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IMPACT OF CRACK-INDUCED DAMAGE ON THE DYNAMIC STABILITY OF A FLUID-TRANSPORTING PIPE SUPPORTED BY A WINKLER ELASTIC FOUNDATION

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Abstract: *The Pipeline transport proves to be a highly efficient means of transporting fluids over long distances. Some of its benefits include continuity of transport, substantial volume capacity, lower energy costs and provides large environmental benefits. Pipelines damage could have negative effects on the economy, the environment and cause health and safety issues for the population. Therefore, the safety issues garner significant attention in both industry and science. The current study explores the dynamic stability of a cracked simply supported pipe. Throughout its entire length the pipe rests on a Winkler elastic foundation. The problem is numerically approached through the Galerkin method. The study provides insights into how both the crack and the parameters of the elastic foundation affect the stability of the system.*

Key words: *pipe, crack, fluid, critical velocity, Winkler elastic foundation, dynamic stability.*

1. INTRODUCTION

In recent years, considerable research have been devoted to investigating the interaction between fluid and structures. The dynamic behavior of pipes conveying fluids represents a fundamental problem in this particular domain. The oscillatory behavior of the pipe is contingent upon both the mass and velocity of the conveyed fluid. The system exhibits stability when flow velocity remains below a specific threshold known as the critical flow velocity. Engineers are responsible for ensuring the safety, reliability and proper functionality of structures over their entire lifespan. However, the presence of a defect in these structures can result in their failure. The most common type of damage found in structures is the presence of cracks. The presence of cracks results in a reduction in the stiffness of the structural element, thereby decreasing its natural frequencies and inducing alterations in its mode shapes. This is why the detection of cracks plays a crucial role in the ongoing structural strength monitoring process during the structure's entire lifespan.

In their study, Chondos and Dimarogonas [1] explored the influence of crack depth on the dynamic characteristics of a cantilevered beam. Their findings demonstrated that an increase in crack depth corresponds to a decrease in the natural frequency of the beam.

Ostachowicz and Krawczuk [2] investigated how both the position and depth of cracks impact the dynamic characteristics of a cantilever beam.

Yoon and Son [3] conducted a study on the dynamic response of a fluid-conveying pipe with a crack, which is fixed at both ends, in the presence of a moving mass. Once more, Yoon and Son [4] explored the same issue, this time employing Timoshenko beam theory.

Eslami et al. [5], delved into the impact of an open crack and the shape of the flow velocity profile within the pipe on the dynamic characteristics of a fluid-conveying pipe immersed in a viscoelastic medium.

The results suggest that the velocity profile arising from the viscosity of real fluids significantly influences the critical flow velocity in both unblemished and cracked pipes.

The utilization of alterations in natural frequencies caused by structural cracks has been

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implemented as a means of detecting cracks. In reference [6], it is demonstrated that the depth of the crack exerts only a minimal effect on the natural frequency of the structure. Consequently, alternative approaches, such as the exploration of harmonic response analysis, have been pursued for the purpose of crack detection [7].

The objective of this study is to assess the impact of an open crack on the critical velocity of a simply supported fluid-conveying pipe on Winkler elastic foundation.

2. PROBLEM FORMULATION

The dynamic stability of a fluid-conveying pipe is investigated in this study using the Euler-Bernoulli beam theory. The static scheme of the pipe is shown in Fig.1. The open edge crack is modeled as a rotational spring [8] (Fig.2).

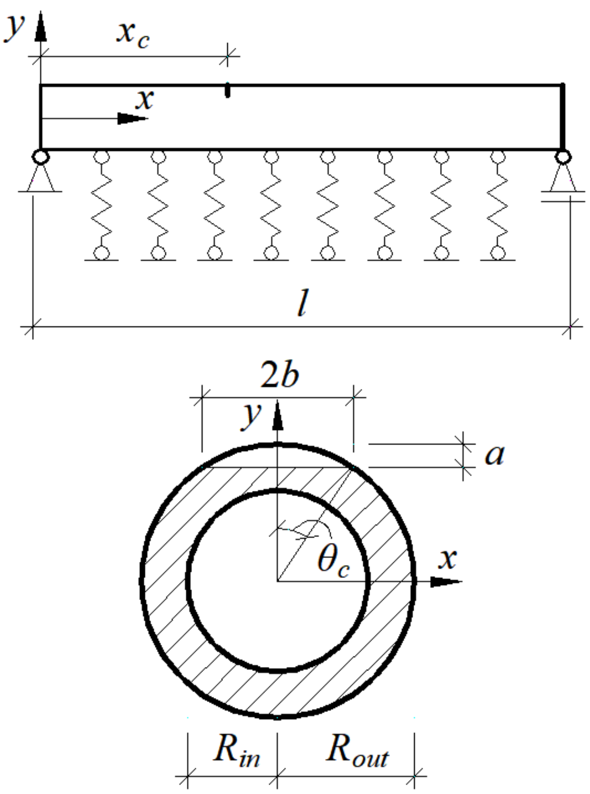

Fig. 1. Static scheme and cross-section of the pipe under investigation

The pipe is divided into two sections: to the left and to the right of the crack.

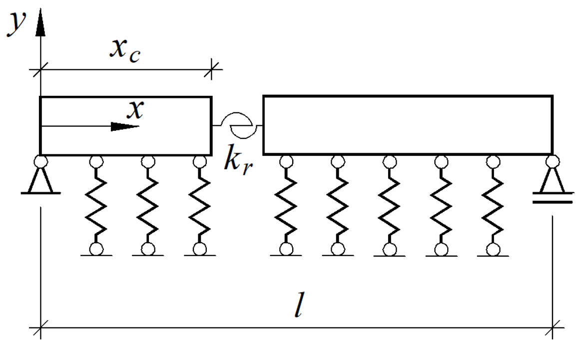

Fig. 2. Mechanical model of the crack

k_r is the stiffness of the rotational spring.

The transverse oscillations of a straight pipe, which transports inviscid fluids and lies on a Winkler elastic foundation, are described by the following differential equation.

$$EI\frac{\partial^4 w}{\partial x^4}+m_f V^2\frac{\partial^2 w}{\partial x^2}+2m_f V\frac{\partial^2 w}{\partial x \partial t}+$$

$$+\left(m_f+m_p\right)\frac{\partial^2 w}{\partial t^2}+k\,w=0\,. \qquad (1)$$

In (1) $w(x,t)$ is the function of the lateral displacment of the axis of the pipe. The remaining symbols in (1) are: the time t, the rigidity of the cross section of the pipe EI, the velocity of the flowing fluid V, the rigidity of the Winkler's foundation k, the mass of the pipe per unit length m_p and the mass of the conveyed fluid per unit length of the pipe m_f.

For the sake of simplicity, we are introducing the following dimensionless parameters.

$$u=lV\sqrt{\frac{m_f}{EI}}\,;\; \beta=\frac{m_f}{m_f+m_p}\,;\; \xi=\frac{x}{l}\,;$$

$$\xi_c=\frac{x_c}{l}\,;\; \tau=\frac{t}{l^2}\sqrt{\frac{EI}{m_f+m_p}}\,;$$

$$\bar{k}_r=\frac{k_r l}{EI}\,;\; \bar{k}=\frac{\kappa l^4}{EI}\,. \qquad (2)$$

The non-dimensional equations that govern the lateral vibrations in the two sections of the pipe are as follows.

$$\eta_n^{IV} + u^2\eta_n^{II} + 2u\sqrt{\beta}\dot{\eta}_n^{I} + \ddot{\eta}_n + \overline{k}\eta_n = 0,$$
$$n = 1{,}2. \quad (3)$$

In equation (3) and the subsequent expressions, dots represent derivatives with respect to the dimensionless time, while derivatives with respect to ξ are indicated as primes.

The solution of the differential equation (3) is approximated using the spectral Galerkin method. The expression for the solution in each segment of the pipe is as follows:

$$\eta_n(\xi,\tau) = \sum_{i=1}^{m} W_{ni}(\xi) q_i(t), n = 1{,}2. \quad (4)$$

where

$q_i(\tau)$ are functions that are not specified or known

$W_{ni}(\xi)$ are fundamental functions that satisfy the boundary conditions. In this study, functions describing the i-th eigenform of a simply supported beam with length l are employed for these functions.

The boundary conditions for the system depicted in Fig.1 are as follows:

- at the left end of the pipe

$$W_{1i}(0) = 0 \text{ and } W_{1i}^{II}(0) = 0. \quad (5)$$

- at the right end of the pipe

$$W_{2i}(1) = 0 \text{ and } W_{2i}^{II}(1) = 0. \quad (6)$$

- at the cracked cross-section of the pipe [9]

$$\begin{gathered} W_{1i}(\xi_c) = W_{2i}(\xi_c) \\ W_{1i}^{II}(\xi_c) = W_{2i}^{II}(\xi_c), \\ W_{1i}^{III}(\xi_c) = W_{2i}^{III}(\xi_c), \\ \left[W_{1i}^{I}(\xi_c) - W_{2i}^{I}(\xi_c)\right]\overline{k}_r = W_{2i}^{II}(\xi_c) \end{gathered} \quad (7)$$

By inserting equation (3) in equation (4), one obtains:

$$|M|\ddot{q} + |C|\dot{q} + |K|q = 0. \quad (8)$$

The components of the matrices in equation (8) are:

$$M_{ij} = \sum_{n=1}^{2} \int_0^{\xi_n} W_{ni}(\xi) W_{nj}(\xi) d\xi. \quad (9)$$

$$C_{ij} = 2u\sqrt{\beta} \sum_{n=1}^{2} \int_0^{\xi_n} W_{ni}(\xi) W_{nj}^{I}(\xi) d\xi. \quad (10)$$

$$K_{ij} = \sum_{n=1}^{2} \int_0^{\xi_n} W_{ni}(\xi) W_{nj}^{IV}(\xi) d\xi + .$$
$$+ \sum_{n=1}^{2} \int_0^{\xi_n} W_{ni}(\xi) W_{nj}^{II}(\xi) d\xi. \quad (11)$$

The general solution for the system (8) is obtained by considering the roots of the characteristic equation.

$$\det|\Delta| = 0. \quad (12)$$

The elements of the matrix in (12) are defined as follows:

$$\Delta_{ij} = \lambda^2 M_{ij} + \lambda C_{ij} + K_{ij}. \quad (13)$$

Conclusions about the system's stability can be drawn from the obtained roots. If the characteristic equation's roots possess negative real parts, the system is regarded as stable.

As the roots of the equation (13) are influenced by the system's parameters, the following procedure could be employed to determine the critical velocity of the conveyed fluid. All system parameters are kept constant except fluid velocity. The fluid velocity is varied from zero to its critical value, at which point one or more roots of equation (13) change the sign of their real part from negative to positive.

3. MODELING OF THE CRACK

The study assumes that the Euler-Bernoulli beam's bending vibrations occur within the plane $x - y$ (as depicted in Fig. 1), which also serves as a plane of symmetry for the cross-section. The crack is considered to be open.

The calculation of the local flexibility in the presence of a crack is performed using Castigliano's theorem [5].

$$c = \frac{\partial^2 U}{\partial M^2} = \frac{1-\nu^2}{E} \int_{-b}^{b}\int_{0}^{a} \frac{\partial^2 \left(K_I^2\right)}{\partial M^2} dx\,dy \quad (14)$$

In (14) U is the potential energy, E is the Young's module, ν is the Poison's ratio and K_I is referred as the stress intensity factor of bending. M is the bending moment in the cracked cross-section. The crack dimensions are a and b (Fig.1).

$$K_I = \frac{M}{\pi R^2 t_p} \sqrt{\pi R \theta_c}\, F(\theta_c) \quad (15)$$

In (15) θ_c is the half central angle of the crack and t_p is the thicknes of the cross-section (Fig.1). The remaining parameters in equation (15) are computed using the following formulas [9]:

$$R = 0{,}5\left(R_{in} + R_{out}\right) \quad (16)$$

$$F(\theta_c) = 1 + A_t \left[4{,}59 \left(\frac{\theta_c}{\pi} \right)^{\frac{3}{2}} + 2{,}64 \left(\frac{\theta_c}{\pi} \right)^{4{,}24} \right] \quad (17)$$

The determination of A_t is derived from the equations:

$$A_t = \sqrt[4]{\frac{1}{8}\frac{R}{t_p} - \frac{1}{4}} \quad \text{for} \quad 5 \le \frac{R}{t_p} \le 10 \quad (18)$$

$$A_t = \sqrt[4]{\frac{2}{5}\frac{R}{t_p} - 3} \quad \text{for} \quad 10 \le \frac{R}{t_p} \le 20 \quad (19)$$

In (16) R_{in} and R_{out} are respectively the inner and the outer radii of the cross-section (Fig.1).

4. RESULTS AND DISCUSSION

Numerical studies have been carried out for the pipe shown in figure 1.

The dimensions and material properties of the pipe are as follows: inner radius of the cross-section $R_{in} = 0.012\ m$, outer radius of the cross-section $R_{out} = 0.014\ m$, modulus of elasticity $E = 210\ GPa$, density of the flowing fluid - $\rho = 1000\,kN/m^3$, density of the material of the pipe - $\rho = 7800\,kN/m^3$, dimensions of the crack - $a = 1\,mm$, and $b = 5\,mm$, coordinate determining the location of the crack along the axis of the pipe $x_c = 1\,m$.

The Finite Element Method (FEM) was employed to derive the fundamental functions $W_{ni}(\xi)$, with these functions being the eigentfunctions for the pipe containing stationary fluid ($V = 0$). In the present study, the first 16 eigenmodes were used in the calculations.

Conclusions about the stability of the system can be deduced from the roots derived from the characteristic equation (12). The stability of the system is determined by the negativity of the real parts of all roots; instability arises if one or more roots have positive real parts.

Moreover, the system approaches instability if any roots of the characteristic equation have real parts equal to zero, and the fluid velocity corresponding to this condition is known as the critical fluid velocity V_{cr}.

These roots are affected by every parameter in the system. By keeping all parameters constant except for the conveyed fluid velocity V, one can compute the associated critical velocity V_{cr}.

The critical velocities for the pipe illustrated in Fig. 1 are evaluated across different values of the Winkler elastic foundation's rigidity.

The calculations are conducted for both the damaged and undamaged pipe. The results presented in Fig.3 depict the relationship between the critical velocity and the rigidity of the elastic foundation.

Based on results shown in Fig.3, conclusions can be drawn regarding how the stiffness of the elastic foundation influences the stability of the two examined pipes.

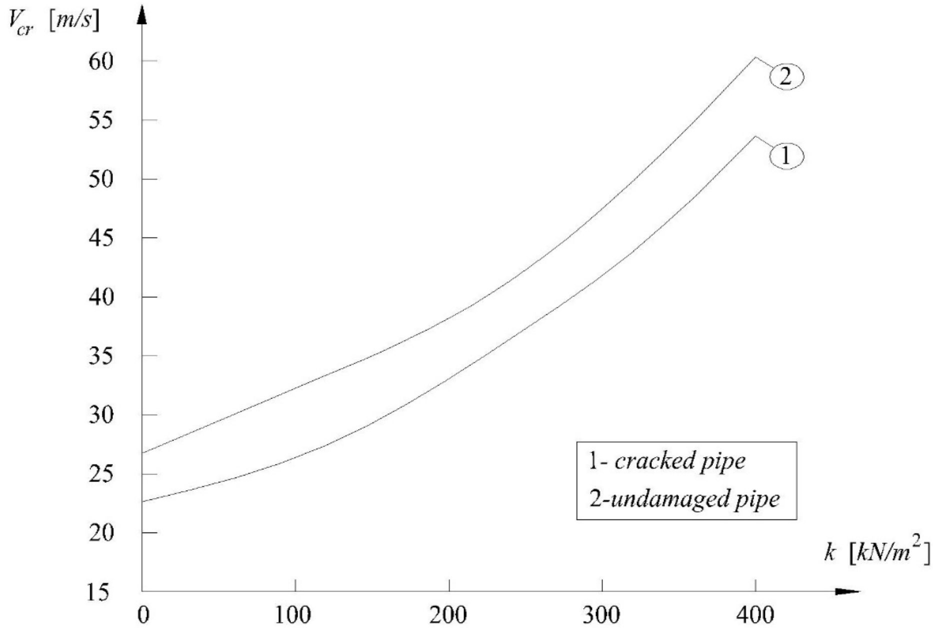

Fig. 3. *Critical velocity versus the rigidity of the Winkler elastic foundation*

5. CONCLUSION

Structural damage in the form of cracks is one of the most common issues encountered in buildings and other structures. When a structure undergoes cracking, its stiffness diminishes, leading to a subsequent reduction in natural frequencies and a shift in natural vibration modes.

The Winkler elastic foundation is commonly used as a model in geotechnical studies. It assumes that the deformation at any point on the surface of an elastic medium is directly proportional to the applied load at that specific point and is not influenced by the loads applied at other points on the surface.

This research aims to evaluate how a Winkler elastic foundation influences the dynamic stability of a cracked fluid-conveying pipe.

The cracked pipe is separated to two sections joined by an elastic rotational spring at the crack's location. The rigidity of this spring is determined using Castigliano's theorem, and it relies on both the geometry of the cross-section of the pipe and the severity of the crack.

The results obtained indicate that the Winkler foundation contributes to stabilizing the pipe—increasing the foundation's rigidity results in an increase in critical velocity. Conversely, the crack destabilizes the system, leading to a decrease in critical velocity.

8. REFERENCES

References have to include whole name and the first letter of the surname. All references will be directly cited in the body of the paper by its number.

[1] Chondros, T., Dimarogonas, D.. *Identification of crack in welded joints of complex structures*, Journal of Sound and Vibration, 69, pp. 531-538, 1980.

[2] Ostachowiicz, M., Krawczuk, M., *Analysis of the effect of cracks on the natural frequencies of a cantilever beam,* Journal of Sound and Vibration, 150, pp. 191-201, 1991.

[3] Yooh, Hi., Son, Is., *Dynamic behavior of cracked simply supported pipe conveying fluid with moving mass*, Journal of Sound and Vibration, 292, pp. 941-953, 2006.

[4] Yooh, Hi., Son, Is., *Dynamic behavior of cracked pipe conveying fluid with moving mass based on Timoshenko beam theory*, KSME International Journal, 18, pp. 2216-2224, 2004.

[5] Eslami, G., et al., *Effect of open crack on vibration behavior of a fluid-conveying pipe embedded in a visco-elastic medium*, Latin

American Journal of Solids and Structures, 13, pp. 136-154, 2016.

[6] Han, Y., et al., *A method for crack detection in structures using piezoelectric sensors and actuators*, Proceedings of the international conference on MEMS, NANO and smart systems ICMENS'05, 2005.

[7] Kim, MB., Zhao, M., *Study on crack detection of beam using harmonic responses*, Proceedings of the international conference on intelligent mechatronics and automation, Chegdu, China, 2004.

[8] Sadettin, O., *Analysis of free and forced vibration of a cracked cantilever beam*, NDT&E International, 40, pp. 443-450, 2007.

[9] In-Soo, S., Sang-Pill, L., Jin-Kyung, L., *Dynamic behavior of forced vibration of elastically restrained pipe conveying fluid with crack and concentrated mass*, 9-th international Conference on Fracture & Strength of Solids, Jeju, Korea, 2013.

EFECTUL DAUNELOR PRIN CRĂPARE ASUPRA STABILITĂȚII DINAMICE A UNEI ȚEVI DE TRANSPORT FLUID AȘEZATE PE O FUNDATIE ELASTICA WINKLER

Rezumat: Transportul prin conducte se dovedește a fi un mijloc extrem de eficient pentru transportul fluidelor pe distanțe lungi. Printre avantajele sale se numără continuitatea transportului, capacitatea substanțială de volum, costuri reduse cu energia și beneficii semnificative pentru mediu. Daunele la conducte ar putea avea efecte negative asupra economiei, mediului și pot provoca probleme de sănătate și siguranță pentru populație. Prin urmare, aspectele legate de siguranță atrag o atenție semnificativă atât în industrie, cât și în știință. Studiul curent explorează stabilitatea dinamică a unei conducte crăpate susținute simplu. Pe întreaga sa lungime, conducta se sprijină pe o fundație elastică de tip Winkler. Problema este abordată numeric prin metoda Galerkin. Studiul oferă perspectiva asupra modului în care atât crăpătura, cât și parametrii fundației elastice influențează stabilitatea sistemului.

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