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A SIMPLIFIED FEM BENDING MODEL FOR CORRUGATED BOARD

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Abstract: *In this paper a study on bending behavior of corrugated board has been performed. An analytical formulation is presented. This analyze is the model for a study including an development of a Finite Element Method code. Through the method and the original code presented is then calculated the deflection and the shear displacement of points of a simplified 2D shape of a corrugated boards, taking into account the two liners and the orthotropic medium material.*
Key words:*corrugated board, stiffness, bending, FEM, simplified simulation*

1. INTRODUCTION

Due to its protective properties, density, low production price and degree of greening, corrugated cardboard is one of the most popular materials used in the production of packaging and other useful applications. More than that, there are also numerous advantages such as efficient material, characteristics, economic advantages, high strength versus a low weight ratio. In the literature corrugated paperboard is shown as a main way used for the manufacture of shipping containers, and, for this purpose, the subject is very important in our days.[1],[2],[3].

2. GEOMETRY AND MODEL

2.1 Geometry of corrugated cardboard model

The most used types of cardboard for packaging are grease-resistant cardboard, “kraft” paper (produced from chemical pulp), duplex and triplex cardboard, corrugated cardboard etal. It is typically manufacture from two facings/surfaces or liners bonded to a corrugated medium which we will name fluting. (Fig.1). There are also many other types of cardboard, as double-wall (two fluted medium plies and three liner plies) used for large appliances or furniture, or triple-wall (three fluted medium plies and four liner plies) used for particularly. In our study we use a structure described in Fig.1

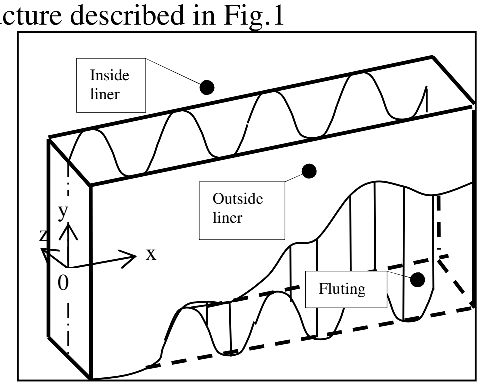

Fig. 1. Single-wall corrugated cardboard configuration

2.2 Characteristics of corrugated cardboard model

The corrugated cardboard is an orthotropic sandwich with the two linear facings (liners) which provide bending stiffness, and, between them, a lightweight corrugated core (fluting)that provides shear stiffness [4].The core medium and the liners are glued along the to form a wide sandwich panel. The linerboards are usually made from recycled paper or as known “Kraft” paperboard which may be bleached white, colored, or pre-printed [5].
Several previous research have studied the behavior of the complex structure of a

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corrugated paperboard. The main objectives in these studies were the modeling of corrugated paperboard components and structures and the analyze of the properties of the structure to tests such as elasticity, bending stiffness, compression strength, crush strength, buckling, etal.[3],[4],[5],[6],[7],[8]. The geometric structure is presented in table Tab.1.

Tab.1

Characteristics of the Single-wall corrugated cardboard

Compo-nent	**Thick-ness [mm]**	**Weight [gr/m^2]**	**E [GPa]**		**G [GPa]**	**ν**
Inside liner	0.263	230	E_1	8.12	1.289	0.34
			E_2	2.75		
Outside liner	0.263	230	E_1	8.12	1.289	0.34
			E_2	2.75		
Fluting	0.252	170	E_1	7.95	1.936	0.34
			E_2	3.15		

Concerning the geometry of the fluting, in Fig.2 the height of the fluting is h=4.75±0.05 mm, the pitch is p_{out}=8.55±0.05 mm at the contact with outside liner and p_{in}=8.19±0.05 mm at the contact with inside liner. There are also indicated the radius of the wave, at the contact with outside liner R_{out}=1.5±0.1 mm and R_{in}=2±0.1 mm at the contact with inside liner

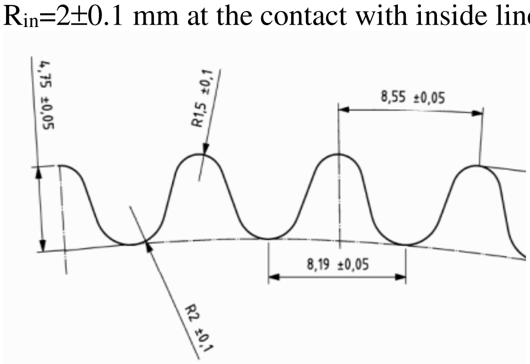

Fig. 2. Wave-wall geometry of fluting the corrugated cardboard

3. ANALITYCAL FORMULATION

The classical theory of plate deformation, under the assumption of orthotropic plates with linear elastic deformation, can also be formulate for thin plates subjected to transverse loads, in the case of the assumption that the transverse deformations are small in relation to the thickness of the plates. The governing equation for the transverse deflection is given by [9], [10]

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$$p(x,y) = S_{11}\frac{\partial^4 w}{\partial x^4} + 2(S_{12} + S_{66})\frac{\partial^4 w}{\partial x^2 \partial y^2} + S_{22}\frac{\partial^4 w}{\partial y^4} \quad (1)$$

where $w = w(x,y)$ is the transverse deflection of the middle plane of the plate, h is the wall thickness, $p(x,y)$ is the transverse pressure loading on the plate.[8], [9] (see Fig.1)

The constitutive terms in this equation are:

$$\left.\begin{aligned} S_{11} &= \frac{E_1 h^3}{12(1-\nu_{12}\nu_{21})} \\ S_{12} &= \frac{\nu_{21} E_2 h^3}{12(1-\nu_{12}\nu_{21})} \\ S_{22} &= \frac{E_2 h^3}{12(1-\nu_{12}\nu_{21})} \\ S_{66} &= \frac{G_{12} h^3}{12} \end{aligned}\right\} \quad (2)$$

where $E_1$and $E_2$are the elastic moduli in x and y direction, ν_{12} and $\nu_{21}$are the Poisson's ratios, and G_{12} is the shear modulus. The values of these properties are shown in Tab.1. The values for the in-plane shear modulus G_{12} were calculated using the relation of Baum's approximation [11]:

$$G = 0.387\sqrt{E_1}\,E_{12} \quad (3)$$

But for our 2D model the moment of inertia I_{yy} could be estimate as $I_{yy}=0$.

As consequence the term S_{11}will be equal to zero. For analytical calculation we must compute separately the moment of inertia for the liners and the fluting. For the liners is not a problem, but for the fluting the moment of inertia of the corrugated medium per unit length can be calculate using:

$$I_{xx}^{flu} = \frac{1}{L}\int_0^L \int_0^h z^2 dz dx \quad (4)$$

Usually two test methods are performed in order to measure the flexural deflection (z direction) and the transverse shear stiffness (x and y direction), which are the principal directions of the board. The associated standards for behavior of flat sandwich constructions are the ASTM standard C393-62 named three-point bend test (TPB) , respective ASTM C273-61 block shear test. Norstrand et al. [12] have evaluated the transverse shear stiffness of such a structure as corrugated cardboard using the methods described in standards. The model of 2D three-point bend test is presented in Fig.3. Concerning the shear test, which has been standardized ASTM propose a line of the load tilted to the specimen mid-plane so that the line of load of shear action is in the sandwich diagonal plane.

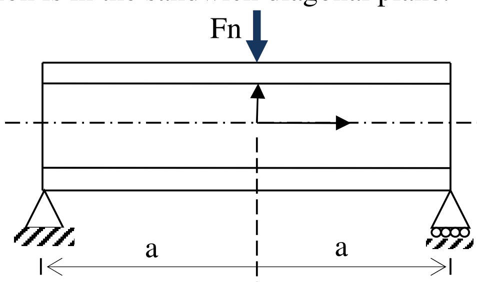

Fig. 3. Three-point bend flexuring test model

Norstrand et al. [12] found that the shear modulus determined by the bending test, are significantly lower than those obtained by the block shear test..

4. FEM METHOD

In this paper we will study the comportment of the carboard structure using Finite Element Mehod. For our study we propose the model presented in Fig.4

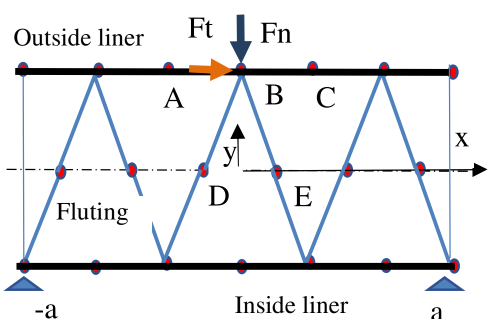

Fig. 4. Three –point bend and shearing model

This model is built in order to evaluate the comportment of a simplified structure with liners/faces and a single wall –fluting. For this structure we built a numerical Finite Element Model in Matlab R2023a [12]. Separate elements are built for liners and for fluting. The elements are Beam type element with three degrees of freedom. (28 elements for each liner and 37 elements for fluting). For the first simulation we put each of main elements of the structure, that means the liner and the fluting under load. In the first step we validate the FEM model with a structure of the flute in the three-point bend model, without any tangential load. The result is shown in Fig.5

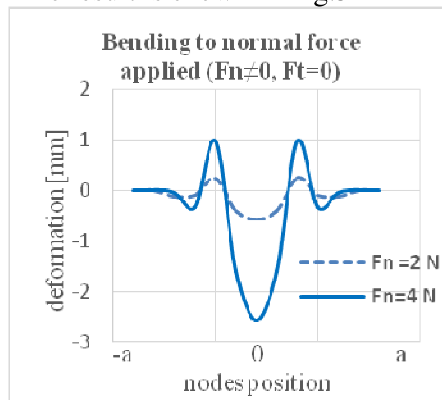

Fig. 4. Three-point bend simulation with FEM method

We found that the results validate the physical comportment of a structure with three DOF, some of points due to specific linkage have a positive displacements but the main characteristic is a symmetrical deflection. In the second step we introduce the same analyze for the structure presented in Fig.4, that means for a sandwich structure with liners and fluting. The stiffness matrix of elements are built using the analytic development presented in Eq.(1), (2), and the tangential load Ft equal to normal load for Fn=4N. (Fig.5)

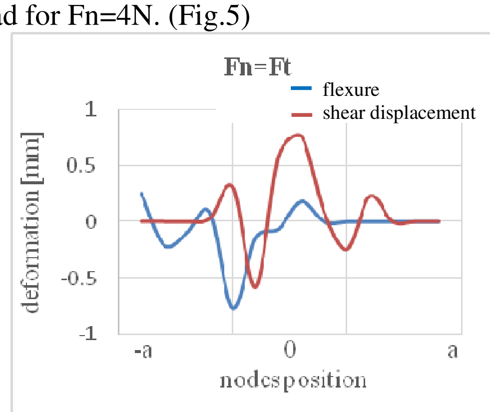

Fig. 5. Three-point bend and shearing simulation with FEM method

As we can see, the influence of the shear effect is obvious on the value but also on the configuration of the flexure due to normal load. The shear displacements also give an idea of the comportment of the fluting structure. In the next step we introduce in the work model the structure presented in Fig.4, with the liners and the fluting. In Fig.6 and in Fig.7 we see the comportment of the fluting structure when the normal load is greater than the tangential load and, respective, the tangential load is greater than the normal load.

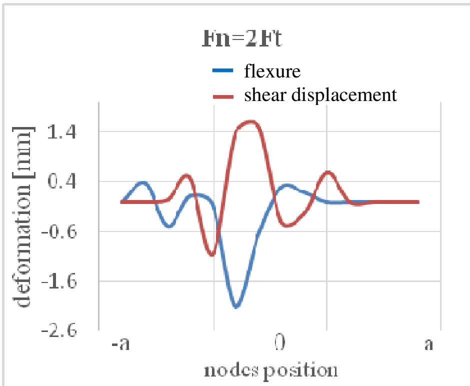

Fig. 6. Three-point bend and shearing simulation with FEM method, Fn=2Ft

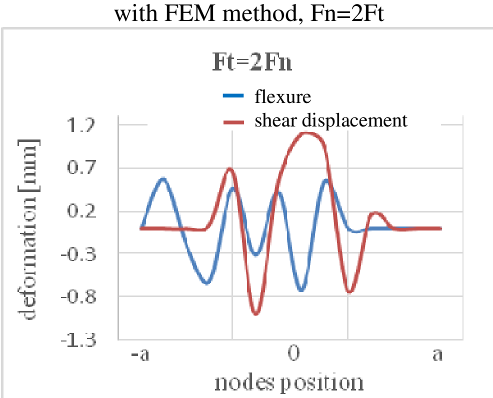

Fig. 7. Three-point bend and shearing simulation with FEM method, Ft=2Fn

In the next step of our study we introduce the same analyze for the structure presented in Fig.4, that means for a sandwich structure with liners and fluting. The stiffness matrix of elements are built using the analytic development presented in Eq.(1), (2).

5. THE HOMEGENOUS MODEL

The problem take it into account in this study is to introduce a simplified model for an homogenous structure which includes the equivalent properties of materials of the three components, the two liners and the fluting. For accomplish this step, we have to find the materials properties used in the sandwich structure. For this we take a unit cell of the structure presented in Fig.8. This structure will be subject of FEM analyze using into account the displacement method. For a unit displacement imposed to this structure in longitudinal and transverse direction (x and y direction) we will find the corresponding load.

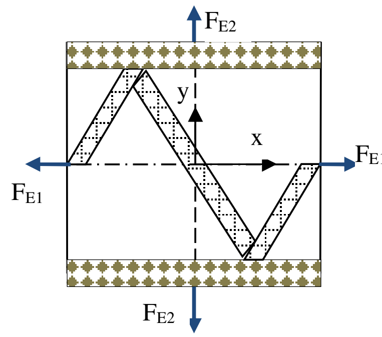

Fig. 8. Unit-Cell structure

Plane 2D elements are used to model the liners and the fluting. The first analyze is made for the structured under the load F_{E1} equal to unit and then under the load F_{E2} equal to unit. In this way we have an approximation of the material behavior. Homogenous structure which includes the equivalent properties of materials of the three components has in his structure a suite of unit-cell. The model of homogenous structure is present in Fig.9.

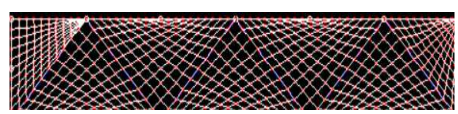

Fig. 9. FEM homogenous cardboard structure

We perform next the same analyze as we did in Chp.4. The results are present in Fig.10, 11,12 and Fig.13. In all these figures, the abbreviation TD is transverse direction (y direction in local system) and LD is longitudinal direction (x direction in local system). As we can see, all, the values of flexure or shear displacement are smaller in the case of homogenous model. We can compare. e.g., Fig.5 and Fig.12 with fig.13. In Fig.5 the value for flexure is 0.75 mm and shear displacement 0.85 mm along the longitudinal axe; in Fig.12 the value for flexure is around 0.3 mm , value taken at inner and outer liner. In Fig. 13 shear displacement is around 0.25 mm at inner liner and 0.35 mm at outer liner. Another observation, the maximum values are near the local point of load application in the FEM model with beams (Fig.4) and displaced on axes, along the direction of load applied in the model of homogenous model. A validation of the homogenous model was perform using the experimental study of Nordstrand and Carlsson.

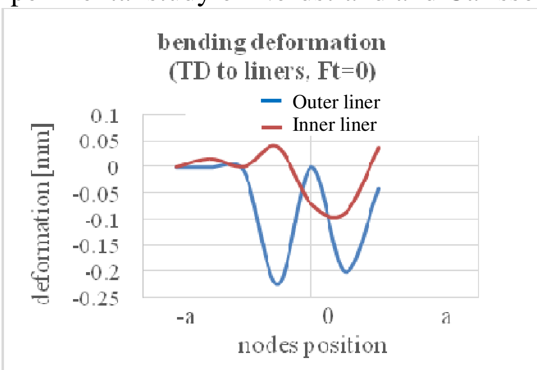

Fig. 10. FEM homogenous cardboard bend

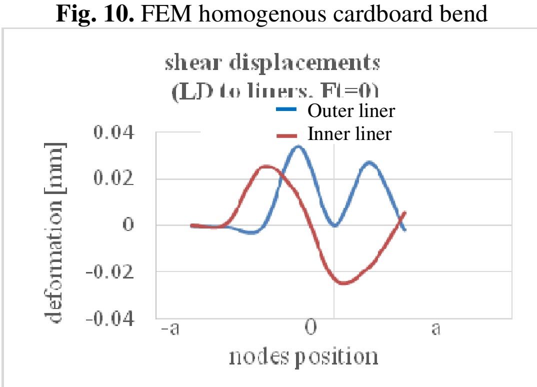

Fig. 11. FEM homogenous cardboard bend

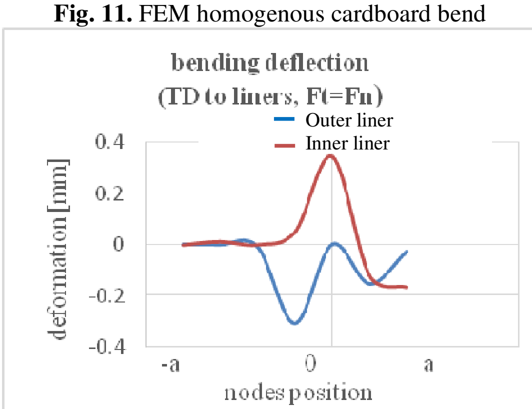

Fig. 12. FEM homogenous cardboard bend and shear

In this paper the authors perform a three-point bend test (TPB) for a 3D structure as presented in Fig. 14.

The material taken into account by authors, for the result present in Fig.14 was 160C, with similar values of material properties and very close values of geometry characteristic of the model in our paper. For our model the result is presented in Fig.15.

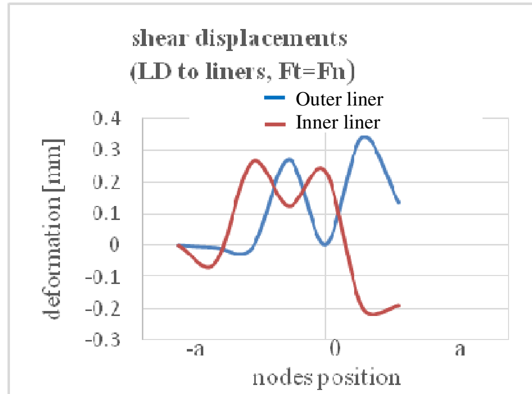

Fig. 13. FEM homogenous cardboard bend and shear

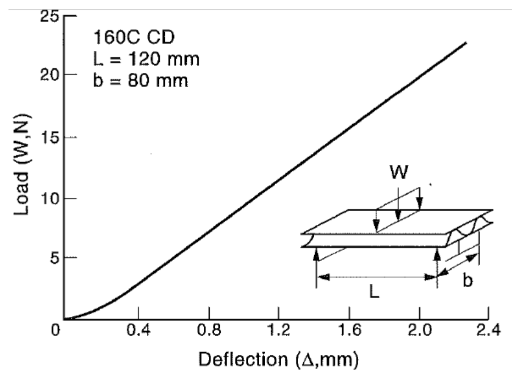

Fig. 14. Load-deflection curve for corrugated board 160C-Nordstrand and Carlsson, pp.150, [12]

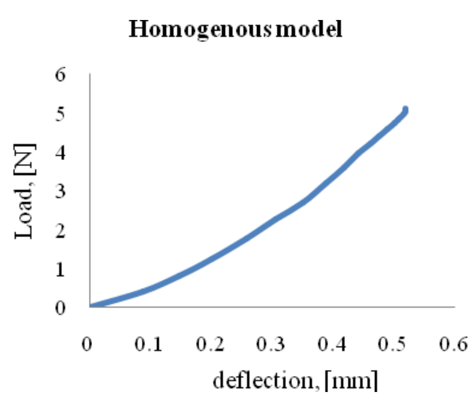

Fig. 15. Load-deflection curve for homogenous corrugated board

As we can see even if the Nordstrand's test was performed fo a 3D structure and our numerical study was performed for 2D model, there is a similar load-deflection curve for the part of the curve presented in Fig.15. In this part of a curve is a small stiffening – the first part of the curve in Nordstrand's figure. After this initial behavior, the load-deflection response is linear.

6. CONCLUSION

In this paper was developed a study regarding the behavior of a corrugated board structure. Was proposed a model for numerical analyze based on a analytic formulation. The model for this study include an development of a Finite Element Method code. Through the method and the original code presented is then calculated the deflection and the shear displacement of points of a simplified 2D shape of a corrugated boards, taking into account the two liners and the orthotropic material of the flute as medium. The results was validate also by comparing with results form the scientific literature.

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Un model MEF simplificat pentru calculul la incovoiere al cartonului mondulat

Rezumat : În această lucrare a fost realizat un studiu privind comportamentul la încovoiere, cu efecte de deplasare tangentiala, al cartonului ondulat. Este prezentată o formulare analitică. Această analiză constituie modelul pentru un studiu numeric care include dezvoltarea unui cod pentru metoda elementelor finite. Prin intermediul metodei și a codului original prezentat se calculează apoi deformarea și deplasarea tangentiala a punctelor unei forme 2D simplificate a unei plăci ondulate, luând în considerare cele două fete și materialul mediu ortotropic

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