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EVALUATION OF THE ENERGETIC PARAMETERS OF VISCOUS DISSIPATION FOR DYNAMIC SYSTEMS EXCITED IN HARMONIC REGIME

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Abstract: *The technological equipments with vibrating action, the anti-seismic devices in the structure of bridges or buildings for base isolation, are evaluated based on the concept of functional performance and length of service behaviour. In this paper, is presented the evaluation of the energy dissipation level based on the linear viscous damping elements. The linear viscous damping can be a viscous shock as part of the internal structure of a deformable material with linear viscoelastic behaviour (elastomers, soils, plastics, composites) as well as in the form of added constructive. The studies are highlighted a linear dissipative behaviour in the engineering fields of interest with the option of evaluating the dissipated energy and the dissipative power of the materials in operation (soils, asphalt mixtures, concrete).*

Key words: *energy dissipation, dissipated energy, average quantities, instantaneous quantitiess*

1. INTRODUCTION

The evaluation of the dissipative energy parameters is necessary for the technological vibratory machines whose dynamic schemes are Voigt-Kelvin modelled in the linear domain. The category of vibratory processing machines is characterized by the fact that mass, rigidity and amortization parameters can become discretely variable depending on the sequential and repetitive technologies. In this case, depending on the discrete variation of one or more parameters, the dynamic response expressed by the amplitude – excitation frequency characteristic leads to essential changes both in the dissipation energy and the capacity of the dissipative system expressed by the dissipative power. In this context, based on the case study of tests carried out within ICECON Bucharest on several categories of vibrating machines, the need to evaluate the dissipative energy parameters arose. Thus, the following categories of vibrating machines were tested in real technological dynamic mode: vibrating feeders for granular and powdery materials, vibrating sifters with two and four sieves, exterior vibrators for concrete compaction, vibrating compactor rollers for soil and asphalt mixtures, vibro-punchers for piles in the ground etc.

For the mentioned categories, at least one mass, elastic or viscous parameter changes during the technological processing operation.

The vibrating machines are usually excited with inertial vibrators based on the rotation of eccentric masses capable of generating the disturbing force $F(t) = m_0 r \omega^2 sin\omega t$ where $m_0 r$ is the static moment of the set of eccentric masses and ω is the angular frequency of rotation. Vibrators in this category are also called vibradines, being made as actuation equipment in various size categories.

The novelty elements of the research outcomes consist in the fact that the established calculation relationships were verified on 8

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families of vibratory machines totalling 15 types and dimensions of equipment approved in Romania in operation for about 20 years [1, 2, 4].

2. EVALUATION OF DISSIPATED ENERGY

For any real-time processing vibrating machine, the dominant model covering the dynamic regime is represented by a linear system with a degree of freedom (m, c, k) excited by a harmonic force as $F(t) = F_0 sin\omega t$, where $F_0 = m_0 r\omega^2$.

The linear dynamic model is characterized by the equivalent parameters according to the dominant unidirectional degree of freedom represented by an instantaneous displacement such as $x = x(t) = Asin(\omega t - \varphi)$
(1)
where A is the amplitude of the movement and φ the phase shift between the amplitude and the disturbing force [4, 5, 6, 7].

2.1. Instantaneous dissipated energy

The evaluation of the dissipated energy is based on knowing the equivalent amortization constant that includes all the dissipative effects of the model related to the viscoelastic support system as well as the technological process with energy losses through internal friction in the processed material (mineral aggregates, soil, concrete, asphalt mixture).

Consequently, the linear viscous model with the constant *c* highlights only the internal energy losses, that is the energy dissipation during the vibratory process without taking into account the frictions in the mechanical couplings of the supporting bodies (bearings, supports).

For the linear - viscous element with constant *c* and dissipative viscous force $F_v = F_d$, we have $F_d = c\dot{x}$, and the infinitesimal mechanical work is $dL_d = F_d dx = c\dot{x}^2 dt$. In this case, the dissipated energy $W_d(t)$ results from $dW_d(t) = dL_d$, thus

$$W_d(t) = \int d\, L_d \qquad (1)$$

or

$$W_d(t) = \int c\dot{x}^2\, dt \qquad (2)$$

where
$dW_d dt = c\dot{x}^2 dt$, iar $\dot{x} = A\omega\cos(\omega t - \varphi)$.
Thus, we have:

$$dW_d(t) = c\omega^2 A^2 cos^2(\omega t - \varphi)dt$$

It emerges that relation (2) can be set as:

$$W_d(t) = c\omega^2 A^2 \int_0 cos^2(\omega t - \varphi)dt$$

or

$$W_d(t) = \frac{1}{2} c\omega^2 A^2 \int_0 [1 + cos2(\omega t - \varphi)]dt$$

from where

$$W_d(t) = \frac{1}{2} c\omega^2 A^2 \left[t + \frac{1}{2\omega}\sin(2\omega t - 2\varphi)\right] \quad (3)$$

According to interval 0-t, we have:

$$W_d = \frac{1}{4} c\omega A^2 [2\omega t + \sin(2\omega t - 2\varphi) + sin2\varphi] \ (4)$$

For $t = T$ from the relations (4), we obtain:

$$W_d(T) = \frac{1}{4} c\omega A^2 2\omega T = \pi c\omega A^2 \qquad (5)$$

The graphical representation of the instantaneous dissipated energy, from (4), in relation to time, is given in image 1, where it was denoted by $W^* = \frac{\pi c\omega A^2}{2}$ for $t^* = T/2$.

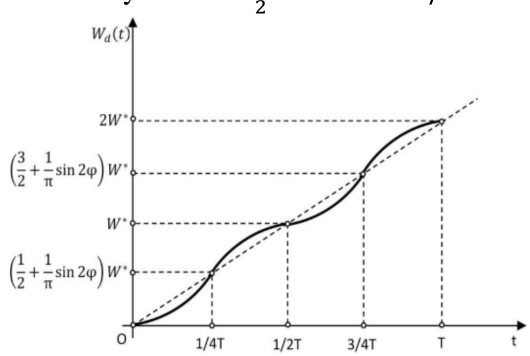

Fig 1. Variation of instant dissipated energy according to time

At resonance

$$W_d^{rez}(t) = c\omega A^2 \left[\frac{1}{2}\omega t - \frac{1}{4}sin2\omega t\right] \quad (6)$$

with representation in fig 2.

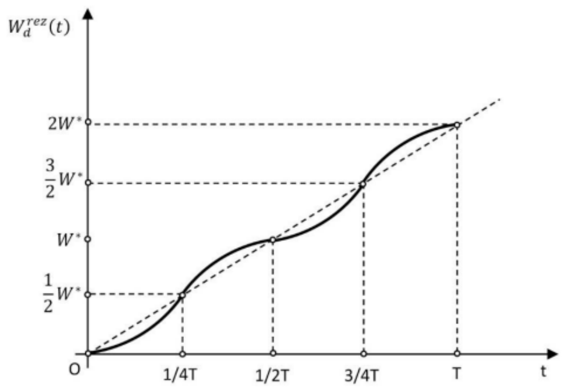

Fig. 2. Variation of instantaneous resonance energy related to time

2.2 Average dissipated energy per cycle

Value of average dissipated energy $\langle W_d \rangle = W_d^{med}$ may be calculated as follows:

$$\langle W_d \rangle = W_d^{med} = \frac{1}{T}\int_0^T W_d\,(t)dt$$

or

$$\langle W_d \rangle = W_d^{med} = \frac{1}{T}\frac{1}{2}c\omega^2 A^2 \int_0^T \left[t + \frac{1}{2\omega}\sin\,(2\omega t - 2\varphi)\right] dt$$

From where

$$\langle W_d \rangle = W_d^{med} = \frac{1}{2T}c\omega^2 A^2 \left[\frac{t^2}{2} - \frac{1}{2\omega}\frac{1}{2\omega}\cos\,(2\omega t - 2\varphi\right]_0^T$$

In the end we obtain

$$\langle W_d \rangle = W_d^{med} = \frac{1}{2T}c\omega^2 A^2 \frac{T^2}{2}$$

Or

$$\langle W_d \rangle = W_d^{med} = \frac{1}{2T}c\omega^2 A^2 \omega T = \frac{1}{2}2\pi c\omega A^2$$

Therefore, the expression of the dissipated energy on cycle is:

$$\langle W_d \rangle = W_d^{med} = \pi c\omega A^2 \quad (7)$$

2.3 Maximum dissipated energy

Maximum value of the dissipated energy W_d^{max} emerges from condition

$$\frac{dW_d(t)}{dt} = 0 \text{ or } W_d(t) = 0,$$

From where we have

$$\dot{W}_d(t) = \frac{1}{2}c\omega^2 A^2[1 + \cos(2\omega t - 2\varphi)] = 0$$

That is for $t^* = \frac{1}{\omega}(\varphi + \frac{\pi}{2})$

Emerges

$$W_d(t^*) = W_d^{max} = \frac{1}{4}c\omega A^2[\pi + 2\varphi + sin2\varphi] \quad (8)$$

At resonance, for $\varphi = {}^{\pi}/_{2}$, we have

$$W_{d,rez}^{max} = \frac{1}{4}c\omega A^2[2\pi + 1] \quad (9)$$

2.4 Maximum dissipated energy per cycle

For $t = T$ we have the dissipated energy for one cycle, as:

$$W_d^{ciclu} = W_d(T) = \pi c\omega A^2$$

Where by replacing amplitude $A = A(\Omega)$ we obtain:

$$W_d^{ciclu} = \pi 2\zeta m\omega_n^2\Omega \frac{F_0^2}{k^2[(1-\Omega^2)^2 + 4\zeta^2\Omega^2]}$$

or

$$W_d^{ciclu} = 2\pi\frac{F_0^2}{k}m\omega_n^2\frac{\Omega\zeta}{(1-\Omega^2)^2 + 4\zeta^2\Omega^2} \quad (10)$$

The maximum value $W_{d,max}^{ciclu}$, in relation to the variation of the system parameter ζ, is obtained from the condition

$$\frac{dW_d^{ciclu}}{d\zeta} = 0 \quad (10)$$

From where it emerges that for $\zeta_{max} = \frac{(1-\Omega^2)}{2\Omega}$, we have

$$W_{d,max}^{ciclu} = \frac{\pi}{2}\frac{F_0^2}{k}\frac{1}{1-\Omega^2} \qquad (11)$$

Where $\Omega = \frac{\omega}{\omega_n}$ is the relative pulsation.

3. DISSIPATIVE POWER

3.1 Instantaneuos dissipative (active) power

The dissipative instantaneous power P_d(t), or the active power is granted by relation:

$$P_d(t) = \dot{W}_d(t) = \frac{1}{2}c\omega^2 A^2[1 + cos2(\omega t - \varphi)] \qquad (12)$$

The graphical representation of the instantaneous dissipative power $P_d(t)$ is shown in fig. 3.

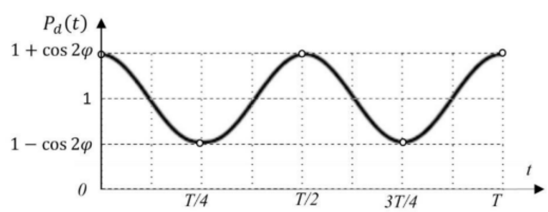

Fig 3. Variation of instantaneous dissipative power in relation to time

3.2 Instantaneous dissipative power at resonance

P_d^{rez} is reached when $\varphi = \pi/2$. Thus, from relation (12) we have:

$$P_d^{rez}(t) = \frac{1}{2}c\omega^2 A^2[1 - cos2\omega t]$$

or considering the trigonometric function

$$haversin\alpha = havsin\alpha = \frac{1 - cos2\alpha}{2}$$

we have

$$P_d^{rez}(t) = c\omega^2 A^2 havsin2\omega t \qquad (13)$$

3.3 Average dissipative power

$P_d^{med} = \langle P_d \rangle$ emerges as:

$$P_d^{med} = \langle P_d \rangle = \frac{1}{T}\int_0^T P_d dt(t) = \frac{c\omega^2 A^2}{2T}\int_0^T [1 + cos2(\omega t - \varphi)]dt$$

From where we have

$$P_d^{med} = \langle P_d \rangle = \frac{c\omega^2 A^2}{2T}\left[t + \frac{1}{2\omega}\sin(2\omega t - 2\varphi)\right]_0^T = \frac{c\omega^2 A^2}{2T}\left\{T + \frac{1}{2\omega}(-sin2\varphi + sin2\varphi\right\}$$

And in the end it is obtained

$$P_d^{med} = \langle P_d \rangle = \frac{1}{2}c\omega^2 A^2 \qquad (14)$$

By replacing $A(\Omega)$, it emerges the relation

$$P_d^{med} = \langle P_d \rangle = \omega_n \frac{F_0^2}{k}\frac{\zeta\Omega^2}{(1-\Omega^2)^2 + 4\zeta^2\Omega^2} \qquad (15)$$

At resonance, for $\Omega = 1$, we have:

$$P_{d,rez}^{med} = \langle P_d^{rez} \rangle = \omega_n \frac{F_0^2}{k}\frac{1}{4\zeta}$$

But

$2\zeta = \frac{c}{m\omega_n}, k = m\omega_n^2,$

as it emerges:

$$P_{d,rez}^{med} = \langle P_d^{rez} \rangle = \frac{1}{2}\frac{F_0^2}{c} \qquad (16)$$

3.4 Maximum dissipative power

For maximum power P_d^{max} it is set the condition $dP_d/dt = 0$ from where we have $t^* = \varphi/\omega$, and $P_d(t^*) = P_d^{max}$

Thus, it emerges

$$P_d^{max} = c\omega^2 A^2 \quad (17)$$

By replacing $c = 2\zeta m\omega_n$ and $A(\Omega)$, we have:

$$P_d^{max} = 2\omega_n \frac{F_0^2}{k} \frac{\zeta\Omega^2}{(1-\Omega^2)^2 + 4\zeta^2\Omega^2} \quad (18)$$

In this case it is found that $P_d^{max} = 2P_d^{med}$.
At resonance, for $\Omega = 1$, we obtain:

$$P_{d,rez}^{max} = 2\omega_n \frac{F_0^2}{m\omega_n^2} \frac{1}{4\zeta}$$

or

$$P_{d,rez}^{max} = \frac{F_0^2}{c} \quad (19)$$

With the specification that it is maintained the correlation

$$P_{d,rez}^{max} = 2P_{d,rez}^{med} \quad (20)$$

3.5 Dissipative power per cycle

Power per cycle P_d^{ciclu} emerges as:

$$P_d^{ciclu} = \frac{1}{T} W_d^{ciclu} = \frac{1}{T} \pi c\omega^2 A^2$$

or

$$P_d^{ciclu} = \frac{1}{2} c\omega^2 A^2 \quad (21)$$

The same as expression (14), that is:

$$P_d^{ciclu} = P_d^{med} = \langle P_d \rangle$$

At resonance, for $\omega = \omega_n$ and $c = 2\zeta m\omega$, and $A_{rez} = {}^{F_0}/_{(2k\zeta)}$, we obtain:

$$P_{d,rez}^{ciclu} = \frac{1}{2} 2\zeta m\omega\omega \frac{F_0^2}{4k^2\zeta^2}$$

From where

$$P_{d,rez}^{ciclu} = P_{d,rez}^{med} = \frac{1}{2} \frac{F_0^2}{c} \quad (22)$$

Identical with relation (16)
Resonance to the left for $\omega = \omega_r = \omega_n\sqrt{1 - 2\zeta^2}$. We have $\Omega_r^2 = 1 - 2\zeta^2$, from where $1 - \Omega_r^2 = 2\zeta^2$ and amplitude $A(\Omega_r)$ will be as:

$$A_r(\Omega_r) = \frac{F_0}{k} \frac{1}{\sqrt{(2\zeta^2)^2 + 4\zeta^2(1 - 2\zeta^2)}}$$

or

$$A_r(\Omega_r) = \frac{F_0}{k} \frac{1}{2\zeta\sqrt{1 - \zeta^2}}$$

It emerges that $P_{d,rez}^{max} = c\omega_r^2 A_r^2$ can be written as:

$$P_{d,rez}^{max} = c\omega_n^2 \Omega_r^2 A_r^2 \quad (23)$$

or

$$P_{d,rez\Omega_r}^{max} = \frac{1}{2} \frac{F_0}{c} \frac{1 - 2\zeta^2}{1 - \zeta^2} \ll P_{d,rez\Omega=1}^{max} \quad (24)$$

The function given by (23) as

$$P_{d,rez\Omega_r}^{max} = \frac{1}{2} \frac{F_0}{c} \frac{1 - 2\zeta^2}{1 - \zeta^2}$$

May be graphically represented in the physical space of the existence domain, as in image 4, that is for $\zeta \leq {}^{\sqrt{2}}/_{2}$

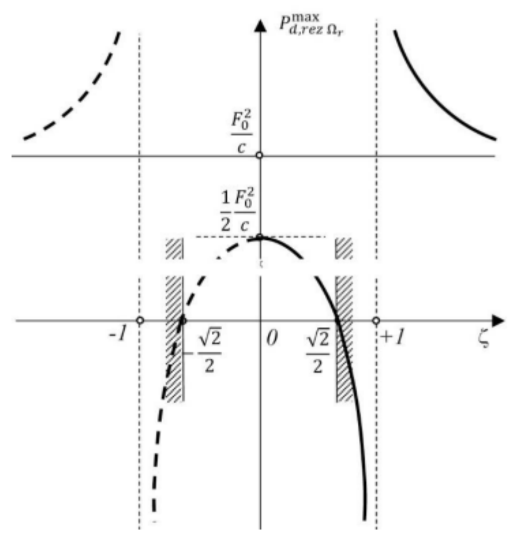

Fig. 4. Variation of the maximum dissipative power at resonance with continuous modification of ζ within the value intervals $[0, \sqrt{2}/2]$.

Comment: Amortisation $\zeta \equiv 0$, as system parameter, when c=const, can be estimated only for product $mk \rightarrow \infty$.

Maximum value of average dissipated power P_d^{med} according to variation of ζ emerges from condition [5, 6, 7]:

$$\frac{dP_d^{med}}{d\zeta} = 0$$

From where we have $(1-\Omega^2) - 4\zeta^2\Omega^2 = 0$, that is for $\zeta_{optim} = \zeta_0 = (1-\Omega^2)/2\Omega$, considering relation (15) and that $m\omega_n = c/2\zeta$, we have:

$$P_{d,\zeta_{optim}}^{med} = \omega_n \frac{{F_0}^2}{k} \frac{(1-\Omega^2)\Omega}{2[(1-\Omega^2)^2 + (1-\Omega^2)^2]}$$

or

$$P_{d,\zeta_{optim}}^{med} = \frac{F_0^2}{m\omega_n} \frac{(1-\Omega^2)\Omega}{2(1-\Omega^2)^2} \qquad (25)$$

In which we replace

$$m\omega_n = \frac{c}{2\zeta_{optim}} = \frac{c\Omega}{1-\Omega^2}$$

And obtain

$$P_{d,\zeta_{optim}}^{med} = \frac{F_0^2}{4c\Omega}(1-\Omega^2)\frac{\Omega}{1-\Omega^2}$$

or

$$P_{d,\zeta_{optim}}^{med} = \frac{1}{4}\frac{F_0^2}{c} \qquad (26)$$

The graphical representation of P_d^{med} after relation (15) is shown in image 5.

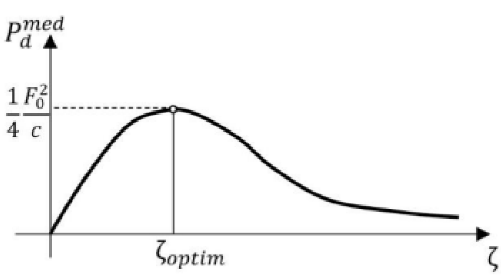

Fig 5. Variation of average dissipative power

For $\Omega = 0{,}7$, we have $\zeta_{optim} = \sqrt{2}/4 \cong 0{,}35$ and

$$P_{d,rez\Omega_r}^{max} = 0{,}86\frac{1}{2}\frac{F_0^2}{c} = 0{,}43\frac{F_0^2}{c}$$

and

$$P_{d,\zeta_{optim}}^{med} = \frac{1}{4}\frac{F_0^2}{c} = 0{,}25\frac{F_0^2}{c}$$

with representation in image 6.

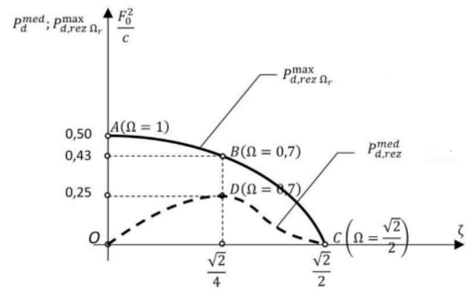

Fig 6. Variation of average dissipative power in relation to ζ

Representative cases:

a) for $\zeta = 0$ and $\Omega = 1$ we have

$P_{d,rez\Omega_r}^{max} = P_{d,rez\Omega=1}^{max} = 0{,}5\frac{F_0^2}{c}$ în $\mathrm{A}(\Omega = 1)$

and

$P_d^{med} = 0$ in $\mathrm{O}(\Omega = 1)$

b) for $\zeta_{op} = \sqrt{2}/2$ and $\Omega = 0{,}7$,

we have:

$P_{d,rez\Omega_r}^{max} = 0{,}43\frac{F_0^2}{c}$ in $B(\Omega = 0{,}7)$

and

$$P_d^{med} = 0{,}25\frac{F_0^2}{c} = \left(P_d^{med}\right)_{max}$$

In $D(\Omega = 0{,}7)$

c) for $\zeta_{op} = \sqrt{2}/2$ and $\Omega = \sqrt{2}/2$ we have

$P_{d,rez\Omega_r}^{max} = 0$ in $C(\Omega = \frac{\sqrt{2}}{2})$

and

$P_d^{med} = 0$ in $C(\Omega = \frac{\sqrt{2}}{2}$.

4. CONCLUSIONS.

Based on the analytical and numerical results obtained, the following conclusions can be drawn [1, 2, 7] :

a) the instantaneous dissipated energy is increasing in relation to time, having oscillations of the values around the evolution line;

b) the maximum energy dissipated per cycle reaches the maximum value for the modification of the dissipation parameter expressed by the amortization rate ζ_{max} according to the relative pulsation Ω in accordance with relation (11).

c) the instantaneous dissipative power is a pulsating harmonic periodic function;

d) the average dissipative power reaches a maximum for ζ_{optim} as in image 5.

e) the original results consist in the fact that for vibrating systems the evaluation of the energy dissipated and the energy required by the actuators can be carried out on a numerical basis and evidenced by time evolution diagrams.

f) the analytical results established on the basis of the adopted mathematical model highlight both the instantaneous values of the actuation energy, the dissipated energies and the synthetic parameters on the actuation capacity based on the required vibrator power as well as the power dissipated in the system in the presence of the vascular damping factors when the excitation and response vibrations are harmonic. The rise of the dissipative power curves in the resonant regime as well as the maximum dissipated power when the fraction of the critical damping changes as a result of changing technological process conditions, is shown in Figure 6.

g) the importance of analytical relations as well as specific energy parameter variation curves in a vibration processing system becomes vital when sequentially changing damping parameters. In this context, any vibration processing system can be monitored by a special sensor system so that energy optimization requirements can be realized based on the analytical relations presented in the paper.

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Evaluarea parametrilor energetici de disipare vascoasa pentru sisteme dinamice excitate in regim armonic

Rezumat*: Echipamentele tehnologice cu acțiune vibrantă, dispozitivele antiseismice ce intră în structura podurilor sau clădirilor pentru izolarea bazei, sunt evaluate pe baza conceptului de performanță funcțională și de comportare de durată în serviciu.*

În lucrare, se prezintă evaluarea nivelului de disipare a energiei pe baza elementelor de amortizare vâscoasă liniară. Amortizarea vâscoasă liniară poate fi reprezentată, fie printr-un amortizor vâscos ca parte a structurii interne a unui material deformabil cu comportament vâscoelastic liniar (elastomeri, pământuri, mase plastice, compozite) fie prin dispozitive constructive adăugate, ca amortizoare vâscoase liniare, structura modelului dinamic.

Cercetarile au evidențiat comportamentul liniar disipativ în domeniile inginerești de interes cu posibilitatea evaluării energiei disipate și a puterii disipative a materialelor puse în operă (pământuri, mixturi asfaltice, betoane).

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