} \tag{43}
\end{equation*}
$$

in which $t_{d_{0}}$ is the duration of the slippage.

### 3.3. Reconstruction of the pre-skid phase

In the pre-skid phase, the vehicle already has an unstable movement, usually generated by inappropriate driving, a technical fault or the unevenness of the road. Its movement is accompanied by the printing of traces with an intensity sufficient to be observable and to
distinguish them from the braking traces both by their number and by their curvilinear shape. The speed $v_{p_{0}}$ at the beginning of the phase shall be determined in accordance with the methodologies set out in the previous section. In this respect, it is necessary to know the distance $\mathrm{S}_{\mathrm{p}_{0}}$ from which the unstable movement occurred, the beginning of the phase being marked by the origin of some brake-skid traces, by an obstacle to the movement of the vehicle (curb, ditch, etc.), by an unevenness of the terrain, by the remnants of some components detached as a result of a technical failure, etc. The speed $v_{p_{0}}$ is determined according to the driving resistance coefficient related to the distance $S_{p_{0}}$.

## 4. NUMERICAL EXAMPLE OF RECONSTRUCTION OF A ROLLING ACCIDENT

The technical characteristics of the vehicle involved in the accident are captured in Table 2. The vehicle was initially on a dry asphalt road with medium wear, with only one lane per direction, and its driver is about to overtake another vehicle traveling in the same direction. When crossing the lane of the other direction, the right rear wheel quickly deflates, which
generates an unstable movement; as a result, the driver braked energetically until the vehicle overturned.

The vehicle thus enters on the verge of the road when the driver makes a sharp turn to the right to rejoin the roadway path. The turning of the front wheels gives the vehicle a rotational movement around the centre of mass while the movement continues on the approach made up of grassy ground with slight bumps. After the skidding of the wheels on the left side, the vehicle starts to overturn, followed by rollover ( 1.25 revolutions), and finally it stops on the left side of the vehicle body.

The findings from the crash site (Table 3) show that the vehicle travelled while braked on the road (after the explosion of the right rear wheel tire) to the entrance to the verge, a distance $S_{1}$, and on the verge, until the start of the rollover, a distance $\mathrm{S}_{2}$ (Fig. 5).

On the distances $S_{1}$ and $S_{2}$, traces produced by a deflated tire and discontinuous braking marks were identified. On the portion at the end of the $S_{2}$, four traces were identified, which are separated by two traces printed at the beginning of distance $S_{1}$, which shows that the vehicle had an unstable movement, characteristic of braking accompanied by skidding.

Table 2
Technical characteristics of the vehicle involved in the rollover accident

|  | Parameter | Notation | Value | Unit of <br> measurement |
| :--- | :--- | :---: | :---: | :---: |
| $\square$ Total length of the vehicle | L | 3.70 | m |  |
| Total width of the vehicle | B | 1.68 | m |  |
| Overall height | $\mathrm{h}_{\mathrm{v}}$ | 1.60 | m |  |
| Wheelbase | A | 2.40 | m |  |
| Track/Gauge | E | 1.35 | m |  |
| Total mass of the vehicle | $\mathrm{m}_{\mathrm{a}}$ | 1250 | kg |  |
| Height of the mass centre | $\mathrm{h}_{\mathrm{g}}$ | 0.75 | m |  |

Table 3

| Data from the crash site |  |  |  |  |
| :--- | :--- | :---: | :---: | :---: |
| Measured parameter | Notation | Value | Unit of <br> measurement |  |
| -Distance travelled after the tyre explosion to the verge entrance <br> Distance travelled after entering the verge until the start of the rollover | $\mathrm{S}_{1}$ | 19.75 | m |  |
| a $\mathrm{S}_{2}$ | 6.25 | m |  |  |
| The distance covered in skidding with the right side wheels detached from | $\mathrm{S}_{\mathrm{d}_{0}}$ | 5.35 | m |  |
| the ground | Distance between the vehicle stopped on the left side and the slippage marks | $\mathrm{S}_{\mathrm{r}_{0}}$ | 10.25 | m |



Fig. 5. Stages of the accident.
1 - deflating the wheel; 2 - exit on verge; 3 - rollover stage.

At the end of the distance $S_{2}$ the traces have changed shape (due to the detachment of the wheels from the right side) and thus continue over a length $S_{\mathrm{d}_{0}}$ after which they also disappear, this aspect delimiting the phase of skidding. After the skid stage, the vehicle rolled over until it stopped. Between the stopped vehicle and the place where the slippages disappear, the distance was measured $\mathrm{S}_{\mathrm{r}_{0}}$.

It starts with the reconstruction of the last phase, that of rolling, where a forward resistance coefficient between the limits is adopted $c_{r}=0.40 \ldots 0.50$ [9], after which it is continued with the reconstruction of the skidding phase, where a forward resistance coefficient is considered $\mathrm{c}_{\mathrm{d}}=1.30$, and the slip coefficient $\varphi^{\prime}$ afferent to the skid distance which relates only to frictional losses shall be considered to be the value of 1.10. Afterwards, it is used to reconstruct the pre-skid phase, in which the vehicle was braked while making a left turn generated by the inequality of the braking forces on the wheels on the sides, where the forward resistance coefficient characteristic of the slip are taken into account, namely $c_{p 1}=0.70$ for dry
asphalt and $c_{p 2}=0.40$ for grassy earth, thus being able to determine the speed $\mathrm{v}_{0}$ from the initial moment of braking [9]:

$$
\begin{equation*}
\mathrm{v}_{0}=\sqrt{\mathrm{v}_{\mathrm{d}_{0}}^{2}+2 \cdot g \cdot\left(\mathrm{~S}_{1} \cdot \mathrm{c}_{\mathrm{p} 1}+\mathrm{S}_{2} \cdot \mathrm{c}_{\mathrm{p} 2}\right)} \tag{44}
\end{equation*}
$$

Vehicle speed $\mathrm{v}_{\mathrm{d}_{0}}$ at the beginning of the slippage can be achieved both with the relation (37), after the reconstitution of the rollover phase, but also with the relation (42) if the skidding and rollover phases are treated unitarily, and the distance on which the skidding and rollover phases took place is considered $\mathrm{S}_{\mathrm{dr}}=\mathrm{S}_{\mathrm{d}_{0}}+\mathrm{S}_{\mathrm{r}_{0}}$.

The overall coefficient $\mathrm{c}_{\mathrm{g}}$ of running resistance for both phases of skidding and rollover shall be taken within the limits of 0.70 to 0.80 .

Also, if the skidding and rollover phases are treated uniformly, then the speed $\mathrm{V}_{\mathrm{r}_{0}}$ is determined with the relation (43), and the duration of the slippage is considered $\mathrm{t}_{\mathrm{d}_{0}}=0.50 \mathrm{~s}$.

The results obtained from numerical modelling are shown in tables 4,5 and 6 .

Table 4
Reconstruction of the rollover phase

| Reconstruction of the rollover phase |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Calculated parameter | Notation | Calculation relation | The result according to the value of the advanced resistance coefficient $\mathbf{c}_{r}$ |  | Unit of measurement |
|  |  |  | 0.43 | 0.47 |  |
| - Speed of the vehicle's centre of mass at the end of the skid phase | $\mathrm{v}_{\mathrm{r}_{0}}$ | 30 | $\begin{gathered} 9.299 \\ (33.477) \end{gathered}$ | $\begin{gathered} 9.722 \\ (35) \end{gathered}$ | $\begin{gathered} \hline \mathrm{m} / \mathrm{s} \\ (\mathrm{~km} / \mathrm{h}) \end{gathered}$ |
| - Vehicle speed at the beginning of the rolling phase | $\mathrm{v}_{\mathrm{r}_{0}}$ | 43 | $\begin{gathered} 11.204 \\ (40.334) \end{gathered}$ | $\begin{gathered} 11.724 \\ (42.206) \end{gathered}$ | $\begin{gathered} \mathrm{m} / \mathrm{s} \\ (\mathrm{~km} / \mathrm{h}) \end{gathered}$ |
| - Rollover duration | $\mathrm{t}_{\mathrm{r}_{0}}$ | 31 | 2.204 | 2.109 | ( |
| - Total roll angle | $\Theta_{t}$ | 33 | 450 | 450 | $\bigcirc$ |
| - Maximum angular velocity | $\omega_{\text {max }}$ | 32 | 277.624 | 290.250 | \% |
| - Average angular velocity | $\omega_{\mathrm{m}}$ | 34 | 185.083 | 193.50 | \% |

Table 5
Reconstruction of the skid phase

| Calculated parameter | Notation | Calculation relation | Result depending on the speed of the vehicle's centre of mass at the end of the skid phase,$\mathrm{v}_{\mathrm{r}_{0}},[\mathrm{~m} / \mathrm{s}]$ |  | Unit of measurement |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | 9.299 | 9.722 |  |
| - The speed of the vehicle's centre of mass at the beginning of the skid phase | $\mathrm{v}_{\mathrm{d}_{0}}$ | 37 | $\begin{gathered} 14.931 \\ (53.751) \end{gathered}$ | $\begin{gathered} 15.198 \\ (54.712) \end{gathered}$ | $\begin{gathered} \mathrm{m} / \mathrm{s} \\ (\mathrm{~km} / \mathrm{h}) \end{gathered}$ |
| - The speed of the vehicle at the beginning of the skid | $\mathrm{v}_{\mathrm{d}_{0}}$ | 42 | $\begin{gathered} 14.637 \\ (52.694) \end{gathered}$ | $\begin{gathered} 15.648 \\ (56.333) \end{gathered}$ | $\begin{gathered} \mathrm{m} / \mathrm{s} \\ (\mathrm{~km} / \mathrm{h}) \end{gathered}$ |
| - Duration of skidding | $\mathrm{t}_{\mathrm{d}_{0}}$ | 38 | 0.442 | 0.429 | s |
| - The angle of rotation from road to NSP | $\Theta_{\text {d }}$ | 25 | 41.987 | 41.987 | - |
| - Angular speed at the end of the skid | $\omega_{\mathrm{d}_{0}}$ | 39 | 190.160 | 195.574 | \% |
| - Angular speed (from the energy balance) | $\omega_{\mathrm{d}_{0}}$ | 41 | 192.101 | 192.101 | \% |

Table 6
Reconstruction of the pre-skid phase


For $\omega_{d_{0}}$, the values obtained on the two paths are approximately the same, the deviation being only of $1 \ldots 1.80 \%$ (see Table 5), which means that the coefficients of friction and running resistance have been chosen accordingly.

Speeds $\quad \mathrm{v}_{\mathrm{d}_{0}}$ obtained with the two methodologies differ from each other only with $2 \ldots 3 \%$ (see table 5), and speeds $\mathrm{v}_{\mathrm{r}_{0}}$ with approximately $20 \%$ (see Table 4).

It is established that for the situation in which the skidding and rollover phases are treated unitarily, the results are less accurate due to the lack of measurable traces from the accident site and the fact that the adoption of less exact parameters (such as, for example, the duration of the slippage, which influences both the value of the speed $\mathrm{v}_{\mathrm{r}_{0}}$ given by the relation (43) (see table 4 ), as well as the value of angular speed at the end of the skid ( $167.949 \%$ s) given by relation (39) has been resorted to, the obtained deviation is of $11 \ldots 14 \%$.

The time developments of the angular velocity $\omega$ and the angle of rotation $\Theta$ are determined with the relation (35) and (36), the results being graphically presented in Figure 6.


Fig. 6. Evolution of the rotation angle $\theta$ and angular velocity $\omega$, in case of an accident with 1.25 revolutions in one rollover.

## 5. CONCLUSIONS

In road accidents with rollover, multiple collisions occur with the vehicle body surfaces and its extremities, their rigidities and different shapes also imposing a vertical movement which together with the rotation generates conditions for the expulsion of the occupants at certain
times of rolling. A complex three-dimensional movement (vertically, horizontally and rotationally) is reached, the modelling of which requires not only the acquisition of a multitude of traces but also their correlation with the parts of the vehicle that produced them, only thus being possible to establish the characteristics of the movement.

The research of the site where a road accident with rollovers occurred must be carried out with great care and this because on the one hand the traces printed especially in the soil are more difficult to notice, and on the other hand the tracks have a great diversity. Of great importance is the professional training of the crash scene investigation team and especially the experience of its members faced with similar situations. For these reasons, the accident reconstruction expert must resume the research and perform the crash site measurements.

The reconstruction of the pre-skid phase is of particular importance, as it determines whether the speed of the vehicle fell within the legal limits. A similar importance must be given both to the causes that caused the unstable movement and to the possibilities on the part of the driver to perceive them in time, in order to undertake manoeuvres to avoid or reduce the consequences, aspects that can also be deduced from the reconstruction of the pre-skid phase.

Through numerical modelling, the limit conditions of stability, limit angles, limit turning radii and critical travel speeds can be determined, so that the overturning of the vehicle does not occur.

The numerical model developed can be applied for the reconstruction of road accidents with rollover and rolling of vehicles, respectively for any type of vehicle involved in such an accident.

## 6. REFERENCES

[1] Altman, S.; Santistevan, D.; Hitchings, C.; Wallingford, J. et al., A Comparison of Rollover Characteristics for Passenger Cars, Light Duty Trucks and Sport Utility Vehicles. SAE Technical Paper 2002-01-

0942, SAE International, 2002, https://doi.org/10.4271/2002-01-0942.
[2] Brach, Raymond M.; Brach, R. Matthew, Vehicle Accident Analysis and Reconstruction Methods, Second Edition. Warrendale, PA, SAE International, 2011.
[3] Bratten, T.A., Development of a Tumble Number for Use in Accident Reconstruction. SAE Technical Paper 890859, SAE International, 1989, https://doi.org/10.4271/890859.
[4] Cristea, D., Abordarea accidentelor rutiere. Piteşti, Editura Universităţii din Piteşti, 2009.
[5] Dahmani, H.; Chadli, M.; Rabhi, A.; El Hajjaji, A., Road bank angle considerations for detection of impending vehicle rollover. IFAC Proceedings Volumes, Volume 43, Issue 7, July 2010, Pages 31-36, ISSN: 1474-6670,
https://doi.org/10.3182/20100712-3-DE2013.00141.
[6] Funk, J.R.; Luepke, P.A., Trajectory Model of Occupants Ejected in Rollover Crashes. SAE Technical Paper 2007-01-0742, SAE International, 2007, https://doi.org/10.4271/2007-01-0742.
[7] Gaiginschi, R.; Drosescu, R.; Rakoși, E.; Sachelarie, A.; Filip, I.; Pintilei, M., Siguranţa circulaţiei rutiere, Vol. I. București, Editura Tehnică, 2004.
[8] Gaiginschi, R.; Drosescu, R.; Gaiginschi, Lidia; Sachelarie, A.; Filip, I.; Pintilei, M., Siguranţa circulaţiei rutiere, Vol. II. București, Editura Tehnică, 2006.
[9] Gaiginschi, R., Reconstructia şi expertiza accidentelor rutiere. Bucureşti, Editura Tehnică, 2009.
[10]Han, I., Rho, K. Characteristic analysis of vehicle rollover accidents: Rollover scenarios and prediction/warning. Int. J Automot. Technol. 18(3), 2017, 451-461, https://doi.org/10.1007/s12239-017-0045-1.
[11]Hassan, M.A.; Abdelkareem, M.A.A.; Moheyeldein, M.M.; Elagouz, A.; Tan, G., Advanced study of tire characteristics and their influence on vehicle lateral stability and untripped rollover threshold. Alexandria Engineering Journal, Volume

59, Issue 3, June 2020, Pages 1613-1628, ISSN: 1110-0168, https://doi.org/10.1016/j.aej.2020.04.008.
[12] Jin, Z.; Li, B.; Li, J., Dynamic Stability and Control of Tripped and Untripped Vehicle Rollover. Morgan \& Claypool Publishers, series Synthesis Lectures on Advances in Automotive Technology (Khajepour, A. Editor, University of Waterloo), 2019.
[13]Luepke, P.A.; Carter, J.W.; Henry, K.C.; Germane, G.J.; Smith, J.W., Rollover Crash Tests on Dirt: An Examination of Rollover Dynamics. SAE Int. J. Passeng. Cars Mech. Syst. 1(1):18-30, April 2008, https://doi.org/10.4271/2008-01-0156.
[14]Orlowski, K.; Moffatt, E.; Bundorf, R.; Holcomb, M., Reconstruction of Rollover Collisions. SAE Technical Paper 890857, SAE International,

1989, https://doi.org/10.4271/890857.
[15] Meyer, S.E.; Davis, M.; Chng, D.; Herbst, B., Accident Reconstruction of Rollovers: A Methodology. SAE Technical Paper 2000-01-0853, SAE International, 2000.
[16] Rivers, R.W., Basic physics: notes for traffic crash investigators and reconstructionists: an introduction for some a review for others. CHARLES C THOMAS • PUBLISHER, LTD, Springfield, Illinois, USA, 2004.
[17]Struble, D.E.; Struble, J.D., Automotive Accident Reconstruction: Practices and Principles, Second Edition, (Ground Vehicle Engineering Series). Boca Raton, CRC Press, Taylor \& Francis Group, LLC, 2020.
[18]Todoruţ, A., Bazele dinamicii autovehiculelor: Algoritmi de calcul, teste,
aplicaţii. Cluj-Napoca, Editura Sincron, 2005.
[19]Todoruţ, A., Dinamica accidentelor de circulaţie. Cluj-Napoca, Editura U.T.PRESS, 2008.
[20] Todoruţ, A.; Cordoș, N.; Bălcău, Monica, Ways to Evaluate the Transversal Stability Parameters of the Vehicles. Cluj-Napoca, Acta Technica Napocensis, Series: Applied Mathematics, Mechanics, and Engineering, Vol. 61, Issue III, September, 2018, pg. 323-332, Editura U.T.PRESS, ISSN 12215872, https://atnamam.utcluj.ro/index.php/Acta/article/view/ 1010/937.
[21]Todoruț, I.-A., Modelarea, simularea și experimentarea comportamentului dinamic al autovehiculelor in sistemul circulației rutiere - Teză de abilitare. Cluj-Napoca, Universitatea Tehnică din Cluj-Napoca, 2020,
http://iosud.utcluj.ro/files/Dosare\ abilit are/Todorut\%20Ioan\%20Adrian/m-Teza-abilitare_Adrian-Todorut_DART-FARMM-UTCN.pdf.
[22]Todoruţ, A.; Cordoş, N.; Barabás, I., Elemente de dinamica autovehiculelor. Cluj-Napoca, Editura U.T.PRESS, 2021.
[23]Untaru, M.; Poțîncu, Gh.; Stoicescu, A.; Pereş, Gh.; Tabacu, I., Dinamica autovehiculelor pe roţi. Bucureşti, Editura Didactică şi Pedagogică, 1981.
[24]Untaru, M.; Câmpian, V.; Ionescu, E.; Pereş, Gh.; Ciolan, Gh.; Todor, I.; Filip, Natalia; Câmpian, O., Dinamica autovehiculelor. Braşov, Universitatea Transilvania din Braşov, Sectorul Reprografie U02, 1988.

## POSIBILITĂȚI DE EVALUARE A PARAMETRILOR STABILITĂȚII AUTOVEHICULELOR ÎN CAZUL ACCIDENTELOR CU RĂSTURNĂRI ŞI ROSTOGOLIRI

Rezumat: În lucrare se evaluează, din punct de vedere fizico-matematic, parametrii stabilității autovehiculelor în accidente rutiere cu răsturnări și rostogoliri. Astfel de accidente au loc frecvent în întreaga lume, iar standardele actuale în acest domeniu nu oferă o metodă acceptabilă de evaluare a stabilității autovehiculului la răsturnare. Stabilitatea autovehiculului în timpul virajului este o etapă riscantă a călătoriei din cauza factorilor suplimentari care acţionează asupra acestuia. Principalul factor de stabilitate este forța centrifugă, care depinde de raza de curbură a drumului și este foarte
sensibilă la viteza autovehiculului, controlată de obicei de conducătorul auto, iar contraforța este produsă la interacțiunea roată-drum, unde diferitele tipuri și stări ale drumului provoacă o mare variație a reacțiunilor între roți ș̦i calea de rulare, respectiv a stabilității și maniabilității autovehiculului. În lucrare se urmărește, în principal, reconstrucţia acestor tipuri de accidente, pornind de la etapa finală la cea inițială. Mărimile inițiale pot fi încadrate în anumite criterii de stabilitate pe baza cărora se pot analiza eventualele erori ale conducătorului auto, posibilităţile de evitare a accidentului şi alte condiții tehnice care pot justifica evoluţia evenimentului rutier, respectiv condițiile posibile de menținere a stabilităţ̧ii transversale la răsturnare și posibilitățile de evitare a accidentelor cu rostogoliri. Astfel, se identifică condiția desprinderii roţilor autovehiculului din interiorul curbei și condiția atingerii poziţiei de stabilitate neutră, după care se procedează la reconstrucția accidentului, care se efectuează în ordine inversă derulării evenimentelor, pornind de la faza rostogolirii, continuând cu faza de derapare și faza de prederapare.

Adrian TODORUŢ, PhD. Eng., Professor, Technical University of Cluj-Napoca, Faculty of Automotive Engineering, Mechatronics and Mechanics, Department of Automotive Engineering and Transports, Romania, adrian.todorut @auto.utcluj.ro, Office Phone 0264401674.
Nicolae CORDOȘ, PhD. Eng., Associate Professor, Technical University of Cluj-Napoca, Faculty of Automotive Engineering, Mechatronics and Mechanics, Department of Automotive Engineering and Transports, Romania, nicolae.cordos@auto.utcluj.ro, Office Phone 0264401779.
Irina DUMA, PhD Student, Eng., Assistant, Technical University of Cluj-Napoca, Faculty of Automotive Engineering, Mechatronics and Mechanics, Department of Automotive Engineering and Transports, Romania, irina.duma@auto.utcluj.ro, Office Phone 0264401779.
István BARABÁS, PhD. Eng., Professor, Technical University of Cluj-Napoca, Faculty of Automotive Engineering, Mechatronics and Mechanics, Department of Automotive Engineering and Transports, Romania, istvan.barabas @auto.utcluj.ro, Office Phone 0264401674.
Nicolae BURNETE, PhD. Eng., Professor, Technical University of Cluj-Napoca, Faculty of Automotive Engineering, Mechatronics and Mechanics, Department of Automotive Engineering and Transports, Romania, nicolae.burnete @ auto.utcluj.ro, Office Phone 0264401779.
Radu-Mihai EFRIM, PhD Student, Eng., Technical University of Cluj-Napoca, Faculty of Automotive Engineering, Mechatronics and Mechanics, Department of Automotive Engineering and Transports, Romania, radu.efrim@auto.utcluj.ro, Office Phone 0264401779.
Marian-Daniel DRAGOSTE, PhD Student, Eng., Technical University of Cluj-Napoca, Faculty of Automotive Engineering, Mechatronics and Mechanics, Department of Automotive Engineering and Transports, Romania, daniel.dragoste @ auto.utcluj.ro, Office Phone 0264401779.

