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ADVANCED GEOMETRICAL MODELING OF A 2RTR SERIAL ROBOT

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Abstract: *In this study, matrix exponential functions will be utilized to formulate the direct geometric equations for a multi-body system. Matrix exponential functions will be employed to analyze a serial robot structure of the 2RTR type. In comparison to conventional algorithms, the advantages of matrix exponential functions are notable in deriving the direct geometric equations. The outcomes derived from the geometric representation will serve as initial input for kinematic modeling. This data is crucial for investigating the dynamic characteristics of any multi-body system.*

Key words: *geometrical modeling, matrix algorithm, matrix exponentials, kinematic modeling, robotics.*

1. INTRODUCTION

The methodologies for geometric modeling of industrial robots, frequently employed in academic literature, are closely linked with the inherent geometry of the robot [1]. In this context, the robot is envisioned as an assembly of rigid structures (multibody system), flawlessly characterized by dimensions and spatial configuration.

To initiate the investigation of kinematics followed by the subsequent dynamic analysis of the robot, it becomes crucial to meticulously examine its geometrical model. This procedure involves the assessment and simulation of the components, limitations, and operational conditions encompassing the entire system.

The core challenge of direct geometric modeling (DGM) can be effectively tackled through a range of techniques [1]-[6]: the location matrix algorithm, the PG-type composite operator algorithm, the DH-type composite operator algorithm, and the exponentials matrix algorithm. Amid the aforementioned alternatives through which the geometric scrutiny of the robot's structure can be executed, the application of matrix exponential functions provides notable benefits in inferring the equations related to direct geometry, as delineated in [1]-[6].

Geometric modeling harnessed with matrix exponential functions represents a more advanced approach that capitalizes on intricate mathematical concepts to clarify and regulate the robot's spatial displacement [1]. This approach empowers the portrayal of intricate motions and meticulous oversight of the robot's path.

The geometric investigation outlined in this document was performed on the suspended Epson RS4-551 robot (figure 1), acknowledged as a multi-elemental system.

Fig. 1 Epson RS4-551 robot [9]

1.1. EPSON RS4-551 ROBOT

The Epson robotic system is a SCARA-type robot, commonly employed within industrial

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automation due to its relatively compact size and minimal footprint upon integration into industrial processes.

The arm's architecture has been meticulously crafted to facilitate the placement of joint 2 beneath joint 1, enabling the entire workspace beneath the arm to be effectively utilized. Unlike alternative robot designs, it boasts a 360-degree operational workspace envelope.

These structural configurations exhibit superior swiftness, accuracy, and reproducibility in contrast to human operators, without necessitating breaks. Consequently, they are increasingly favored within industrial automation [12]. Their prevalent applications include enhancing the efficiency of Pick&Place tasks, as well as facilitating the intricacies of small-to-medium assembly procedures, fastening, precise dosing, and manipulation of diverse components throughout various phases.

The Epson robot consists of four joints, more accurately it has two rotation joints (joint 1 and joint 2) and a roto-translation joint (joint 3+4) which is capable of performing both a rotation and a translational movement on the Z axis according to Fig. 2.

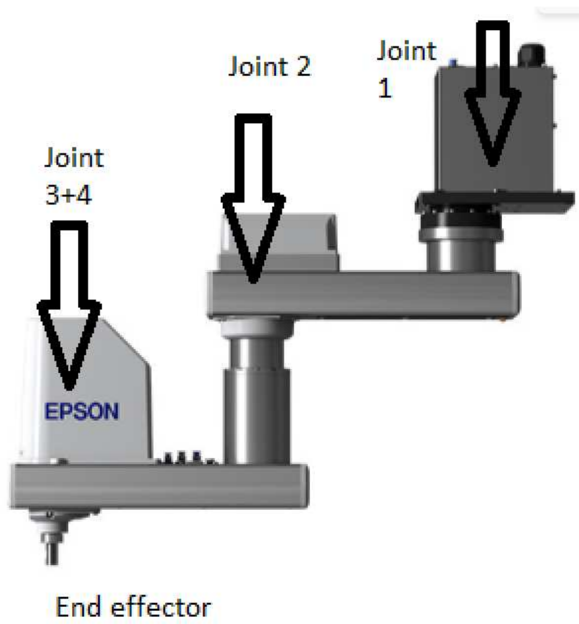

Fig. 2 Epson RS4-551 robot [9]

For the purpose of investigating the roto-translation joint, it will be treated as two distinct components: specifically, joint 3 will be identified as a translational articulation, while joint 4 will be characterized as a rotational joint. The subsequent illustration provides enhanced clarity on the roto-translation joint, which incorporates a ball screw mechanism powered by two motors. One of these motors imparts rotational motion, resulting in a reciprocating movement along the ball screw and consequently inducing linear motion in the robot arm. The second motor generates the rotational movement around the Z-axis.

Fig. 3 Epson RS4-551 robot

The roto-translational articulation provides the benefit of exceptional accuracy and dependable functionality, with the ball screw mechanism renowned for possessing these attributes. An additional advantage lies in the fact that this roto-translation joint can be effortlessly managed via the robot's software, enabling the attainment of rapid and precise motions during operational utilization.

2. THE STRUCTURE OF THE ROBOT

The kinematic schematic of the robot was crafted using SolidWorks software, aiming to ensure a comprehensible delineation of its structure, facilitating the subsequent geometric analysis.

Within the robotic system, there exist two rotational joints and a singular roto-translational joint. To facilitate the geometric investigation, the roto-translation joint is perceived as comprising two separate components: specifically, a translational joint (joint 3) and a rotational joint (joint 4), as depicted in figure 4.

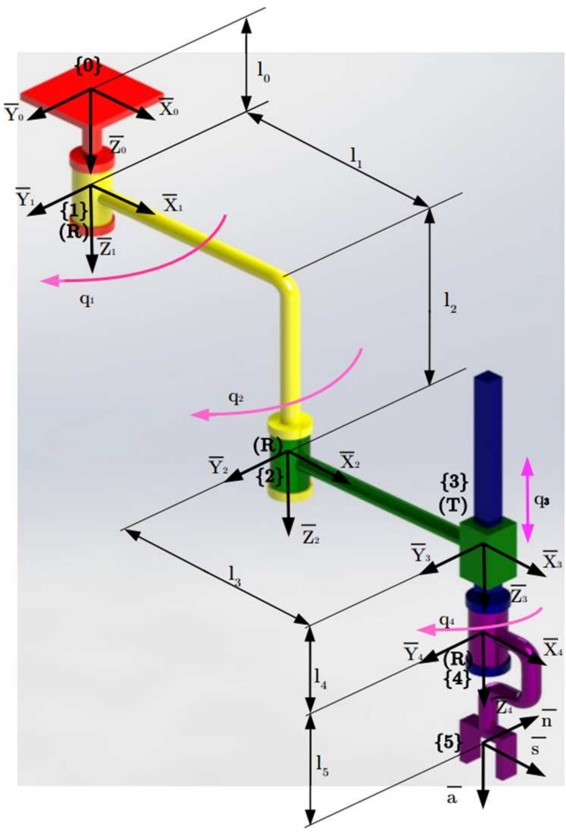

Fig. 4 The kinematic diagram of the 2RTR structure

For the purpose of the geometric exploration, direct your attention to the kinematic configuration of the 2RTR-style robot (rotation-rotation-translation-rotation) as illustrated in figure 2. Through the application of the matrix exponentials algorithm in the context of direct geometry, the equations governing direct geometric attributes are established, offering representation of the end-effector's position and orientation within Cartesian space.

By employing the matrix exponential algorithm, the exponential values of the location matrices across the $\{0\} \rightarrow \{5\}$ systems are ascertained. These matrices elucidate the spatial orientation (both position and orientation) of the end-effector concerning the $\{0\}$ coordinate system, which is affixed to the immobile base of the robot. To achieve this, a sequence of steps, as detailed in [1], is executed, and these stages are elaborated upon subsequently.

3. INPUT DATA

The matrix of the nominal geometry $M_{vn}^{(0)}$, specific to the configuration $\bar{\theta}^{(0)}$ of the robot, is given, the kinematic scheme is presented in figure 2, the matrix of the nominal geometry is completed with the skew parameters $\left\{\bar{k}_i^{(0)}; \bar{v}_i^{(0)}\right\}$ according to [1]-[7]. Thus, the new matrix corresponding to the 2RTR structure of the nominal geometry is symbolized by $M_{vn}^{(0)**}$ and is presented in table 1.

$$M_{vn}^{(0)**} \in 2RTR$$

Table 1

$1 \rightarrow 4$	Joint $\{R;T\}$	$\bar{k}_i^{(0)T}$			$\bar{p}_i^{(0)T}$			$\bar{v}_i^T = \left\{\begin{matrix} \Delta_i\left(\bar{p}_i^{(0)} \times \bar{k}_i^{(0)}\right) + \\ +(1-\Delta_i) \cdot \bar{k}_i^{(0)} \end{matrix}\right\}^T$		
1	*R*	0	0	1	0	0	l_0	0	0	0
2	*R*	0	0	1	l_1	0	z_2	0	$-l_1$	0
3	*T*	0	0	1	x_3	0	z_3	0	0	1
4	*R*	0	0	1	x_4	0	z_4	0	$-l_1-l_3$	0
5	-	-	-	-	-	-	-	-	-	-

Where: $x_3 = l_1+l_3$; $z_2 = z_3 = l_0+l_2$
$x_4 = l_1+l_3$; $z_4 = l_0+l_2+l_4$
$x_5 = l_1+l_3$;

4. DETERMINATION OF THE SCREW PARAMETERS MATRIX

The skew parameter matrix is employed to depict the geometric alteration executed by a robotic joint at a specific location within its operational area. This matrix comprises an array of skewness parameters that delineate the structure and motion characteristics of the robot.

The homogeneous transformation matrix A_i maintains the same expression for both configurations of the robot $\bar{\theta}^{(0)}$ and $\bar{\theta}$ according to [1], this matrix is determined for each joint of the robot and is necessary to further calculate the derivative of the matrix exponentials.

Thus, the matrices of skew parameters are determined for each element of the robot: (i=1); (i=2); (i=3) and (i=4), as follows:

The matrix A_i for the first joint (i=1), which is a rotation joint, is:

$$A_1 = \begin{bmatrix} \left\{ \overline{k}_1^{(0)} \times \right\} & \overline{v}_1^{(0)} \\ 0\ 0\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (1)$$

The matrix A_i for the second joint (i=2), which is a rotation joint, is:

$$A_2 = \begin{bmatrix} \left\{ \overline{k}_2^{(0)} \times \right\} & \overline{v}_2^{(0)} \\ 0\ 0\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & -l_1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (2)$$

The matrix A_i for the third joint (i=3), which is a translation joint, is:

$$A_3 = \begin{bmatrix} \left\{ \overline{k}_3^{(0)} \times \right\} & \overline{v}_3^{(0)} \\ 0\ 0\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (3)$$

The matrix A_i for the fourth joint (i=4), which is a rotation joint, is:

$$A_4 = \begin{bmatrix} \left\{ \overline{k}_4^{(0)} \times \right\} & \overline{v}_4^{(0)} \\ 0\ 0\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & -l_1 - l_3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (4)$$

5. THE EXPONENTIAL OF THE ROTATION MATRIX

Within the realm of industrial robotics, the exponentiation of the rotational matrix finds utility within control algorithms, enabling computation of the robot arm's spatial arrangement and orientation within the workspace milieu. Furthermore, this matrix holds vital significance in devising and refining the robot's motion strategies, allowing for pre-execution simulations prior to their practical implementation in the actual surroundings.

The exponential of the rotation matrix is determined according to [1]-[6] with the following expression, that is:

$$\begin{Bmatrix} e^{\left\{\overline{k}_i^{(0)} \times\right\} q_i \cdot \Delta_i} \\ R\left(\overline{k}_i\ ;\ q_i \cdot \Delta_i\right) \end{Bmatrix} = \left\{ \begin{array}{l} \begin{Bmatrix} I_3 + \left\{\overline{k}_i^{(0)} \times\right\} s\left(q_i \cdot \Delta_i\right) + \\ + \left\{\overline{k}_i^{(0)} \times\right\}^2 \cdot \left[1 - c\left(q_i \cdot \Delta_i\right)\right] \end{Bmatrix} \equiv \\ \equiv \begin{Bmatrix} I_3 \cdot c\left(q_i \cdot \Delta_i\right) + \left\{\overline{k}_i^{(0)} \times\right\} \cdot s\left(q_i \cdot \Delta_i\right) \cdot \\ \cdot \overline{k}_i^{(0)} \cdot \overline{k}_i^{(0)T} \cdot \left[1 - c\left(q_i \cdot \Delta_i\right)\right] \end{Bmatrix} \end{array} \right\} \quad (5)$$

where $\overline{k}_i^{(0)}$ is the versor that expresses the orientation of the motor rotation axis of the studied robot.

By substituting into the previous expression, are obtain expressions for each kinematic element as follows:

For (i=1), specific to the first kinetic element of the robot under study, is:

$$e^{\left\{\overline{k}_1^{(0)} \times\right\} q_1 \cdot \Delta_1} = \begin{bmatrix} cq_1 & -sq_1 & 0 \\ sq_1 & cq_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (6)$$

For (i=2), specific to the second element within the robot structure, is:

$$e^{\left\{\overline{k}_2^{(0)} \times\right\} q_2 \cdot \Delta_2} = \begin{bmatrix} cq_2 & -sq_2 & 0 \\ sq_2 & cq_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (7)$$

For (i=3), specific to the third element within the robot structure, is:

$$e^{\left\{\overline{k}_3^{(0)} \times\right\} q_3 \cdot \Delta_3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (8)$$

For (i=4), specific to the fourth element within the robot structure, is:

$$e^{\left\{\bar{k}_4^{(0)}\times\right\}q_4\cdot\Delta_4}=\begin{bmatrix} cq_4 & -sq_4 & 0 \\ sq_4 & cq_4 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{9}$$

Next, applying the definition expression of the column vector of size (3x1) denoted by b_i, according to [1]-[4]:

$$\bar{b}_i=\begin{Bmatrix} \begin{Bmatrix} l_3\cdot q_i+\left\{\bar{k}_i^{(0)}\times\right\}\left[1-c\left(q_i\cdot\Delta_i\right)\right]+ \\ +\left\{\bar{k}_i^{(0)}\times\right\}^2\left[q_i-s\left(q_i\cdot\Delta_i\right)\right] \end{Bmatrix}\cdot\bar{v}_i^{(0)}\equiv \\ \equiv\begin{Bmatrix} l_3\cdot sq_i+\left\{\bar{k}_i^{(0)}\times\right\}\left[1-c\left(q_i\cdot\Delta_i\right)\right]+ \\ +\bar{k}_i^{(0)}\cdot\bar{k}_i^{(0)T}\cdot\left[q_i-s\left(q_i\cdot\Delta_i\right)\right] \end{Bmatrix}\cdot\bar{v}_i^{(0)} \end{Bmatrix} \tag{10}$$

By substituting in the previous expression, is obtained expressions for each element, as follows:

For (i=1), specific to the first element of the robot under study, is:

$$\bar{b}_1=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{11}$$

For (i=2), specific to the second element within the robot structure, is:

$$\bar{b}_2=\begin{bmatrix} -l_1\cdot cq_2-1 \\ l_1\cdot\left(q_2-2\cdot sq_2\right) \\ 0 \end{bmatrix} \tag{12}$$

For (i=3), specific to the third element within the robot structure, is:

$$\bar{b}_3=\begin{bmatrix} 0 \\ 0 \\ q_3 \end{bmatrix} \tag{13}$$

For (i=4), specific to the fourth element within the robot structure, is:

$$\bar{b}_4=\begin{bmatrix} -\left(cq_4-1\right)\cdot\left(l_1+l_3\right) \\ \left(q_4-2\cdot sq_4\right)\cdot\left(l_1+l_3\right) \\ 0 \end{bmatrix} \tag{14}$$

6. THE MATRIX EXPONENTIALS

The matrix exponentials is determined according to [1]-[5] using the following expression:

$$e^{A_i q_i}=\exp\left(\begin{bmatrix} \left\{\bar{k}_i^{(0)}\times\right\} & \bar{v}_i^{(0)} \\ 0\ \ 0\ \ 0 & 0 \end{bmatrix}q_i\right)= \\ =\begin{bmatrix} \exp\left\{\left\{\bar{k}_i^{(0)}\times\right\}q_i\cdot\Delta_i\right\} & \bar{b}_i \\ 0\ \ 0\ \ 0 & 1 \end{bmatrix} \tag{15}$$

For (i=1), specific to the first element of the robot under study, is:

$$e^{A_1 q_1}=\begin{pmatrix} cq_1 & -sq_1 & 0 & 0 \\ sq_1 & cq_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{16}$$

For (i=2), specific to the second element within the robot structure, is:

$$e^{A_2 q_2}=\begin{pmatrix} cq_2 & -sq_2 & 0 & -l_1\cdot cq_2-1 \\ sq_2 & cq_2 & 0 & l_1\cdot\left(q_2-2\cdot sq_2\right) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{17}$$

For (i=3), specific to the third element within the robot structure, is:

$$e^{A_3 q_3}=\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & q_3 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{18}$$

For (i=4), specific to the fourth element within the robot structure, is:

$$e^{A_4 q_4}=\begin{pmatrix} cq_4 & -sq_4 & 0 & -\left(cq_4-1\right)\cdot\left(l_1+l_3\right) \\ sq_4 & cq_4 & 0 & \left(q_4-2\cdot sq_4\right)\cdot\left(l_1+l_3\right) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{19}$$

The exponentials expressions that characterize the location matrices and that express the position and orientation of the

$$R_{x0} = \exp\left\{\sum_{i=1}^{n}\left\{\bar{k}_i^{(0)}\times\right\} q_i\cdot\Delta_i\right\}\cdot R_{x0}^{(0)} \qquad (20)$$

systems $\{n\}$ and $\{n+1\}$ in relation to the system $\{0\}$, according to [1]-[4] are obtained as follows:

By substituting into R_{x0} from the previous expression, yilds the rotation matrices for each kinematic elements as follows:

For (i=1), specific to the first element of the robot, the rotation matrix is:

$$R_{10} = \begin{pmatrix} cq_1 & -sq_1 & 0 \\ sq_1 & cq_1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad (21)$$

For (i=2), specific to the second element within the robot structure, is:

$$R_{20} = \begin{pmatrix} c(q_1+q_2) & -s(q_1+q_2) & 0 \\ s(q_1+q_2) & c(q_1+q_2) & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad (22)$$

For (i=3), specific to the third element within the robot structure, is:

$$R_{30} = \begin{pmatrix} c(q_1+q_2) & -s(q_1+q_2) & 0 \\ s(q_1+q_2) & c(q_1+q_2) & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad (23)$$

For (i=4), specific to the fourth element within the robot structure, is:

$$R_{40} = \begin{pmatrix} c(q_1+q_2+q_4) & -s(q_1+q_2+q_4) & 0 \\ s(q_1+q_2+q_4) & c(q_1+q_2+q_4) & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad (24)$$

For the initial configuration $\bar{\theta}^{(0)}$, the location matrix between the systems $\{0\}\rightarrow\{5\}$ is given by the following expression:

$$T_{50}^{(0)} \equiv \begin{bmatrix} R_{50}^{(0)} & \bar{p}^{(0)} \\ 0\ 0\ 0 & 1 \end{bmatrix} \qquad (25)$$

Substituting in the previous expression results:

$$T_{50}^{(0)} = \begin{bmatrix} 0 & 1 & 0 & l_1+l_3 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & l_0+l_2+l_4+l_5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (26)$$

To determine the orientation matrix of the end-effector of the robot according to [1] the following expression is used:

$$\begin{aligned} {}_5^0[R] &= \left[\begin{pmatrix} n_x \\ n_y \\ n_z \end{pmatrix} \begin{pmatrix} s_x \\ s_y \\ s_z \end{pmatrix} \begin{pmatrix} a_x \\ a_y \\ a_z \end{pmatrix}\right] = \\ &= \left\{\exp\left\{\sum_{i=1}^{4}\left\{\bar{k}_i^{(0)}\times\right\} q_i\cdot\Delta_i\right\}\right\}\cdot R_{50}^{(0)} \end{aligned} \qquad (27)$$

Thus, the orientation matrix of the end-effector is:

$$\begin{aligned} {}_5^0[R] &= \left[\begin{pmatrix} n_x \\ n_y \\ n_z \end{pmatrix} \begin{pmatrix} s_x \\ s_y \\ s_z \end{pmatrix} \begin{pmatrix} a_x \\ a_y \\ a_z \end{pmatrix}\right] = \\ &= \left\{\begin{bmatrix} s(q_1+q_2+q_4) & c(q_1+q_2+q_4) & 0 \\ -c(q_1+q_2+q_4) & s(q_1+q_2+q_4) & 0 \\ 0 & 0 & 1 \end{bmatrix}\right\} \end{aligned} \qquad (28)$$

To determine the position matrix of the end-effector of the robot according to [1] the following expression is used:

$$\left\{\begin{aligned} &\bar{p} \equiv \bar{b}_1 + \exp\left\{\left\{\bar{k}_1^{(0)}\times\right\} q_1\cdot\Delta_1\right\}\cdot\bar{b}_2 + \\ &+\prod_{i=1}^{2}\exp\left\{\left\{\bar{k}_2^{(0)}\times\right\}\cdot q_2\cdot\Delta_2\right\}\cdot\bar{b}_3 + \\ &+\prod_{i=1}^{3}\exp\left\{\left\{\bar{k}_3^{(0)}\times\right\}\cdot q_3\cdot\Delta_3\right\}\cdot\bar{b}_4 + \\ &+\prod_{i=1}^{4}\exp\left\{\left\{\bar{k}_4^{(0)}\times\right\}\cdot q_4\cdot\Delta_4\right\}\ \bar{p}^{(0)} \end{aligned}\right\} \qquad (29)$$

Substituting into the previous expression yields the end-effector position vector for the robot:

$$\bar{p} = \begin{bmatrix} l_3\cdot c(q_1+q_2) + l_1\cdot cq_1 \\ l_3\cdot s(q_1+q_2) + l_1\cdot sq_1 \\ l_0+l_2+l_4+l_5+q_3 \end{bmatrix} \qquad (30)$$

Based on direct modeling and utilizing the orientation algorithm [1], in order to determine the autonomous orientation parameters while taking into account that each articulation of the robot solely operates along the z-axis, the subsequent matrix identity is formulated:

$$R(\overline{z};\gamma_z) = {}^{0}_{5}[R] \qquad (31)$$

Where: $R(\overline{z};\gamma_z) = \begin{pmatrix} c\gamma_z & -s\gamma_z & 0 \\ s\gamma_z & c\gamma_z & 0 \\ 0 & 0 & 1 \end{pmatrix}$

The matrix expresses rotations around the mobile z axis. Thus, the matrix identity is written developed as follows:

$$\begin{aligned} {}^{0}_{5}[R] &\equiv \begin{bmatrix} c\gamma_z & -s\gamma_z & 0 \\ s\gamma_z & c\gamma_z & 0 \\ 0 & 0 & 1 \end{bmatrix} \equiv \\ &\equiv \begin{bmatrix} s(q_1+q_2+q_4) & c(q_1+q_2+q_4) & 0 \\ -c(q_1+q_2+q_4) & s(q_1+q_2+q_4) & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{aligned} \qquad (32)$$

Given that it is only z-axis rotation, the rotation angle of the end-effector in the robot under study is:

$$\gamma_z = -\frac{\pi}{2} + (q_1 + q_2 + q_4) \qquad (33)$$

The orientation angle, is generally used to simulate and program the movements of the robot, but it is essential to control the movements of a robot and also to achieve the desired results in its practical applications.

7. CONCLUSIONS

In conclusion, matrix exponential functions present a modern alternative to classical geometric analyses when examining multibody systems. This advanced approach, driven by complex mathematical principles, offers enhanced capabilities for describing and controlling robotic motion within a spatial context. It enables the modeling of intricate maneuvers and precise motion management, thus yielding a range of valuable advantages. Notably, these matrix functions prove highly effective in deducing the direct geometric equations of the system.

Upon scrutinizing the matrix exponentials characterizing the previously established direct geometric model for the 2RTR configuration, several notable advantages come to light compared to the PG and DH type operators introduced in [1]-[6]. These benefits stem from their concise formulation, clear geometric interpretation, inherent independence, and the avoidance of reliance on specific motor pair systems.

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Modelarea geometrică avansată a structurii de robot de tipul 2RTR

În acest studiu, funcțiile exponențialelor de matrice vor fi utilizate pentru a stabili ecuațiile geometriei directe pentru un sistem multi-corp. Funcțiile exponențialelor de matrice vor fi folosite pentru a analiza o structură de robot serial de tipul 2RTR. În comparație cu algoritmii convenționali, avantajele funcțiilor exponențialelor de matrice sunt notabile în derivarea ecuațiilor geometrice directe. Rezultatele derivate din reprezentarea geometrică vor servi ca intrare inițială pentru modelarea cinematică. Aceste date sunt cruciale pentru investigarea caracteristicilor dinamice ale oricărui sistem multicorp.

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