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# MODELLING, SIMULATION AND INTEGRATION OF A 3R SERIAL ROBOT INTO A WORKING PROCESS 

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#### Abstract

The process of trajectory generation is executed through the application of polynomial interpolation functions, with their complexity defined by the constraints imposed by the technological protocol in which the robot is utilized. The central objective of this study is to deduce polynomial interpolation functions tailored to the structural configuration of the 3 R serial robot. This robotic system has been seamlessly integrated into a technological workflow encompassing the sequential operations of selection, optical scanning, precise placement, and subsequent transportation of diverse-colored finished products into a container. Commencing the procedure, the structure of the three-degree-of-freedom robot was initially modeled using Solidworks design software, followed by subsequent simulation of its operational protocol. Drawing on the acquired data, which was inputted into MATLAB, the polynomial interpolation functions were derived utilizing a matrix housing unknown coefficients, the subsequent inversion of this matrix, a column vector, coefficients pertinent to the trajectories and graphical representations meticulously detailing the generalized positional, velocity, and acceleration parameters.


Key words: process, polynomial interpolation functions, serial robot, modelling

## 1. INTRODUCTION

According to [1], industrial robots have been developed to perform a variety of tasks across a wide range of fields, such as industrial manufacturing, healthcare, research and development or surgical intervention. One of the key advantages of industrial robots is their ability to execute highly precise and complex movements, making them ideal for tasks that require a high degree of accuracy and control. This is achievable due to the large number of joints and their individual control through actuators and sensors, enabling the robots to adapt to different environments and situations. Industrial robot modeling represents the process of designing and simulating the behavior and performance of these machines in the production environment. This involves the use of specialized software to develop virtual models of the robots, which are later used to analyze and optimize various aspects of their operations in a factory. To achieve an accurate modeling of an industrial robot, several key
factors are taken into consideration. The first step involves defining the robot's geometry, including its dimensions and configuration. This includes the length of the robot's arms, the number of axes, motion limitations, and other physical characteristics, in accordance with [1].

In this particular article, the focus is on a $3 R$ robot, a manipulator robot with three joints that allow it to move in three different directions. In this case, the three degrees of freedom are achieved through rotations. This robot can be manually or automatically controlled using a programmable controller. This robot is designed to be used in a variety of industrial applications, offering flexibility and versatility, capable of manipulating larger objects and performing complex tasks with ease. It is equipped with force and torque sensors, allowing it to detect and react to human presence. The personal contribution consists of designing the 3 R robot using the Solidworks design software and implementing it in a process of selection, scanning using the Keyence optical scanner camera, and arranging
the finished products. The workflow was generated in the same program used for design, where motion parameters were defined for each joint or moving element of the robot.

## 2. DESCRIPTION OF WORKING PROCESS

According to Fig. 1, which describes the working cycle of the 3 R robot (1), it will be programmed to start from the initial zero configuration, descend towards the workpiece (3). The camera equipped with a color sensor (2) selects the first workpiece and the robot picks up the workpiece which is placed on the table (4) and places it in the container (5), this process being repeated for each workpiece separately.


Fig. 1. 3R robot used in a technological process
The working cycle of the robot is divided into seven phases and each phase into trajectory segments, according to Figure 2. By imposing initial conditions at the beginning of the phases, final conditions at the end of the phases and continuity conditions in coordinates, velocities, and accelerations at the transition between trajectory segments, the polynomial interpolation functions of the generalized coordinates will be obtained of degree 4 for the end segments and degree 3 for the intermediate segments corresponding to each phase. By deriving these with respect to time, the polynomial functions describing the generalized velocities and accelerations in the joints of the 3 R robot are obtained.

### 2.1 Phases of the technological process

In Figure 2., the robot configurations were tagged as follows: ( 0 ) - the zero configuration
of the robot; (1) - the gripping configuration of the workpiece; (i)- the release position of the workpiece in compartment $\mathrm{i}-1$. The state of the gripper device (GD) was marked with a solid line for the loaded GD and a dashed line for the released GD. The gripping phase of the workpiece (closing the gripper fingers) was denoted with GC (grip closed), while the release phase (opening the gripper fingers) was denoted with GO (grip open). Additionally, the operation periods of the joints were labeled with solid lines corresponding to the respective time segments.


Fig. 2. Cyclogram of 3 R robot
For the study of the work process, with the help of the previously presented notions, the numerical values imposed for the coordinates and the travel times of the intervals are presented in the table above.

For the phase 1, described in Fig. 3, moving from configuration (0) to configuration (1) and clamping the part, the time interval in total: T1 $=0-6.2 \mathrm{~s}$. The robot starts from configuration (0) at time 0 . In the interval $0-1.5 \mathrm{~s}$, the joints perform the following synchronized movements: joint 1 rotates the entire arm with $\mathrm{q} 1=\pi / 2$, joint 2 performs a rotation movement with $\mathrm{q} 2=\pi / 4$, so as to raise the robot arm in the plane of the part to be clamped. Joint 3 makes a rotation with $\mathrm{q} 3=2 \pi$, so that the fingers of the gripper reach the plane of the piece to be transported. The robot arrives in configuration (1). In the interval 2-6.2 s, the gripper fingers close, the part is gripped, and this is the end of phase 1.


Fig. 3. The first phase

The second phase are represented in Fig.4. where the robot moving from configuration (1) to configuration (2) and releasing the part into the container. The time interval corresponding to the second phase: $\mathrm{T} 2=6-2-14.5 \mathrm{~s}$.

Starting from the configuration (1) with the part clamped in the pliers, in the interval 6.2-8.5 s . In the interval $8.5-10.75 \mathrm{~s}$, the synchronized movements of the robot couplers are as follows: joint 1 rotates the entire arm with $\mathrm{q} 1=\pi / 3$ for positioning near the compartmentalized packaging, joint 2 causes the arm to rotate with $\mathrm{q} 2=\pi / 3$, to reach the packaging plane and joint 3 is stationary, $q 3=0$. At this stage, the robot has reached configuration (2). On the interval $10.75-14.5 \mathrm{~s}$ the fingers of the pliers are relaxed, the piece is released in compartment 1.


Fig. 4 The second phase
The 3rd phase, described from Fig. 5 shows the serial robot 3 R returning from configuration (2) to configuration (1) and grabbing the next piece. Time interval corresponding to this phase is $\mathrm{T} 3=14.5-20.5 \mathrm{~s}$.


Fig. 5. The 3rd phase
Phase 4th in Fig. 6 is related to the transition from configuration (1) to configuration (3) and
the release of part 2 into the container. Time interval of phase 4 : $\mathrm{T} 4=20.5-25.5 \mathrm{~s}$.


Fig. 6. The 4th phase
The 5th phase is described in Fig. 7. by the return to the configuration (3) to the configuration (1) and the clamping of the part 3. The time interval reaches the 5 th phase: T5 $=25.5-34 \mathrm{~s}$.


Fig. 7. The 5th phase
The 6th phase, shown in Fig. 8. is about moving the robot from configuration (1) to configuration (4) and releasing part 3 into the container. Time interval of phase 6: T6 $=34-$ 42.5 s .


Fig. 8. The 6th phase
The 7th phase describes the return of the robot from configuration (4) to configuration
(0). The time interval corresponding to the 7th phase: $\mathrm{T} 7=42.5-48 \mathrm{~s}$.

### 2.2 The general algorithm of 4-3-4 movement trajectories

As outlined in references [2-3], the investigation of the robot's motion trajectory encompasses the path between two distinct workspace points, representative of the initial and final time instances. To preempt collisions with external obstructions, a finite array of intermediary points is introduced between these pivotal points. The process of generating motion trajectories typically leverages polynomial interpolation functions, formulated symbolically as depicted:

$$
\bar{\theta}(\tau)=f^{-1}\left[{ }^{0} \bar{X}(\tau)\right](1)
$$

$$
\begin{align*}
& j=1 \rightarrow n ; \quad i=1 \rightarrow m \text { (segments of trajectory) }  \tag{2}\\
& \qquad q_{j i}(\tau)=f_{j i}^{-1}\left[{ }^{0} \bar{X}(\tau)\right] \tag{3}
\end{align*}
$$

For kinematical constraints:

where $\tau_{i}$ is the real time corresponding to position $h$

Trajectory generation is obtained using polynomial interpolation functions, the degree of which depends on the restrictions imposed by the technological process in which the robot is implemented. In the case of trajectories of the type (4-3-4), the end segments are interpolated with polynomials of degree 4 , and the intermediate segments with polynomials of degree 3 (cubic spline functions)

For intermediate segments $i=\overline{2, n-1}$, the interpolation functions are of the form:

$$
\begin{aligned}
h_{j 1}(t)=q_{j 1}(t) & \\
& =a_{j 14} t^{4}+a_{j 13} t^{3}+a_{j 12} t^{2} \\
& +a_{j 11} t+a_{j 10} t
\end{aligned}
$$

$$
\begin{align*}
& v_{j 1}(t)=\frac{\dot{q}_{j 1}(t)}{t_{1}} \\
& =\frac{1}{t_{1}}\left(4 a_{j 14} t^{3}+3 a_{j 13} t^{2}\right. \\
& \left.+2 a_{j 12} t+a_{j 11}\right) \\
& a_{j 1}(t)=\frac{\ddot{q}_{j 1}(t)}{t_{1}^{2}}=\frac{1}{t_{1}^{2}}\left(12 a_{j 14} t^{2}+6 a_{j 13} t+\right. \\
& \left.2 a_{j 12}\right) \tag{5}
\end{align*}
$$

where $h_{j 1}(t)$ normalized interpolation function the movement trajectory of the analyzed robot.

According to [2], time functions for generalized variables, velocities and accelerations are :

$$
\begin{array}{r}
\ddot{q}_{j i}(\tau)=\frac{\tau_{i}-\tau}{t_{i}} \cdot \ddot{q}_{j i}\left(\tau_{i-1}\right)+\frac{\tau-\tau_{i-1}}{t_{i}} \cdot \ddot{q}_{j i}\left(\tau_{i}\right), \\
\dot{q}_{j i}(\tau)=-\frac{\left(\tau_{i}-\tau\right)^{2}}{2 \cdot t_{i}} \cdot \ddot{q}_{j i-1}+\frac{\left(\tau-\tau_{i-1}\right)^{2}}{2 \cdot t_{i}} \\
\quad+\left(\frac{q_{j i}}{t_{i}}-\frac{t_{i j}}{6} \cdot \ddot{q}_{j i}\right)-\left(\frac{q_{j i-1}}{t_{i}}-\frac{t_{i}}{6} \cdot \ddot{q}_{j i-1}\right) .
\end{array}
$$

where, represents the real time variable.
For determination of the unknowns (generalized accelerations), the equation bellow are used:

$$
\begin{aligned}
& X_{j}=\left[\ddot{q}_{j i} ; i\right.=1 \rightarrow n-1]^{T} \\
& X_{j}=A_{j}^{-1} \cdot B_{j} ; \underset{(n-1) \times(n-1)}{A_{j}}= \\
& A_{j}(\tau) ; \operatorname{Det}\left[A_{j}(\tau)\right] \neq 0(9)
\end{aligned}
$$

where $X_{j}$ is the column vector of the coordinates operational, which expresses the position and the orientation of the system \{i\} in relation to $\{\mathrm{j}\}$ and $B_{j}$ the column vector of the free terms.

## 3. THE DETERMINATION OF POLYNOMIAL INTERPOLATION FUNCTIONS

3.1 The use of MATLAB tools for the calculation of polynomial coefficients

The following is the code fragment from the script file RRR_path.m, which contains the numerical values of the phases in the technological process of the 3R robot obtained in Solidworks and with their help the polynomial coefficients of the interpolation functions are determined, based on the call of the inter (4-3-4) function, for the data specific to couple 1 , in phase I of the technological process. In the following, you will be presented with the interfaces for the MATLAB program in which the values of the times obtained in Solidworks, shown in Fig. 9. for joint 1, in Fig. 10 for joint 2 and in Fig. 11 for joint 3.


Fig. 9 MATLAB interface for joint 1

```
4% Covia 2
Soint 1d = 2t [ititiol conditions - P[4]
```



```
n21 = p1/3; ; {rad]
v21=0%
M21-0;
init_oond = [talu, h21, v21,221],
$4 continolty condit
M22 - pi/3; I Iral]
```



```
ocost1_cond = [tas4, h22];
4ten5-10.75: \ [s]
n23 = p1/3: | [rad]
452 = tau5 - tan9;
cont2 cond = [tau5, h23);
4t final conditions - P[7]
\tan6 = 14.5; [ [9] 
M24=p1/2; + [rad!
v24-0;
024-0:
f1n_cond = {tav6, n24, v24,a24];
[h2tv, v2tv, a2tv, -]-R_ joint_434 (joint_1d, intt_cond, cont1_cond, cont2_cond, tin_cond, trtep):
```

Fig. 10 MATLAB interface for joint 2

## ${ }^{45}$ Cupla 3

\$ont_iditial conditions - Pf
4 initial condit
thaus $-6.2 F:+10$
$\begin{array}{ll}\text { nan } & 0 ; 2 ; 1 \text { (rod) } \\ \text { n31 }\end{array}$
Y)
v31 $=0 ;$
a31 $=0 ;$
init_cond = (tau3, b31, v31, a31):
It continnity conditions - P[5]


cont1_cond $=[$ tau4, n32 $] ;$
4t continalty conditions - P[6]
${ }^{\text {tenus }}=20.75 ;+[\mathrm{B}]$
${ }^{\mathrm{n} 33}=0 ; 1$ [rad]

coant2 coond $=[$ teaus, h33];

h34-0; 1 (tad)
tiss - cauc - taus
v34-0;
a34 $=0$ :
fin_oond $=(t a v 6$, h34, v34 ,a34) $;$
[h3stv, v3ty, a3tv, -] = T_joint_434 (joint_id, init_cond, cont1_cond, cont2_cond, fin_cond, tatep);

Fig. 11 MATLAB interface for joint 3

The 7 x 7 matrix belongs to a system of 7 equations with 7 unknowns and the results in MATLAB are obtained. The matrix of coefficients of the unknowns, given by numerical values, is:

$$
A_{1}=\left[\begin{array}{cccccccc}
1.0000 & 1.0000 & 0 & 0 & 0 & 0 & 0 &  \tag{10}\\
1.3043 & 1.7391 & -0.4444 & 0 & 0 & 0 & 0 \\
1.1342 & 2.2684 & 0 & -0.3951 & 0 & 0 & 0 \\
0 & 0 & 1.0000 & 1.0000 & 1.0000 & 0 & 0 & \\
0 & 0 & 0.4444 & 0.8889 & 1.3333 & -0.8000 & 1.0667 \\
0 & 0 & 0 & 0.3951 & 1.1852 & 0.4267 & -0.8533 \\
0 & 0 & 0 & 0 & 0 & 1.0000 & -1.0000 &
\end{array}\right]
$$

The inverse of the matrix of the coefficients of the unknowns is :

$$
A_{1}^{-1}=\left[\begin{array}{ccccccc}
2.7436 & -0.8551 & -0.5539 & -0.4837 & 0.2332 & 0.1458 & 0.1244 \\
-1.7436 & 0.8551 & 0.5539 & 0.4837 & -0.2332 & -0.1458 & -0.1244  \tag{11}\\
1.2291 & -1.4135 & 0.5418 & 0.4732 & -0.2281 & -0.1426 & -0.1217 \\
-2.1348 & 2.4550 & -0.9411 & 1.3887 & -0.6695 & -0.4185 & -0.3571 \\
0.9057 & -1.0415 & 0.3992 & -0.8619 & 0.8977 & 0.5611 & 0.4788 \\
0.5391 & -0.6199 & 0.2376 & -1.1083 & 1.8736 & -1.1727 & 2.9993 \\
0.5391 & -0.6199 & 0.2376 & -1.1083 & 1.8736 & -1.1727 & 1.9993
\end{array}\right]
$$

The column vector of the free terms is :

$$
B_{1}=\left[\begin{array}{c}
0  \tag{12}\\
0 \\
-5.2360 \\
0 \\
0 \\
1.3090
\end{array}\right]
$$

The polynomial coefficients are five for polynomials of the fourth degree and four for polynomials of the third degree. For the segment 1 in joint 1 unknown polynomial coefficients are:

$$
\begin{align*}
& a_{113}=2.6954 ; a_{114}=2.6954 ; \quad a_{121}=0  \tag{13}\\
& a_{122}=0 ; a_{123}=6.2832
\end{align*}
$$

For the segment 2 of $j=1$ the coefficents are:

$$
\begin{align*}
& a_{113}=5.1394 ; a_{114}=-7.7386 ; \quad a_{121}=-2.6368  \tag{14}\\
& a_{122}=6.2832 ;
\end{align*}
$$

and finally for the segment 3 are presented:

$$
\begin{align*}
& a_{113}=8.4198 ; a_{114}=9.7288 ; \quad a_{121}=0  \tag{15}\\
& a_{122}=0 ; a_{123}=2.3562 ;
\end{align*}
$$

For the joint 2 of 3 R robot, $\mathrm{j}=2$, the column vector of the free terms are:

$$
B_{1}=\left[\begin{array}{c}
0  \tag{16}\\
0 \\
0 \\
0 \\
0 \\
0.5326
\end{array}\right]
$$

The unknown polynomial coefficients for the couple $\mathrm{j}=2$ are obtained and coefficients for the first segment:
$a_{113}=-0.0651 ; a_{114}=0.0651 ; \quad a_{121}=0$
$a_{122}=0 ; a_{123}=1.0472$;
Coefficients fot the second segment:
$a_{113}=0.2507 ; a_{114}=-0.1870 ; a_{121}=-0.0637$
$a_{122}=1.0472$
Coefficients fot the third segment:
$a_{113}=1.0468 ; a_{114}=1.5704 ; a_{121}=0$
$a_{122}=0 ; a_{123}=1.5708$
The inverse of the matrix of the coefficients of the unknowns for joint 2:

$$
A_{1}^{-1}=\left[\begin{array}{ccccccc}
2.7436 & -0.8551 & -0.5539 & -0.4837 & 0.2332 & 0.1458 & 0.1244  \tag{20}\\
-1.7436 & 0.8551 & 0.5539 & 0.4837 & -0.2332 & -0.1458 & -0.1244 \\
1.2291 & -1.4135 & 0.5418 & 0.4732 & -0.2281 & -0.1426 & -0.1217 \\
-2.1348 & 2.4550 & -0.9411 & 1.3887 & -0.6695 & -0.4185 & -0.3571 \\
0.9057 & -1.0415 & 0.3992 & -0.8619 & 0.8977 & 0.5611 & 0.4788 \\
0.5391 & -0.6199 & 0.2376 & -1.1083 & 1.8736 & -1.1727 & 2.9993 \\
0.5391 & -0.6199 & 0.2376 & -1.1083 & 1.8736 & -1.1727 & 1.9993
\end{array}\right]
$$

The column vector of the free terms and polynomial coefficients for the 3 segments is 0 to joint 3 .

### 3.2 The graphical representation of kinematical parameters from MATLAB

Using as input data, the running time for trajectory, and the coordinate at the beginning and end of the sequence, the expressions of the generalized coordinates and kinematic parameters, are represented :

For the $\mathrm{j}=-1$, graphic for generalized coordinate is:


Fig. 12 Generalized coordinate joint 1

For the $\mathrm{j}=1$., joint graphic for generalized acceleration is:


Fig. 13 Generalized acceleration joint 1 For the $\mathrm{j}=1$, joint graphic for generalized velocity is a parable:


Fig. 14. Generalized velocity joint 1
For the second joint graphic for generalized coordinate is:


Fig. 15. Generalized coordinate joint 2

For the second joint graphic for generalized acceleration in real time is :


Fig. 16. Generalized acceleration joint 2
For the second joint graphic for generalized velocity in real time is :


Fig. 17. Generalized velocity joint 2
The results obtained in Solidworks were of great importance, because based on them the polynomial interpolation functions and the graphs for the generalized positions, velocities and accelerations were obtained in real time.

## 4. CONCLUSION

The objectives of this paper is the integration of the 3 R robot in a work process in Solidworks and the determination of polynomial interpolation functions with the help of the MATLAB program. Also, graphs were obtained for positions, velocities, generalized accelerations for rotational joints. These results will be used to obtain the driving joints graphs

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## Modelarea, simularea și integrarea unui robot serial 3R într-un proces de lucru

Rezumat: Scopul acestei lucrări este de a integra robotul serial 3R într-un proces de lucru și de a obține funcț̦ii polinomiale de interpolare. Generarea traiectoriei se obține utilizând funcții polinomiale de interpolare, al căror grad depinde de restricțiile impuse de procesul tehnologic în care este implementat robotul. Acest robot a fost integrat într-un proces tehnologic de selectare, scanare ș̦i plasare a produselor finite de diferite culori și transportarea acestora într-un container.Pentru inceput s-a modelat structura robotului cu trei grade de libertate în Solidworks, iar mai apoi s-a facut simularea procesului de lucru. Pe baza rezultatelor obtinute care s-au introdus in MATLAB, s-au obtinut functiile polinomiale de interpolare prin matricea coeficientilor necunoscutelor, inversa matricei, vectorul coloana, coeficientii pentru cele trei segmente de traiectorie si grafice pentru pozitii, viteze, acceleratii generalizate .

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