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TECHNICAL AND ECONOMIC STUDY ON MAXIMIZING THE WORKSPACE AND DETERMINING THE MANUFACTURING COST OF THE 3RUU PARALLEL MANIPULATOR

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Abstract: *The paper presents both a technical study on maximizing the workspace of the 3RUU spatial manipulator (based on the influence of its constructive parameters) and an economic study on determining the manufacturing cost of the manipulator, happily combining notions of robotics and economics in an interdisciplinary study. The 3RUU parallel structure studied is made of duralumin, equipped with stepping motors, gearboxes and a parallel port board for computer control of end-effector trajectories.*
Key words: *parallel manipulator, kinematic chain, workspace, discretization method, input-output equations, manufacturing cost.*

1. INTRODUCTION

The kinematic diagram of the 3RUU space manipulator is presented in figure 1. The manipulator has three degrees of freedom in translation and three identical kinematic chains. Its symbolic notation is related to its characteristics: 3 – the number of degrees of freedom, RUU – the type of successive kinematic joints from one kinematic chain starting from the base to the end effecting element (R – rotation joint, U – universal joint).

The rotational motor joints from the base are positioned in the middle of the equilateral sides of a triangle (with the length b). The axes of motor joints are coincident with the base triangle sides.

The passive universal joints from the levels II and III are arranged in the kinematic chains as in figure 1. In order for the manipulator to have 3 degrees of freedom in translation, the joint axes must be arranged as in figure 1.

The calculations presented in the paper are made for the case where there is only one fork (of length L) that joins the universal joints on levels II and III of a kinematic chain. The scale manipulator (figure 2) will have for each kinematic chain two such forks, of equal length (L) and offset between them so as to form a parallelogram-type mechanism, the opposite sides always remaining parallel (thus explaining the parallelism of the base with mobile platform). This construction leads to a variant of the Delta-type robot [1]. The generalized coordinates of the mobile platform (operational coordinates) are: Z_P, Ψ, θ meaning the coordinates of the point P of the center of the gripper with respect to the fixed system OXYZ, the precession angles and notation of Euler between the 2 platforms (mobile and fixed).

By varying the coordinates q_i, i = 1,2,3, the object manipulated in space can be positioned according to the phases of the manipulation operation.

2. DESCRIPTION OF THE SCALE MODEL OF THE 3RUU MANIPULATOR

The components of the manipulator (figure 2) were executed on universal machine tools, the material being duralumin (to have a reduced weight).

Component 1 was made in 3 pieces, being a connecting piece, a coupling, between the electric motor shaft and the first shaft of the motor joint of the manipulator. The transmission of the rotation movement from the gearbox shaft

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to the motor joint shaft is done through the M4 screws (figure 2).

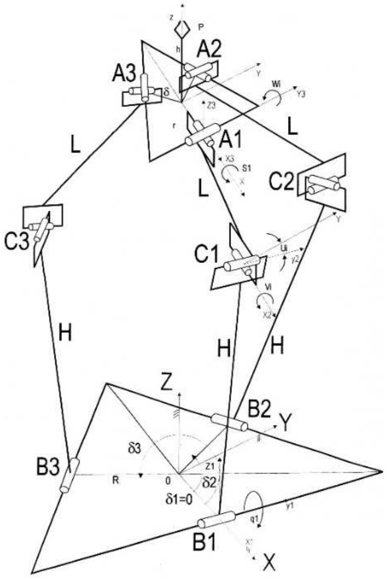

Fig. 1. Kinematic scheme of the 3RUU parallel manipulator

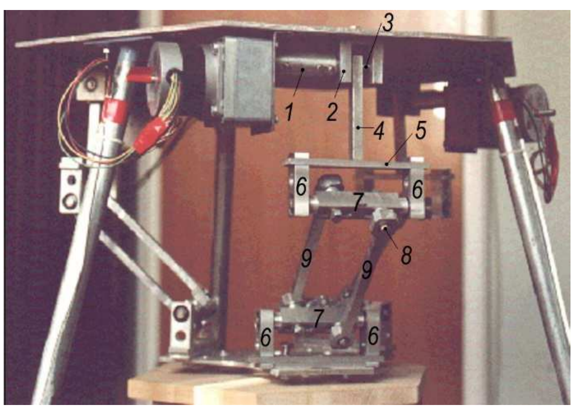

Fig. 2. The 3RUU parallel manipulator

Component 2 represents the shaft bearing of the motor joints of the manipulator. 3 such pieces of duralumin were made (having 30 mm and 35 mm sides respectively, and 20 mm recess inside each semi-finished product, as well as the M6 holes needed to fix component 2 on the base platform).

Component 3 was executed in 3 pieces, being the first motor shaft of the manipulator rotational joint.

Component 4 was executed in 3 pieces, being the motor arm (the arm that leaves the motor joint of the robot). This piece is assembled together with piece 3 and piece 5.

Component 5 was made of 3 pieces of duralumin, being the upper part of the universal joint in the kinematic chain. The piece has the shape of a rectangular parallelepiped with three millings.

Component 6 was executed in the number of 12 pieces, being the part of the universal joint in which its axis will be mounted. It is assembled together with component 5. This part actually represents the bearing of the shaft 7. The hole of ϕ 10 is given only to lighten the part and its execution precision is not important.

Component 7 was executed in 6 pieces (having a square section of 16 mm^2 and slots of 8 mm). This part is assembled in components 6 being the bearing of component 7 and is also assembled with components 8 through the shape in the 8 mm slots. The components 7,8,9 must form a parallelogram when running, so that the components 9 always remain parallel to each other regardless of how the component 7 rotates.

Component 8 was executed in 12 pieces. On this part, component 9 is mounted and the whole assembly is fixed on component 7.

Component 9 was executed in 6 pieces, important being the distance of 147 mm between the two bores of ϕ 16 (bearing dimensions) and having recesses in order to lighten the part.

3. TECHNICAL CALCULATIONS REGARDING THE WORKSPACE ANALYSIS OF THE 3RUU MANIPULATOR

The scientific literature [2], including medical robotics [3], presents in addition to the many advantages of parallel structures (very good ratio "manipulated weight / robot weight" (2-10), increased rigidity, exceptional positioning precision (5-10 µm), special dynamic behavior (velocities of 6 m/s, accelerations up to 22 g), simple geometric modeling) and the main disadvantage of these structures: the reduced workspace, an aspect that requires serious studies regarding its maximization in any way.

For the 3RUU parallel mechanism, there is the problem of determining the workspace in translation because the platform of the mechanism only executes translations in space [5], [6], [7]. It is obtained in discrete form (the discretization method is theoretically presented by J.P. Merlet in [2]) with the help of the computer (the program designed in AutoCAD is presented in paper [5]). The stages of the discretization method are based on the input-output equations, deduced in paper [4] and illustrated by equations (1):

$$2H(h-Z_P)\cos q_i + 2H[(X_P + r\cos\delta_i' - R\cos\delta_i)\cos\delta_i + (Y_P + r\sin\delta_i' - R\sin\delta_i)\sin\delta_i]\sin q_i = L^2 - H^2 - (X_P + r\cos\delta_i' - R\cos\delta_i)^2 - (Y_P + r\sin\delta_i' - R\sin\delta_i)^2 - (Z_P - h)^2 \equiv F(q_i, X_P, Y_P, Z_P) \quad i = 1,2,3 \tag{1}$$

The notations are introduced:

$$\begin{cases} a_i = 2H(h-Z_P) \\ b_i = 2H\begin{bmatrix}(X_P + r\cos\delta_i' - R\cos\delta_i)\cos\delta_i + \\ (Y_P + r\sin\delta_i' - R\sin\delta_i)\sin\delta_i\end{bmatrix} \\ c_i = L^2 - H^2 - (X_P + r\cos\delta_i' - R\cos\delta_i)^2 - \\ \quad (Y_P + r\sin\delta_i' - R\sin\delta_i)^2 - (Z_P - h)^2 \end{cases}$$

The coefficients a_i, b_i, c_i $(i=1,2,3)$ depend only on the variables X_P, Y_P, Z_P, which are considered known when solving the inverse problem of the mechanism. With these notations, equations (1) can be put in the form of trigonometric equation (2):

$$a_i \cos q_i + b_i \sin q_i = c_i \qquad i = 1,2,3 \tag{2}$$

whose solution (if the fundamental equation of trigonometry $cos^2 q_i + sin^2 q_i = 1$ is attached) has the form:

$$\sin q_i = \frac{b_i c_i \mp a_i\sqrt{a_i^2 + b_i^2 - c_i^2}}{a_i^2 + b_i^2} \tag{3}$$

$$\cos q_i = \frac{a_i c_i \pm b_i\sqrt{a_i^2 + b_i^2 - c_i^2}}{a_i^2 + b_i^2} \tag{4}$$

$$q_i = Atan\,2\,(\sin q_i, \cos q_i) \tag{5}$$

In conclusion, relations (3), (4) and (5), respectively, represent the analytical solutions of the inverse geometric model of the 3RUU parallel mechanism (meaning, these relations allow the determination of the joint coordinates of the manipulator q_i $(i=1,2,3)$ when it's known the position of the manipulated object, respectively its generalized coordinates: X_P, Y_P, Z_P).

The representation of the workspace in 3D and its various sections by the discretization method (with the help of a program written in the AutoCAD language), was presented in detail in the paper [5] and thesis [6], for the following constructive configuration: b = 676.325 mm; a = 353.338 mm; H = 110 mm; L = 150 mm; h = 45 mm; $\delta_1 = \delta_1' = 0^0$; $\delta_2 = \delta_2' = 120^0$; $\delta_3 = \delta_3' = 240^0$; $q_{imin} = 0^0$; $q_{i\,max} = 180^0$.

4. THE INFLUENCE OF THE CONSTRUCTIVE PARAMETERS OF THE 3RUU MANIPULATOR ON ITS WORKSPACE

For the 3RUU manipulator, it's sought to highlight how the variation of the constructive parameters *(R)*, *(r)*, and *(L)* contribute to the modification of the volume of the workspace of the mechanism.

Based on the discretization method, the workspace volume of the 3RUU parallel manipulator was determined with the following constructive data: R = 195.238 mm, r = 1/3R, 1/2R, 2/3R, 5/6R, R; L = R, 3/2 R, 2R, and the following input data: $q_{i\,min} = 0^0$, $q_{i\,max} = 180^0$.

For those 15 combinations *(r, L)*, 15 values of the volumes of the 15 variants of 3RUU manipulators were determined, and in the calculation program the dimensions of the parallelepiped in which the work space is calculated were increased accordingly with the increase of *(r)* and *(L)*. The results are presented in table 1.

Table 1

Volume of the workspace [dm^3] of the 3RUU manipulator

L/R	r/R 1/3	r/R 1/2	r/R 2/3	r/R 5/6	r/R 1
1	6.867	7.45	8.168	8.645	8.973
3/2	7.375	8.141	9.98	12.403	12.93
2	0.933	0.608	1.12	1.969	2.859

5. ECONOMIC CALCULATIONS REGARDING THE DETERMINATION OF THE MANUFACTURING COST OF THE 3RUU MANIPULATOR

Those 15 values of the workspace volume of the 3RUU parallel manipulator provided in Table 1 correspond to the values of the constructive parameters *(r)* and *(L)* in Table 2.

Table 2

The values of the constructive dimensions *(r)* and *(L)* [mm] for the 15 volumes

	r/R=1/3	**r/R=1/2**	**r/R=2/3**	**r/R=5/6**	**r/R=1**
L/R=1	r1= 65.07 L1= 195.23	r2= 97.61 L2= 195.23	r3= 130.15 L3= 195.23	r4= 162.69 L4= 195.23	r5= 195.2 L5= 195.2
L/R=3/2	r6= 65.079 L6= 292.85	r7= 97.619 L7= 292.85	r8= 130.15 L8= 292.85	r9= 162.69 L9= 292.85	r10= 195.2 L10= 292.8
L/R=2	r11= 65.079 L11= 390.47	r12= 97.619 L12= 390.47	r13= 130.15 L13= 390.47	r14= 162.69 L14= 390.47	r15= 195.2 L15= 390.4

With the help of the *(r)* values, the size of the mobile platform was determined for each of the 15 constructive variants of the manipulator, and the *(L)* values show the length of the driven fork from the 15 variants.

In order to establish the manufacturing cost, it was started from the premise that some of the components of the manipulator are already on the market, as such they will be purchased (Table 3), and the other components that are particular to the analyzed case will be processed in a workshop which is why when determining the cost of each component (Table 4) those specific expenses of the workshop will be taken into account (utilities, worker's salary, etc.).

Table 3

Manipulator components purchased from the market

No. crt.	**Label**	**Pieces**	**Price / piece [Euro]**
1	Stepping motor	3	175.26
2	Gearbox	3	205.66
3	The passive universal joints from the level II (C_i)	3	7.63
4	The passive universal joints from the level III (A_i)	3	7.63
5	Robot gripper h	1	60.27
6	Various materials (bolts, nuts, stepping motor cables, etc.)		5.03

The other dimensions for the fork of length L are 8.24 mm thick and 9.25 mm wide, and for the fork of length H they are 9.83 mm thick and 9.91 mm wide, dimensions that will be obtained by processing some 10x10 mm square aluminum bars that can be purchased in different lengths (1000 mm, 2000 mm, up to 6000 mm). For the fork of length H, a square bar of 1000 mm is purchased from which the 3 necessary H forks are obtained. For the 15 variants of the fork of length L, 2000 mm, 3000 mm and 4000 mm square bars will be purchased.

In order to obtain the base platform and the 15 constructive variants of the mobile platform, an aluminum plate of 2000 mm length, 1000 mm width and thickness of 5 mm will be purchased (taking into account that in the composition of the manipulator, for each of its constructive variants, there will be only one piece of each platform type).

The waste resulting from the processing of the forks and platforms as well as the unused pieces of the purchased bars and plates will be handed in as waste at the price of 1.21 Euro/kg.

Table 4

The components of the manipulator processed in the workshop

No. crt.	**Label**	**Pieces**	**Price / piece [Euro]**
1	Base platform	1	55.79
2	Rotational motor joints from the base (B_i)	3	20.6
3	Fork of length H	3	0.73
4	Fork of length L	3	Table 5
5	Mobile platform	1	Table 6
6	Foot – metal pipe	3	1.41

Each rotational motor joints from the base (B_i) will be composed of the following components purchased from the market and processed to fit:

- Guide axle support SK 12 – 2 pieces
- Shaft WRA10 – 1 piece
- Bearing Union CB-060 6700 2RS – 2 pieces

The manufacturing cost for the components corresponding to positions 1, 3, 4 and 5 from table 4, table 5 and table 6 was determined using relations (6) and (7), established in [8]:

- the cost of the material:

$$M = [M_b \cdot P_m - (M_b - M_n) \cdot P_{des}] \cdot \left(1 + \frac{Aprov}{100}\right) \quad [Euro/piece] \qquad (6)$$

- the cost at the workshop level:

$$C_a = M + \sum S_j \cdot \left(1 + \frac{Ra_j}{100}\right) \quad [Euro/piece] \qquad (7)$$

Table 5

Manufacturing cost (Euro) for the fork of length L

	r/R= 1/3	**r/R= 1/2**	**r/R= 2/3**	**r/R= 5/6**	**r/R= 1**
L/R=1	1.19	1.19	1.19	1.19	1.19
L/R=3/2	1.71	1.71	1.71	1.71	1.71
L/R=2	2.23	2.23	2.23	2.23	2.23

Table 6

Manufacturing cost [Euro] for the mobile platform according to the values of the constructive dimensions *(r)* and *(L)* for the 15 volumes

	r/R= 1/3	**r/R= 1/2**	**r/R= 2/3**	**r/R= 5/6**	**r/R= 1**
L/R=1	6.28	14.02	24.85	38.78	55.79
L/R= 3/2	6.28	14.02	24.85	38.78	55.79
L/R= 2	6.28	14.02	24.85	38.78	55.79

By adding up the manufacturing costs for the above components and also taking into account the number of pieces of these components that make up a manipulator, the manufacturing cost of the 3RUU parallel manipulator for the 15 volumes (constructive variants) will be obtained.

Table 7

Total manufacturing cost [Euro] of the 3RUU parallel manipulator according to the values of the constructive dimensions *(r)* and *(L)* for the 15 volumes

	r/R= 1/3	**r/R= 1/2**	**r/R= 2/3**	**r/R= 5/6**	**r/R= 1**
L/R=1	1363.8	1371.9	1383.2	1397.8	1415.6
L/R= 3/2	1365.3	1373.4	1384.	1399.4	1417.2
L/R= 2	1366.95	1375.05	1386.4	1400.99	1418.82

6. CONCLUSIONS

The workspace of parallel structures is relatively small and singularities can occupy an important area of it [2]. The paper presents the influence of the constructive dimensions of the 3RUU robot on its volume, together with economic calculations regarding the manufacturing cost of the robot.

The scale realization of the 3RUU robot, its equipping with stepping motors, gearboxes and a parallel port board, as well as the design and realization of the computer control software of the robot, allowed the following conclusions to be established:

- The inverse problem of the positions is verified experimentally, which highlights the validity of its establishment.
- The maximum volume of the workspace is achieved when the dimensions of the mobile platform are equal to those of the base platform (when r/R = 1), for a ratio L/R = cst. The increase of this ratio (r/R > 1) leads to the decrease of the volume of the workspace of the manipulator (table 1).
- The increase in the ratio r/R has the greatest influence on the increase in the volume of the manipulator's workspace when L/R = 3/2, less when L/R = 1 and the least influence when L/R = 2 (table 1).
- The lowest manufacturing cost (1363.82 Euro) is achieved for the configuration L/R = 1 and r/R = 1/3 in which the volume of the workspace is 6.867 dm^3 and the highest manufacturing cost (1418.82 Euro) is achieved for L/R = 2 and r/R = 1 having the workspace volume of 2.859 dm^3.
- The largest workspace volume (12.93 dm^3) is achieved for the second highest manufacturing cost (1417.25 Euro).
- Changing the r/R ratio has a greater influence in increasing the manufacturing cost of the manipulator than changing the L/R ratio (table 7).

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STUDIUL TEHNIC ȘI ECONOMIC PRIVIND MAXIMIZAREA SPAȚIULUI DE LUCRU ȘI DETERMINAREA COSTULUI DE FABRICAȚIE AL MANIPULATORULUI PARALEL 3RKK

Rezumat: *Articolul prezintă atât un studiu tehnic privind maximizarea spațiului de lucru al manipulatorului spațial 3RKK (pe baza influenței parametrilor constructivi ai acestuia), cât și un studiu economic privind determinarea costului de fabricație al manipulatorului, îmbinând fericit noțiuni de robotică și economie într-un studiu interdisciplinar. Structura paralelă 3RKK pe care s-a făcut studiul este construită din duraluminiu, prevăzută cu motoare pas cu pas, reductoare și o placă port-paralel pentru comanda prin calculator a traiectoriilor end-efectorului.*

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