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NUMERICAL AND EXPERIMENTAL STIFFNESS EVALUATION OF QUILLING INSPIRED STRUCTURES

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Abstract: *The present paper is a study of mechanical behavior done upon a range of structures inspired by the paper art of quilling. All the models are similar in geometry, having the same type of parameters and all are variations of one model, that is used as a base for comparison. The purpose is to understand how the stiffness of the structure is influenced by the geometrical parameters of the models. For this study have been chosen four geometrical parameters to be modified, corresponding stiffness being computed as the displacements under a compressive force based on numerical simulations and experimental evaluation of 3D printed structures.*
Key words: *Quilling Structures, Stiffness, 3D printed models, numerical simulation*

1. INTRODUCTION

The starting point for the current study has been the paper art of quilling. The challenge was to create a pattern of a metamaterial inspired by a paper quilling design. The decision to look into this particular area of art was made upon observations done on origami-inspired metamaterials [1-3].

In the latest 10 years origami tessellated patterns have become subjects of interest in the field. Traditional origami consists of folding a sheet of paper into a sculpture without cutting, stretching, gluing and taping [1]. From here appeared the idea that another material can take the shape of an origami pattern, by manufacturing complex 3D forms by out of plane deformation [1]. Researchers inspired by origami art began to study and discover new ways of applying this paper folding art to different structures and obtaining interesting results. Patterns such as Miura-ori and Ron Rech are the most representative of origami metamaterials [2-3].

Miura-ori origami pattern is a design composed of a series of chevron-shaped folds [2]. The kinematics of the folding depends on the geometry and is scale-independent [1].

Ron Resch origami metamaterials are star-like patterns joined together [3]. This type of patterns gives the material significant abilities of geometry forming, shape flexibility and potential mechanical performance. Due to these abilities the structures present load dissipation, damping and can perform high deformation [2]. Another interesting research was conducted on origami composed of folded ribbons that can be snapped together to assemble extruded polyhedral [3]. This type of paper folding led to creation of transformable metamaterials [4]. Further on, the experiments continued to the point of creating metamaterials mechanisms and 3D objects by combining the patterns with polyhedrons shapes [5]. Currently these metamaterials are used in soft robotics, aerospace technology, actuators, biomedical devices, furniture and even planning to develop self-healing machines and devices [6, 7].

While the science is looking into the direction of origami and kirigami [8-10], the art of quilling is yet to be explored. The art of quilling, dates back to somewhere around 13th Century and is a form of art that involves the use of paper strips that are rolled and glued together to obtain decorative designs [11]. The art of quilling is able to provide curved shapes that have the

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ability to auto-sustain in a tessellated pattern or in 3D multilayered metamaterials [12]. Several studies in recent years presented mechanical behavior and new applications of origami-based structures and metamaterials [13-18]. This is the reason why quilling art has been chosen as an inspiration for the current study.

The purpose of the current workpaper is to develop variations of models, based on one model, a new structure, which has been designed inspired by the art of paper quilling. The proposed structures were analyzed by numerical and experimental methods. The paper presents the geometry of the structure and the main parameters that define it followed by finite element analyses of the displacements under a compressive force. Similar models were manufactured by 3D printing and tested in similar supporting and loading conditions. The value of stiffness of the structures is compared in the Results section, the paper underlying the main findings in the Conclusions.

2. MATERIALS AND METHOD

The first step was done on paper, by trying different models of quilling and deciding upon a model that was suited for the study. Have been obtain simple spirals that can be joined together with different number of arms. To obtain a repetitive pattern in this case was selected the model with 4 arms joined together as presented below in figure 1.

Fig. 1. The first model made from paper

While experimenting with design and searching for a feasible way of manufacturing the material, the model was deviated from the initial quilling structure, so that it can be reproduced with 3D printing technology and available materials.

The transition was done from paper models to a 3D printed layer of patterns through design studies and has been obtained the base model of spirals as shown in pictures below, named S1.

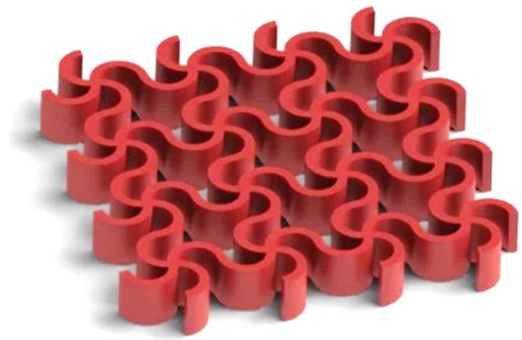

Fig. 2. Model S1 – The base model in perspective

The material chosen for the study to be applied on all of the models for FEA simulation is high density polyethylene. Polyethylene is a light, versatile synthetic resin made from the polymerization of ethylene. Polyethylene is a member of family of polyolefin resins and is one of the most widely used plastic in the world. It can also be slit or spun into synthetic fibers or modified to take on the elastic properties of rubber. High density polyethylene (HDPE) is representative for polyolefin thermoplastics, the most important in world scale plastics production, environmentally polymerization processes, recycling, and sustainability. The material named Z-ultrat, a HDPE filament used in 3D printing has been chosen due to the fact that is accessible, easy to apply, maintaining sophisticated structures, only through one process of manufacturing. Also, an advantage in using Z-ultrat is that can be used in many types of physical test, by giving precise and relevant information upon the response to environment depending on the shape of the part.

In order to understand how Z-ultrat behaves after 3D printing, tensile samples of material have been manufactured for tests. The specimens presented cross-sectional dimensions of 9.57X4 mm and have been subjected to a tensile maximum force of 1100 N, with a rate of 4 mm/min. Testing results have shown a tensile strength of 28.75 MPa, Poisson’s ration 0.3 and

Young's modulus of 1703 MPa [19]. The density 1.179 g/cm^3 and the melting temperature of 106.4°C was conveniently chosen from the specifications given by the manufacturer of the filament [20]. The properties identified above will be used in the current study for all the FE simulations.

In order to understand the model S1, in the Fig. 3 is presented only one cell of the material.

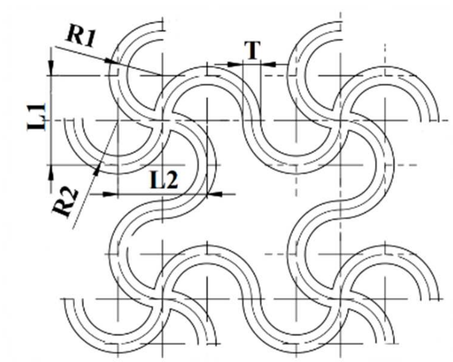

Fig. 3. Model S1 – One cell of the material top view

The cell is represented by repeated geometric shapes obtained by joining together 4 spirals. Analyzing the cell, there are found six parameters that can be modified to produce a variety of versions of the structure. The parameters of the cell, as presented in Fig. 3 are denoted by the middle curves of the cell and are: the middle radius of each spiral (identified in this case with R1, R2), the distance between the origin of two opposite radius (L1 and L2), the thickness of the pattern in cross-section (T) and the thickness of one layer of the solid model (D) (not visible in the plane sketch).

For the given case, there are a few initial conditions established before variating the model so that a real structure can be studied and manufactured. The conditions are the constraints chosen to define the limits of the study samples, in order to focus and understand the impact of only two of the parameters upon the behavior. It was agreed that:

- All the models will have the same transversal thickness D. In the current case has been chosen 10mm.
- Two opposite radiuses will have the same value.
- The lengths between the center of two radiuses will be perpendicular and always intersect in the middle point.
- There will be used the same thickness T for all the patterns and all the models (T=2mm).
- In the current study all the layers of metamaterials will be used in a pattern with three cells on the row and three on the column so that the models can be compared between each other while simulating the mechanical behavior.

The first model developed (S1) was a symmetric model in each quadrant. From this base model, by playing with the parameters have been obtained deviated shapes that present modifications of the mechanical properties and response to the loading conditions. For this study have been chosen two parameters to be modified, more precisely, the opposite radiuses positioned vertically on columns denoted by R1 and the length between them L1, while the rest of the parameters have been kept unchanged.

These parameters influence the shape and the behavior of the model in response to the environment. In order to see how a model behaves and creates linking relationships between parameters and the metamaterial response to stress, during the study, have been modified the parameters one by one, resulting in deviated simple structures, born from the initial S1 model. As a result, have been obtained another 25 versions of the model, respecting the geometry of S1, but with different dimensions. To all the models has been assigned the same material as above described and the samples have been subjected to the same compression load of 100 N. The behavior has been simulated in Ansys Workbench 2022R1 through finite element method.

For all the models have been extracted from the software directional deformation in the force direction (δ). The results have been compared to see what are the differences and what is the influence of the parameters over the mechanical behavior.

For S1 in order to run the FEA simulations and compare the results with the experiments have been chosen the following values for the parameters: equal middle radiuses (R1=R2=5 mm); the equal length between the center of the radiuses (L1=L2=10 mm) and the same point of tangency between the radiuses of the spirals.

The simulation for the study of the metamaterial behavior consists in applying at one end of the structure a fixed support and on the other end a compressive force of 100 N.

For experiments a sample of S1 model was printed on 3D printer and tested on the universal testing machine Instron 3366-10kN. The sample, similar as in FEA simulation was a pattern of 3x3 cells, with the dimensions of 82X82X10 mm. The results comparison between the experiment and simulation pointed out a good convergence between the results.

In the table below are presented the results from FEA physical testing of the model S1.

Table 1

Comparison between experimental and FE stiffness of S1 sample

Test type	Directional Deformation [mm]	Stiffness [N/mm]	Relative Deviation [%]
FEA	2.25	44.44	4.54
Experiment	2.11	47.39	

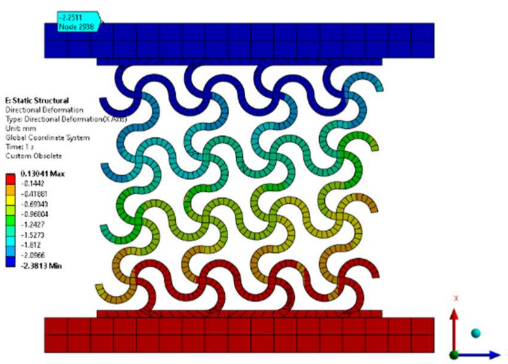

Fig. 4. Results of the directional deformation (x) of S1 structure

In the table from figure 4 are shown the simulation results of directional deformation, when applying a compressive load of 100 N upon model S1. The simulation generated a displacement on X axis, of 2.25 mm. The deformation inside the pattern was uneven having a displacement inside the pattern variating from 0.13 mm on the bottom row to 2.38 mm on the top row.

In the figure 5 are presented the results of equivalent elastic strain of S1 under the same force magnitude. The maximum strain presented by the pattern was of 1.21%.

Figure 6 illustrates the results of equivalent (von Mises) stress under the same load of 100 N. The maximum stress value found in the structure was of 20.5 MPa.

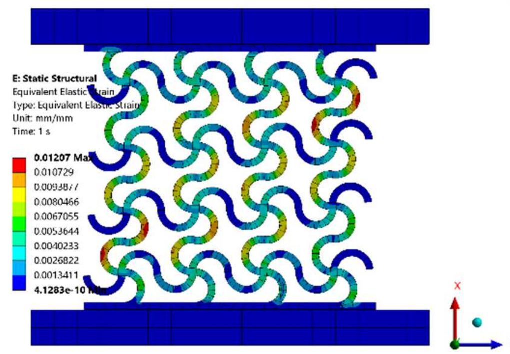

Fig. 5. Results of the equivalent elastic strain of S1 under a force of 100 N

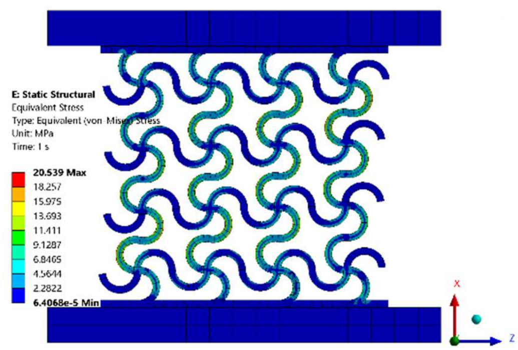

Fig. 6. Results of the equivalent stress of S1 under a force of 100 N

The second step of the study was to create new models of structure, by changing design parameters. There have been chosen two parameters: opposites radiuses denoted by R1 and length between radiuses centers denoted by L1. These parameters have been modified, but only with a few units successively by diminishing and maximizing the values between limits of the design, in such manner that the cell is not too deviated from initial design so that the arms of the spirals result in tangency.

At this stage have been obtained 25 deviated models that can be classified in three groups:

- First group denoted by S2 keep the same radiuses values but have different lengths between the radiuses (L1≠L2).

- The second group, denoted by S3 has different radiuses values, for the pair R1 (R1≠R2), but are keeping the same length between radiuses R1, as in the model S1 (L1=L2).
- The third group denoted by S4 has different radiuses values, for the pair R1 (R1≠R2) and all the radiuses are tangent (L1≠L2).

The first group, S2, is a group containing 8 variations, deviated from S1 that kept the same radiuses values of 5 mm, for this study, but the lengths between the vertically disposed radiuses L1 are different for each model (Fig. 7-8). The first model has a length L1 of 6 mm and after each successive model grows in length with 1 mm, up to an acceptable distance of 14 mm. The purpose was to see what is the influence of length L1 over the metamaterial behavior.

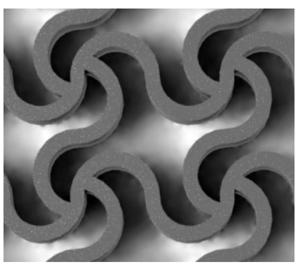

Fig. 7. Model S2.1

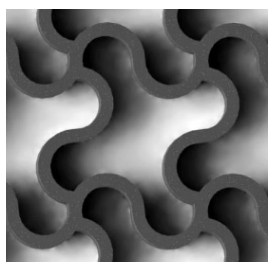

Fig. 8. Model S2.8

All the models have been simulated in Ansys to understand the behavior under a compressive load. The models have been subjected to a force of 100 N exactly like in the case of S1 model and reported the values of directional deformation and stiffness. In the Table 2 are presented the models with the associated length between the center of the radiuses, the results given by the FEA simulation being found in Table 3.

Table 2

Values of the parameters of model from group S2

Model no.	R1 (mm)	R2 (mm)	L1 (mm)	L2 (mm)
S2.1	5	5	6	10
S2.2	5	5	7	10
S2.3	5	5	8	10
S2.4	5	5	9	10
S1	5	5	11	10
S2.5	5	5	12	10
S2.6	5	5	13	10
S2.7	5	5	14	10

As can be observed the values in Table 3 and illustrated on graph (Fig. 9) the stiffness results of the model's simulations are in the range of 43 to 52 N/mm. It can be concluded that by modifying the lengths between radiuses has a low influence over the stiffness of the structure.

Table 3

Results of FEA simulation upon the models from group S2

Model no.	Directional Deformation [mm]	Stiffness [N/mm]
S2.1	1.9	52.63
S2.2	1.99	50.25
S2.3	2.086	47.94
S2.4	2.184	45.79
S1	2.25	44.44
S2.5	2.288	43.71
S2.6	2.298	43.52
S2.7	2.3	43.48
S2.8	2.3	43.48

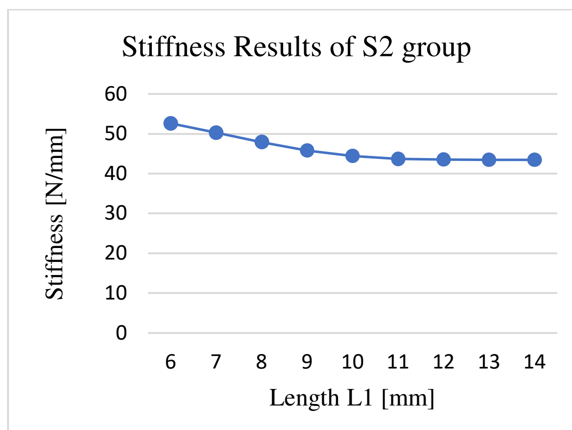

Fig. 9. Stiffness vs. length L1 of the models from group of structures S2

The second group is denoted by S3 and presents different dimensions for radius R1 but keeps the same distance between the center of the middle radiuses as in the initial model S1, in this case is 10 mm (Fig. 10-11).

As presented in the Table 4, the smallest radius was chosen 4 mm and the largest 7.5. Each model undergoes an increase in radius with 0.5 mm from the previous one.

Analyzing the results, as per the graph in figure 12, has been observed that in this case clearly by increasing the radius, a significant decrease in stiffness.

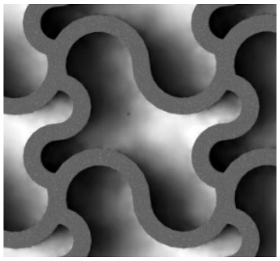

Fig. 10. Model S3.1

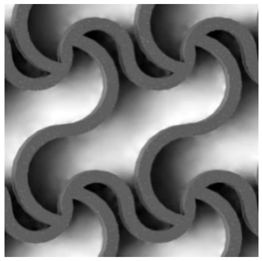

Fig. 11. Model S3.9

Table 4

Values of the parameters of models from group S3

Model no.	R1 (mm)	R2 (mm)	L1 (mm)	L2 (mm)
S3.1	3	5	10	10
S3.2	3.5	5	10	10
S1	4	5	10	10
S3.3	4.5	5	10	10
S3.4	5	5	10	10
S3.5	5.5	5	10	10
S3.6	6	5	10	10
S3.7	6.5	5	10	10
S3.1	7	5	10	10
S3.2	7.5	5	10	10

Table 5

Results of FE simulation upon the models from group S3

Model no.	Directional Deformation [mm]	Stiffness [N/mm]
S3.1	1.22	81.97
S3.2	1.69	59.17
S1	2.25	44.44
S3.3	2.89	34.60
S3.4	3.59	27.86
S3.5	4.41	22.68
S3.6	5.35	18.69
S3.7	6.43	15.55

The last studied structures, a group of models denoted by S4, that have different middle radius values, for the pair R1 (R1≠R2), but in this case the tangency was respected between all of the radiuses. The last condition requires the horizontal and vertical lengths to differ (L1≠L2). The hierarchy starts with a model with a minimum allowable radius (3.5 mm), for the next models the radius grows successively with 0.5 mm per each model until the last model with 7 mm radius (Table 6).

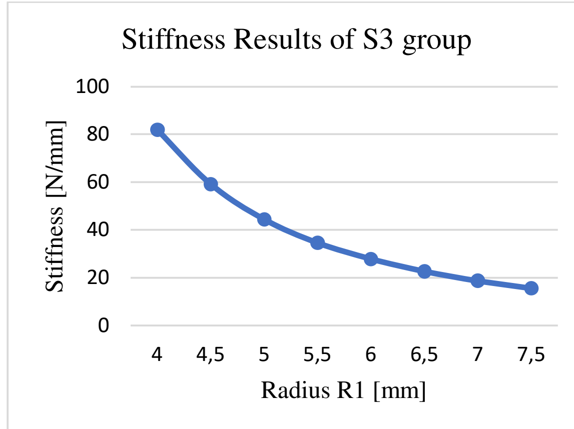

Fig. 12. Stiffness vs radius results of the models from group S3

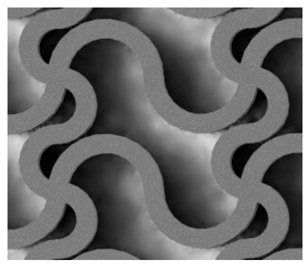

Fig. 13. Model S4.1

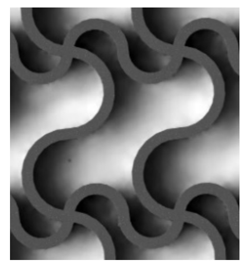

Fig. 14. Model S4.8

Table 6

Values of the parameters of model from group S4

Model no.	R1 (mm)	R2 (mm)	L1 (mm)	L2 (mm)
S4.1	3.5	5	7	10
S4.2	4	5	8	10
S4.3	4.5	5	9	10
S1	5	5	10	10
S4.4	5.5	5	11	10
S4.5	6	5	12	10
S4.6	6.5	5	13	10
S4.7	7	5	14	10
S4.8	7.5	5	15	10

After all the samples have been subjected to a load of 100 N, the results gathered have shown a direct relationship between radius and respectively the deformation and stiffness,. In the table 7 are presented the values of deformations and stiffness for each model.

By analyzing the graph presented in Fig. 15, results have shown that by increasing the radius together with distance between the centers of the semicircles decreases the stiffness.

So, in order to obtain a much more elastic pattern is enough to give a larger difference between radiuses R1 and R2, for sure in the limits allowed by the pattern.

Table 7

Results of FEA simulation upon the models from group S4

Model no.	Directional Deformation [mm]	Stiffness [N/mm]
S4.1	0.848	129.75
S4.2	1.08	92.59
S4.3	1.48	67.57
S1	2.25	44.44
S4.4	2.975	33.61
S4.5	3.69	27.10
S4.6	4.72	21.19
S4.7	6.16	16.23
S4.8	7.28	13.74

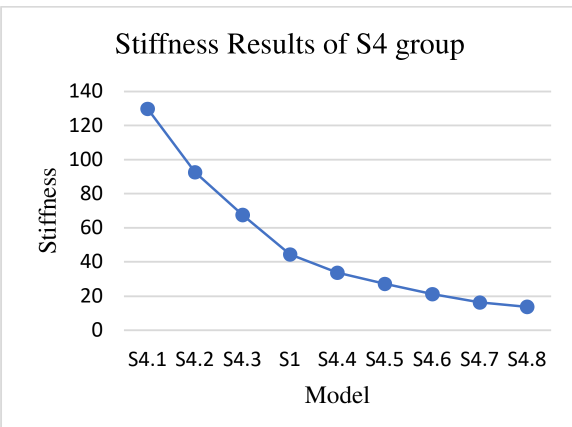

Fig. 15. Stiffness results of the models from group S4

3. CONCLUSION

Comparing the studies between the three groups can be drawn a few conclusions that are essential to understand the behavior of these type of structure and stands as proof that there is a direct link between geometry and the response to loads. But in the current study was important to understand which of the parameters have the highest impact over the mechanical behavior.

The first case (group S2) shows that taking only the lengths between the radius centers to modify them, has a low influence over the stiffness of the structure.

The simulations done upon second group (S3) reveals that the parameter that determines significant stiffness changes is the radius and has shown the possibility to increase or decrease the stiffness even 14 times when doubling the radiuses dimensions if there is to compare the 1st model with the 10th model from group S1.3. For example, at an increase in radius with 11%, stiffness decreases with 27%. From one model to the next one in sequence can be concluded that by keeping the distance and modifying only the radius, at an increase in radius with 0.5 mm gives a decrease of stiffness with 1.3-1.4 times.

The last group S4 differs in reactions than the third one by showing a significant decrease in stiffness when comparing the models. When increasing the radius proportional with the length, gives a more nonlinear transition from one model to another. The tangency of the radiuses provides a more stable structures as shown in results at each increase of 0.5 mm, decreases the stiffness with around 1.2-1.5 times than the previous model.

Furthermore, the study will be continued with integration of this structures to obtain a metamaterial with tunable stiffness.

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Evaluarea experimentală și numerică a rigidității structurilor inspirate din arta filigranării hârtiei

Rezumat: Lucrarea de față este un studiu realizat pe o serie de structuri inspirate din arta filigramării hârtiei. Toate modelele sunt similare în geometrie, având aceiași parametri și reprezintă variații ale modelului inițial, utilizat ca reper de comparație. Scopul comparației este de a pune în contrast modul în care rigiditatea structurii este influențată de parametrii geometrici ai modelelor. Pentru acest studiu au fost aleși patru parametri geometrici ce urmează a fi modificați, rigiditatea corespunzătoare fiind calculată ca deplasarea produsă de o forță de compresiune. Raportul se bazează pe simulări numerice, prin metoda elementului finit și evaluarea experimentală a unor structuri realizate prin imprimare 3D.

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