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# IN-PLANE PERMEABILITY VARIATION WITH COMPRESSION FOR TWO-LAYERED POROUS MATERIALS IMBIBED WITH LIQUIDS

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***Abstract:*** *The expulsion of a fluid from a layer of highly compressible porous material generates important load carrying capacity. The mechanism called eX-Poro-HydroDynamic (XPHD) depends on the permeability of the porous material and its variation with porosity, which in turn depends on compression level. This paper proposes a simple analytical model for calculating the permeability variation with thickness for compressed, two-layered porous materials. The analysis is made for the case of axially symmetric in-plane flow (disc-on-plane configuration). The equations are developed assuming the permeability–porosity correlation, governed by the classic Kozeny-Carman law. A possible simplified approach based on the calculation of equivalent permeability determined using the equivalent porosity of the two layers is also evaluated. The proposed model allows a parametric analysis carried out for the characteristic parameters of open-cell foams, tridimensional fabrics, and other soft, porous sandwich materials.*

***Key words:*** *permeability, porosity, two-layered, porous, analytical model, in-plane flow, parametric analysis, Kozeny-Carman.*

## 1. INTRODUCTION

Studies aiming an original lubrication mechanism based on highly deformable porous layers imbibed with fluids, are in continuous progress.

The expulsion of a fluid from a layer of highly compressible porous material generates important load carrying capacity. This mechanism, called ***E**x-**P**oro-**H**ydro-**D**ynamic* lubrication (XPHD), depends on porous material permeability and its variation with porosity, which in turn depends on the level of compression.

Specifically, imbibed fluid generates hydrodynamic pressure when forced to flow through the pores. The theoretical models considered various contact configurations, allowing the calculation of the forces generated by expulsion in two cases: (i) with constant speed [1][2] and (ii) by impact [3][4][5]. All these models were ***developed for a single layer of porous material***.

In recent years, new non-homogeneous soft, porous materials like sandwich or tridimensional fabrics were developed. As a result, studies on the effect of the imbibed materials and consequently the XPHD model, need to be extended for the case of multi-layered porous materials. In this context, the present analysis aims to expand the existing XPHD model for disc-on-plane configuration [5], for a pack of a two-layered material with different initial porosities and different response on compression (rigidity). By similitude with the behavior of springs mounted in series, it is expected that a pack of two layers could improve the damping behavior when compressed.

Only a few studies can be found in literature related to the problem of in-plane flow through multi-layered porous structures, but deformation is not considered in either one. Bear [6] analyzed the planar flow of a homogeneous fluid through a finite number of porous layers with different but constant thickness and permeability. To determine the permeability of an equivalent porous layer (with the thickness equal with that of the pack) he considered the same pressure gradient in each layer and applied Darcy law and flow conservation. Thus, he obtained an

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equivalent permeability, eq. (1), based on permeability and the thickness of each layer.

$$\phi_e = \frac{1}{h}\sum_i \phi_i \cdot h_i \qquad (1)$$

Almalki et al. [7] analyzed theoretically the planar flow of an incompressible fluid through a package of two porous materials with different thickness. A perfectly flat interface exists between them, and velocities and tangential stress are the same next to it. The layers are isotropic and have different porosities. Their model assumes unidirectional (axial) flow between two rigid and impermeable walls that bound the pack. Darcy-Brinkman equation was used for flow inside each layer and velocity distribution was found. Unfortunately, the proposed mathematical model is not followed by a parametric analysis and does not include comparisons with other similar analyses.

More recently, Ford et al. [8] presented a complex numerical analysis model of flow through a finite series of layers and channels (free-flow zones). The mathematical model is established on the general form of the flow equations. By changing a series of parameters that consider different fluid flow in different regimes (Darcy, Darcy-Brinkman, etc.), it is reduced to particular forms. Similar numerical approach was conducted by Allan and Hamdan [9] and Alharbi et al. [10] by taking into consideration more complex Darcy–Lapwood–Brinkman nonlinear model, respectively fluid viscosity variation. All these complex models allow analytical solutions and consider an interface with solid walls that delimit the flow domain.

Adams & Rebenfeld [11] presented a technique that allowed experimental quantification of in plane permeabilities and of flow anisotropies used to characterize a widespread range of fibrous networks. Specifically, they studied the effective in-plane permeabilities of multilayer assemblies in terms of in-plane permeability of their constituent layer. They evaluated experimentally [12] and analytically [13] the in-plane permeability characteristic of homogeneous and heterogeneous multilayer materials. They stated that the in-flow process is influenced by the interlaminar pore system created by the stacking of multiple materials. Their theoretical model identified three flow regimes: (i) the layers act independently and are governed by the single layer solution, (ii) the layers act like a unified homogeneous system and (iii) an intermediate region connecting the previous two.

Fluid flow through layered, homogeneous porous structures is treated in the literature only considering simpler theoretical models based on Darcy or Darcy-Brinkman. This allows analytical solutions as well as more complex models, in which inertial terms or free flow between porous layers also appear. For multi-layered models, the solutions can only be obtained numerically. The analyzed models considered that the flow is axial (unidirectional) and laminar and is produced by a pressure gradient that is the same for all layers The boundary conditions at the interface of two layers, respectively at the interface with the rigid and impermeable surfaces that bound the flow domain, are identical in all analyzed approaches.

A model for squeeze flow through layered porous structures with compression dependent porosity is not found in literature. Therefore, the development of a simple model for the case of radial (axisymmetric) flow through two porous layers, subjected to compression, is appropriate. The equations are developed assuming the permeability – porosity dependance, governed by the classic Kozeny-Carman equation. Also, for comparison it is evaluated a simplified approach based on the calculation of the equivalent permeability determined using the equivalent porosity of two layers.

## 2. THE MODEL

It is considered a relatively thin, porous structure, composed of two layers with uniform thickness, completely imbibed (saturated). The porous structure is packed between two (one stationary and one moving) rigid, flat and impermeable discs (Fig. 1).

At macroscopic level, the two layers are considered homogeneous and isotropic, with different initial thickness ($h_{01}$ and $h_{02}$ respectively), porosity ($\varepsilon_{01}$ and $\varepsilon_{02}$) and rigidity ($k_1$ and $k_2$). The imbibed fluid is expulsed out of the porous structure, during compression produced by the vertical displacement of the moving disc. For the sake of simplicity, layer 2 is assumed always the more rigid one.

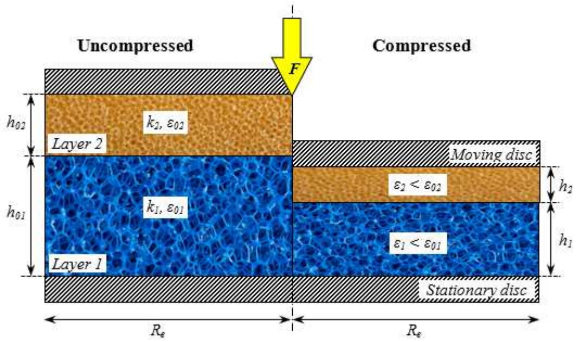


**Fig. 1.** Schematic of the two imbibed layers model subjected to compression

All the classical assumptions of XPHD lubrication are admitted for this model [1][2][5]:

1) The flow is laminar and can be characterized by a sub-unitary permeability-based Reynolds number (Darcy flow).
2) The flow is steady state and isothermal.
3) The fluid is Newtonian.
4) Pressure is constant across each layer.
5) Poisson's ratio is negligible so that the surface normal to the direction of compression does not change.
6) The elastic forces generated by the solid structure are small with respect to hydrodynamic force generated during squeeze motion and, consequently, are neglected.
7) All the pores are connected until the complete compactness is reached (ε=0);
8) Solid phase conservation is considered, and it is used as a function of porosity, $\varepsilon$, and dimensionless thickness, $H=h/h_0$;
$$(1-\varepsilon)H = (1-\varepsilon_0) \tag{2}$$
9) Permeability variation with porosity is governed by Kozeny-Carman law.
$$\phi = D\frac{\varepsilon^3}{(1-\varepsilon)^2} \tag{3}$$
where parameter $D = d^2/k_{KC}$ is a function of a characteristic dimension of the porous structure, $d$, and the Kozeny-Carman coefficient, $k_{KC}$.
10) The rigid discs being permanently parallel allows the consideration of an axisymmetric model.
11) Due to relative low thickness to outer radius ratio, the transverse flow is neglected and only the in-plane flow is considered.

In addition, it is assumed elastic behavior of the two layers, defined by the constant rigidity coefficients, $k_1$ and $k_2$, similar to a package of springs in series. This extra assumption is needed to define the level of compression of each layer, produced by the vertical motion of the moving disc (Fig. 1).

Flow through each layer occurs under the action of a pressure gradient. It is assumed that the pressure at the axis of symmetry does not vary across the thickness of ***layers 1*** and ***2***.
$$\left.\frac{\mathrm{d}p}{\mathrm{d}r}\right|_1 = \left.\frac{\mathrm{d}p}{\mathrm{d}r}\right|_2 \tag{4}$$

The constant velocity expulsion model for disc on plane configuration proposed by M. D. Pascovici [2] and later improved by M. Radu [5] was the starting point for the two-layered analytical model. In this model, by replacing the total flow rate through the porous layer with the fluid flow rate expelled at the vertical displacement and integrating the radial pressure distribution [5], one can obtain the expression of the force resisting the squeeze:
$$F = \frac{\pi\eta w R^4}{8h\phi} \tag{5}$$

Equating the total flow rate with the sum of the flow rates in each layer (at any radial position), we get:
$$q_{tot} = q_1 + q_2 = \frac{2\pi r}{\eta}\frac{\mathrm{d}p}{\mathrm{d}r}\left(h_1\phi_1 + h_2\phi_2\right) \tag{6}$$
where: $h_{1,2}$ and $\phi_{1,2}$ are the thickness and the permeability of each layer at a certain compression level and $\eta$ is the dynamic viscosity, assumed constant.

Neglecting transverse flow, one can define an equivalent permeability, $\phi_e$, based on the equations written for expulsion for each layer, eq. (1) [6].

The applied normal force is equally transmitted to each layer; hence we have:
$$k_1\Delta h_1 = k_2\Delta h_2 \tag{7}$$
where: $\Delta h_{1,2}$, is the compression of each layer, with $\Delta h_1=h_{01}-h_1$ and $\Delta h_2=h_{02}-h_2$.

The contribution of each layer to the total compression of the pack ($\Delta h_1+\Delta h_2=\Delta h$) is:
$$\Delta h_1 = \frac{k_2}{k_1+k_2}\Delta h \tag{8}$$
$$\Delta h_2 = \frac{k_1}{k_1+k_2}\Delta h \tag{9}$$

For the sake of simplicity, as well as for the parametric analysis, dimensionless forms of the equations will be used, with $K = \frac{k_2}{k_1}$, relative rigidity, $H = \frac{h}{h_0}$, relative thickness and $h_0$, the initial (undeformed) total thickness of the pack. Based on these notations, equations (8) and (9) become:

$$\Delta h_1 = h_0(1-H)\frac{K}{K+1} \quad (10)$$

$$\Delta h_2 = h_0(1-H)\frac{1}{K+1} \quad (11)$$

Using the relative initial thickness (Fig. 1), $T = \frac{h_{01}}{h_0}$, and rearranging eqs. (10) and (11) one can obtain:

$$H_1 = \left[1 - \frac{(1-H)}{T}\frac{K}{(K+1)}\right] \quad (12)$$

$$H_2 = \left[1 - \frac{(1-H)}{(1-T)(K+1)}\right] \quad (13)$$

Applying eqs. (2) and (3) to each layer and combining them with eqs. (12) and (13), followed by simple algebraic calculations, the formulas for porosity of each layer are obtained:

$$\varepsilon_1 = \frac{T(K+1)\varepsilon_{01} - K(1-H)}{T(K+1) - K(1-H)} \quad (14)$$

$$\varepsilon_2 = \frac{(1-T)(K+1)\varepsilon_{02} - (1-H)}{(1-T)(K+1) - (1-H)} \quad (15)$$

Finally, bringing together eqs. (14) and (15) with Kozeny-Carman expression (3), one can find the permeability of each layer as a function of the compression level of the pack, expressed by its dimensionless thickness, $H$.

$$\phi_1 = \frac{[T(K+1)\varepsilon_{01} - K(1-H)]^3}{[T(K+1) - K(1-H)][(1-\varepsilon_{01})T(K+1)]^2} \quad (16)$$

$$\phi_2 = \frac{[(1-T)(K+1)\varepsilon_{02} - (1-H)]^3}{[(1-T)(K+1) - (1-H)][(1-\varepsilon_{02})(1-T)(K+1)]^2} \quad (17)$$

To obtain the expression of the equivalent permeability, $\phi_e$, eqs. (12) and (13) must be substituted in eq. (1) and combined with eqs. (16) and (17):

$$\phi_e = \frac{D_1}{(1-\varepsilon_{01})^2} \cdot \left\{\frac{[T(K+1)\varepsilon_{01} - K(1-H)]^3}{H\,T^2(K+1)^3}\right\} + \frac{D_2}{(1-\varepsilon_{02})^2}\left\{\frac{[(1-T)(K+1)\varepsilon_{02} - (1-H)]^3}{H(1-T)^2(K+1)^3}\right\} \quad (18)$$

An extension of the proposed analytical model is a matter of generalization but involves a large number of parameters (three for each layer).

Eq. (18 is valid as long as both layers deform during compression; from the moment when one of the layers reaches total compactness *($\varepsilon$=0, i.e. H=1- $\varepsilon_0$)* the flow occurs only through the other layer; mathematically, eq. (18 will have a null term.

The condition that both layers subjected to compression allow fluid flow, can be written mathematically as follows:

$$1 - \varepsilon_{01} < H_1 < 1 \quad (18)$$

$$1 - \varepsilon_{02} < H_2 < 1 \quad (19)$$

If the inequalities from eqs. (18) and (19) are applied to eqs. (12) and (13), it will be obtained the limit values for the total relative thickness, $H_{lim}$, up to which each of the two layers can be compressed:

$$H > H_{lim\_1} = 1 - \varepsilon_{01}T\frac{K+1}{K} \quad (20)$$

$$H > H_{lim\_2} = 1 - \varepsilon_{02}(1-T)(K+1) \quad (21)$$

If the two layers have different thickness and rigidity, there will always be one that will fully compact first. Hence, eqs. (20) and (21) are applied simultaneously, until this happens. Consequently, the two-layered model applies as long as condition (22) is fulfilled.

$$H \geq H_{lim} = max\,(H_{lim\_1}, H_{lim\_2}) \quad (22)$$

At compression levels beyond this limit, the analytical model is reduced to the one with a single layer i.e. the one that did not reach its compressibility limit (the one with the minimum value of the two limits).

Based on eqs. (20)–(22) it can be written:

$$H_{\text{lim\_2}} - H_{\text{lim\_1}} = \frac{1+K}{K}[\varepsilon_{01}T - K\varepsilon_{02}(1-T)] \quad (23)$$

If $H_{\text{lim\_2}} - H_{\text{lim\_1}} > 0$, layer 2 reaches its compactness, then:

$$[\varepsilon_{01}T - K\varepsilon_{02}(1-T)] > 0 \quad (24)$$

Further on, the analytical model can be split into two cases, depending on which layer reaches first, total compaction. Only one case will be presented in detail, while for the others it is just a matter of algebraic calculations.

***<u>Hypothesis:</u>*** *Layer 1 is compacted first.*

In this situation, eqs. (8) and (9) become:

$$\Delta h_1 = \Delta h_{1max} = \varepsilon_{01}h_{01} = \varepsilon_{01}Th_0 \quad (25)$$

$$\Delta h_2 = h_0[(1-H) - T\varepsilon_{01}] \quad (26)$$

In dimensionless form it is considered $H_1 = H_{1\_min} = 1 - \varepsilon_{01}$. Calculating the relative thickness of layer 2, $H_2$ (eq. (12)), using $H_{lim\_1}$ from eq. (20), it is found:

$$H_{lim\_2} = 1 - \varepsilon_{01}\frac{T}{(1-T)K} \quad (27)$$

From this moment on, the compression is taken over only by layer 2, and the total thickness of the pack is, $h = h_{1\_min} + h_2$, where $h_{1\_min} = h_0 T(1-\varepsilon_{01})$.

$$\Delta h_1 = 0 \quad and \quad \Delta h_2 = \Delta h \tag{28}$$

In dimensionless form, the relative thickness of layer 2, $H_2$, varies between the $H_{lim\_2}$ and 1-$\varepsilon_{02}$.

$$H_2 = \frac{H - T(1-\varepsilon_{01})}{1-T} \tag{29}$$

$$H_{2\_lim} > H_2 > 1 - \varepsilon_{02} \tag{30}$$

Finally, when layer 2 is also compacted, the relative thickness of the multilayer $H_{\text{min}} = H_{\text{lim\_1}} + H_{\text{lim\_2}}$ is:

$$H_{min} = T(1-\varepsilon_{01}) + (1-T)(1-\varepsilon_{02}) \tag{31}$$

To summarize, the model can be split into two stages:

1) *Relative permeability calculation when the flow occurs through two parallel layers.* This regime is defined by eq. (22), for relative thickness being $H_{lim} < H < 1$. Relative equivalent permeability is calculated with eq. (18 and the compression level of each layer is determined using eqs. (10) and (11) or (12) and (13).
2) *Relative permeability calculation when the flow occurs through one layer.* This regime is divided into two possible situations, depending on which layer is compacted first:

- If *layer 1* is compacted first and only *layer 2* is compressed further, the equivalent permeability is the permeability of *layer 2*, based on eq. (2) and Kozeny-Carman law:

$$\phi_e = \phi_2 \frac{h_2}{h} = D_2 \frac{\left(1 - \frac{1-\varepsilon_{02}}{H_2}\right)^3}{\left(\frac{1-\varepsilon_{02}}{H_2}\right)^2} \frac{H_2(1-T)}{H} \tag{32}$$

If eq. (32) is rewritten considering dimensionless form of layer 2 thickness (eq. (29)), then:

$$\phi_e = \frac{D_2}{(1-\varepsilon_{02})^2} \frac{(H - H_{min})^3}{H(1-T)^2} \tag{33}$$

- In the opposite case, when *layer 2* is compacted first and the multilayer deformation is taken over only by *layer 1*, the equivalent permeability is the permeability of *layer 1*:

$$\phi_e = \frac{D_1}{(1-\varepsilon_{01})^2} \frac{(H - H_{min})^3}{HT^2} \tag{34}$$

## 3. PARAMETRIC ANALYSIS

The equivalent permeability, $\phi_e$, is a function of the initial porosity, as well as of three dimensionless parameters: the dimensionless total thickness, $H$, the relative thickness of layer 1, $T$, and the relative rigidity, $K$.

An alternative solution for the equivalent permeability can be obtained if a homogenous material having the same total thickness and an equivalent porosity is considered:

$$\varepsilon_{ehm} = \frac{\varepsilon_1 h_1 + \varepsilon_2 h_2}{h} \tag{35}$$

This is the solution at hand when the separation of the layers and their individual analysis is not easy to achieve. If the component layers with different structure (different rigidity) can be analyzed separately, the evaluation of the compression effect (of each layer) is hard to accomplish. In this situation only initial porosity $\varepsilon_{01}$ and $\varepsilon_{02}$ can be determined, thus the definition of an equivalent initial porosity is justified. The equivalence is based on the observation that the solid phase of the layers sums up in the structure, and the cross-sectional area does not change with compression. Furthermore, the equivalent porosity is used in Kozeny-Carman equation:

$$\phi_{ehm} = D_{ehm} \frac{{\varepsilon_{ehm}}^3}{(1-\varepsilon_{ehm})^2} \tag{36}$$

In this case, a sensible problem is how to define the equivalent complex parameter $D_{ehm}$. If the pores/wires have similar sizes for the two layers, the reasonable solution is to define an average value:

$$D_{ehm} = \frac{D_1 + D_2}{2} \tag{37}$$

Obviously, other assumptions can be proposed (using the average diameter of the pores/wires, or an average diameter weighted by the thickness of the layers).

The first analysis is dedicated to the limit of applicability of the two-layered model, respectively, the definition of the dimensionless thickness limit at which one of the layers becomes completely compact ($\varepsilon$ =0).

The numerical applications presented further on are dedicated to a relatively wide domain of initial porosity (0.8÷0.95) specific to a large group of materials (open-cell foam materials, sandwich materials, tridimensional fabrics, etc).

***Fig. 2–a*** shows the variation of $H_{lim}$ as a function of relative thickness, $T$, for materials with identical initial porosity ($\varepsilon_0$=0.95). If the materials have the same rigidity ($K$=1) and equal thickness ($K$=1, $T$=0.5), the compression limit is, as expected, $H_{lim}=1-\varepsilon_0=0.05$. For $T$=0.5, in the case of materials with different rigidity ($K$=1.5 and $K$=2), the compression limit, $H_{lim}$, is higher. Thus, if the two layers have the same thickness ($T$=0.5), and material 2 has a rigidity two times higher than material 1 ($K$=2), from ***Fig. 2–a*** the compression limit becomes $H_{lim}$=0.3.

For layers with different thicknesses ($T\neq0.5$), the stiffer material must be thinner to reach the same deformation as the more elastic one. From ***Fig. 2–a*** results the range (see grey marked area for $K$=2) of values of relative thickness, $T$, for which the two-layered model can be used, at different values of relative rigidity. For smaller values of $H$ (up to $H$=0.05) the calculation is performed with the single layer model.

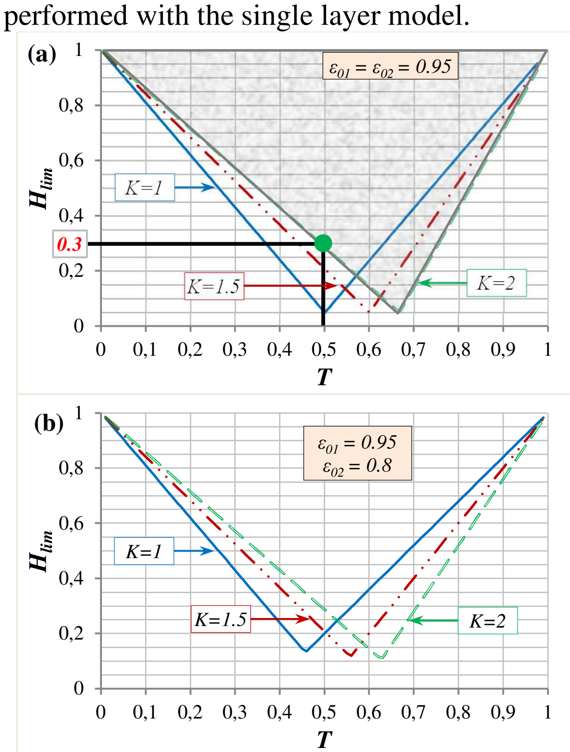


**Fig. 2.** Compression limit for the two-layered model

The case of two layers with different initial porosity is shown in ***Fig. 2–b***. The value of $H_{lim}$ is different and lower than that presented before, and this is due to the increased compression limit of the material with the higher porosity.

A parametric analysis was done to highlight the performance of the two-layered model (respectively reaching the maximum compactness in one of the layers) as well as the errors introduced by using the simplified model with a single equivalent layer.

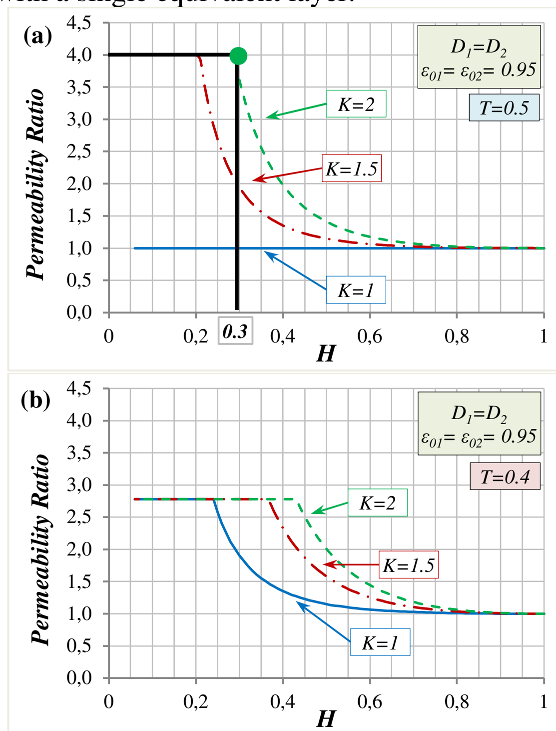


**Fig. 3.** Permeability ratio, as a function of relative thickness and relative stiffness

The comparison in ***Fig. 3*** is presented as a function of dimensionless thickness, $H$. To simplify the analysis, the first case considered is that of two materials with identical structure (the same initial porosity, $\varepsilon_{01}=\varepsilon_{02}$, respectively the same diameters of the pores/wires, $D_1=D_2$). The results are presented in terms of permeability ratio ($\phi_e/\phi_{ehm}$) between the equivalent permeability calculated with two-layered model, eq. (18, and that calculated with the equivalent initial porosity, eq. (36).

A first remark would be, that for materials with the same structure, rigidity and thickness ($\varepsilon_{01}=\varepsilon_{02}$, $K$=1, $T$=0.5), the two models obviously lead to the same result, the permeability ratio being 1 (***Fig. 3–a***). If materials have the same thickness and different rigidity ($K$=2) the permeability ratio between the two models increases. The point when it stops increasing and it becomes constant (four times higher than the permeability obtained with one equivalent layer), corresponds to the value of $H_{lim}=0.3$, previously discussed in ***Fig. 2–a***. For other values of rigidity, the behavior is similar, with $H_{lim}$ having a different value.

The case for materials with the same structure ($\varepsilon_{01}= \varepsilon_{02}$) but different thicknesses ($T$=0.4) is presented in ***Fig. 3–b***. The permeability obtained with two-layered model is almost three times higher for all three situations. The difference between them is in terms of the changes that occur to the relative thickness limit, $H_{lim}$.

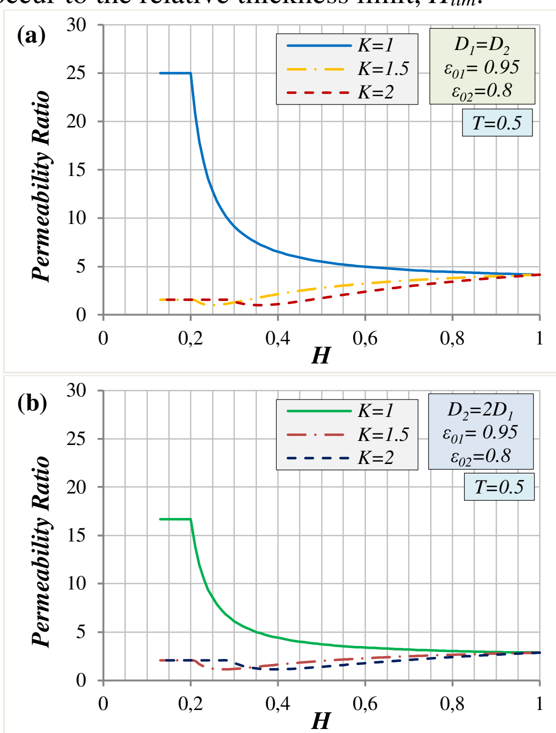


**Fig. 4.** Permeability ratio ($\phi_e/\phi_{ehm}$), as a function of relative thickness and relative stiffness for materials with different structure

For materials with different structure and the same thickness and rigidity ($T$=0.5, $K$=1) the permeability ratio drawn in ***Fig. 4–a*** has an increasing evolution, up to a difference of 25 times, between the two models. This behavior is valid only up to a point, determined by the value of $H_{lim}$, calculated with eq. (22). Further on, the analytical model is reduced to a single layer (the one that did not reach total compactness), hence the ratio between the two permeabilities remains constant. For materials with different rigidities ($K$=1.5, $K$=2) this behavior changes: the permeability ratio decreases with the increase in compression level. The high porosity material is fully compacted early and the second layer influence on permeability is dominant. This means that one layer model highly underestimates the permeability variation.

However, if we consider the fact that pores/wires diameter, $d$, influences the Kozeny-Carman parameters, $D_1$ and $D_2$, we can see that the permeability ratio between the two models decrease when $D_2=2D_1$, for equal rigidity, $K$=1 (***Fig. 4–b***). Analyzing the other two cases presented in ***Fig. 4–b***, and comparing them with their correspondences in ***Fig. 4-a*** ($K$=1.5, $K$=2), one can observe that the differences between the two models slightly increase when Kozeny-Carman parameter is considered.

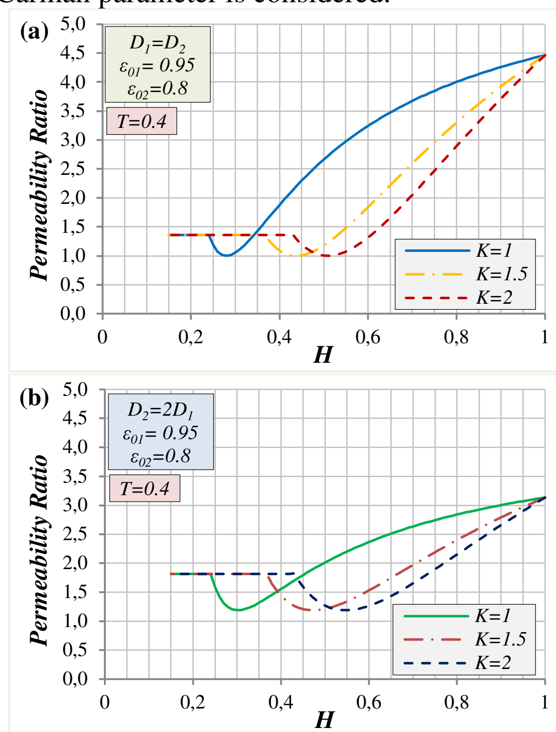


**Fig. 5.** Permeability ratio ($\phi_e/\phi_{ehm}$), as a function of relative thickness and relative stiffness for materials with different structure ($\varepsilon_{01} \neq \varepsilon_{02}$) and $T$=0.4

A similar comparison is presented in ***Fig. 5-a, b***, but this time the analyzed case assumes that the materials have different thickness ($T$=0.4). The first difference noticed is that for all three cases permeability ratio decreases up to a point, being then followed by an increasing slope, until $H_{lim}$ is reached. Considering the influence of Kozeny-Carman parameters (***Fig. 5–b***), leads to the same behavior as the one presented before: the differences between the two models decreased.

The porosity variation with relative thickness for materials with different structure and different thickness is depicted in ***Fig. 6*** and ***Fig. 7***. Basically, these graphs present the point in which the two-layered model is reduced to a single layer. We can observe in these figures, that the layer with the initial porosity higher is the one that reaches total compactness ($\varepsilon=0$) first. From that moment on, the two-layered model is reduced to a single layer. On these graphs this point is signaled by an inflexion of the curve of the uncompressed material (see the *Detail* from ***Fig. 6–a,b***).

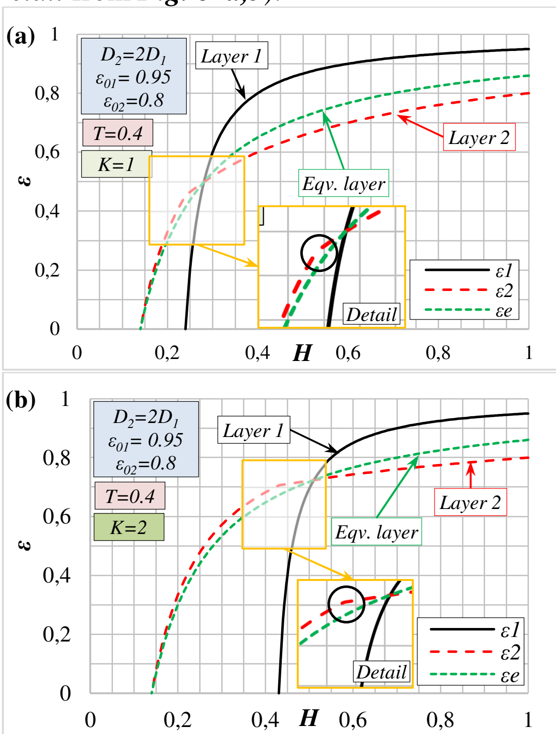


**Fig. 6.** Porosity variation with relative thickness

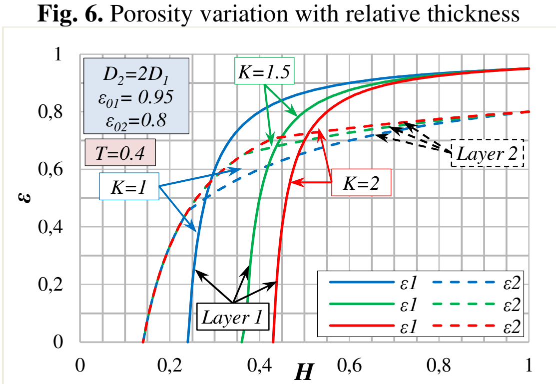


**Fig. 7.** Porosity variation with relative thickness

For a better understanding of the differences between the two models, ***Fig. 8*** presents the permeability variation determined with the two proposed models ($\phi_e$ and $\phi_{ehm}$) with respect to relative thickness, $H$. For low levels of compression, the differences between the two analytic models are not significant. Approaching the limit, $H_{lim}$, small differences between them are noticed. If rigidity influence is considered (i.e.: $K_2$=1.5$K_1$), the differences between the two models are amplified.

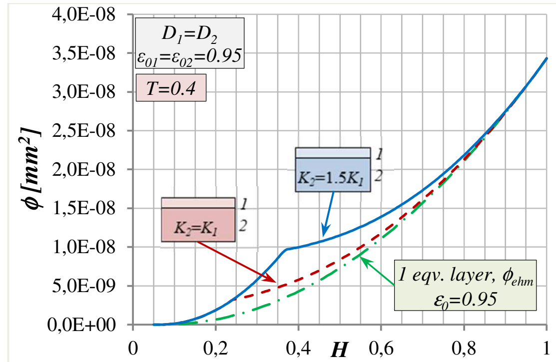


**Fig. 8.** Permeability variation with relative thickness

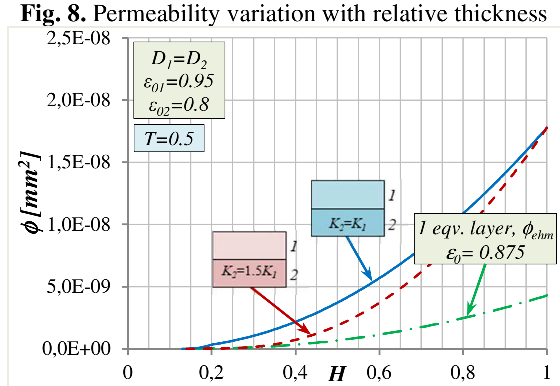


**Fig. 9.** Permeability variation with relative thickness

Studying the graph in ***Fig. 9***, one can see clearly that using the model with equivalent initial porosity ($\phi_{ehm}$) gives differences of one order of magnitude more, when compared to the two layered analytical model ($\phi_e$).

To present a comprehensive parametric analysis two cases were considered: (i) $D_1 \neq D_2$, which is more likely to be found for materials with different porosity, and (ii) $D_1$=$D_2$, which represents a theoretical idealization of porous materials, being very useful to understand the accuracy of the proposed model.

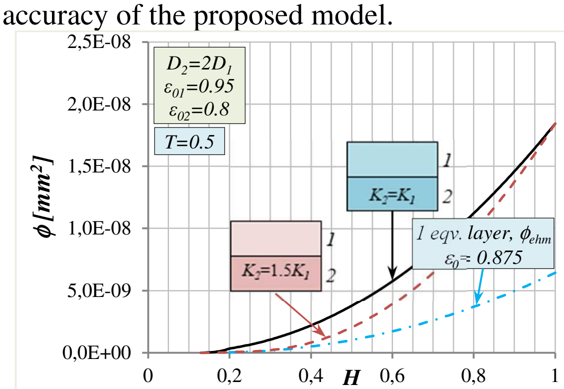


**Fig. 10.** Permeability variation with H, for different structured materials and $D_2$=2$D_1$

The permeability variation with $H$, in the case of $D_2$=2$D_1$, is represented in ***Fig. 10***. The differences between one equivalent layer and two-layered model are reduced significantly.

From this graph it can be stated the fact that the Kozeny-Carman parameter *D*, plays an important role in the permeability–porosity variation.

Writing eq. (5) in dimensionless form, it is found:

$$\bar{F} = \frac{\pi}{8}\frac{D}{H\phi} \tag{38}$$

where, *D* and $\phi$ are applied for both models ($D_e$ and $\phi_e$, respectively $D_{ehm}$ and $\phi_{ehm}$).

Eq. (38) shows that dimensionless contact force at a given level of compression, *H*, depends on equivalent permeability.

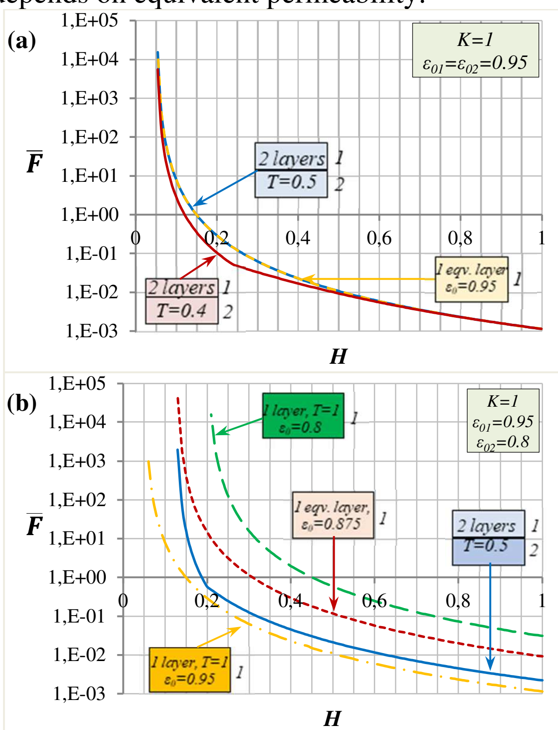


**Fig. 11.** Force variation with relative thickness

***Fig. 11–a***, presents dimensionless force, $\bar{F}$, variation with relative thickness, *H*, for materials with the same structure ($\varepsilon_{01}=\varepsilon_{02}$). Obviously, for materials with the same rigidity and thickness (*K*=1, *T*=0.5), the force is equal when computed with both models. If the layers have different thickness, *T*=0.4, the value of the force is slightly smaller. For different structured materials (***Fig. 11–b***), using an equivalent layer model led to very different results. In ***Fig. 11-b***, it is presented the situation when instead of two layers a single homogeneous layer with *T*=1 (the thickness of the single homogeneous layer is equal with the thickness of the two-layered material) is used. The homogenous layer has the structure and porosity of either material from the analyzed porous pack ($\varepsilon_0=\varepsilon_{01}$, respectively $\varepsilon_0=\varepsilon_{02}$). One can see that the force response of two-layered material is comprised between the response of each constituent material with the same thickness.

## 4. CONCLUSIONS

An analytical model for in-plane axisymmetric squeeze based on the equivalent permeability – porosity variation of a two-layered highly deformable porous pack was proposed. The aim of this paper was to check through comparative assessment if an equivalent layer offers better results than the two-layered model. Permeability–porosity variation was governed by Kozeny-Carman equation.

This model admitted all the classical assumptions of XPHD lubrication. The numerical analysis was dedicated (but not limited) to open-cell foams, tridimensional fabrics and other soft, porous sandwich materials, which are of interest for our research.

It was observed that the two-layered model behaves differently than one equivalent layer, in terms of permeability–porosity variation and dimensionless force.

This can be extended relatively easily for structures with more than two layers. The numbers of layers will increase the difficulties encountered in solving much more complex formulas and possibly relatively difficult to manipulate.

To determine the total force generated during compression, the solid structure force of deformation must be added to fluid squeeze force.

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## VARIAȚIA PERMEABILITĂȚII CU NIVELUL DE COMPRIMARE AL MATERIALELOR POROASE DUBLU STRAT ÎMBIBATE CU LICHIDE

**Rezumat:** Expulzarea unui fluid dintr-un strat de material poros ușor deformabil generează forțe portante importante. Mecanismul denumit eX-Poro-HidroDinamic (XPHD) depinde de permeabilitatea materialului poros și variația ei cu porozitatea, care la rândul ei este dependentă de nivelul de comprimare. Lucrarea propune un model analitic simplu, de calcul al variației permeabilității cu grosimea a 2 straturi de materiale poroase diferite, suprapuse și comprimate. Analiza abordează cazul curgerii plane, axial simetrice (configurația disc/plan). Ecuațiile sunt dezvoltate admițând corelația permeabilitate-porozitate guvernată de clasica lege Kozeny-Carman. Se evaluează și o posibilă abordare simplificată bazată pe calculul permeabilității echivalente în funcție de porozitatea echivalentă a 2 straturi. Modelul propus permite o analiză parametrică realizată pentru valori ale parametrilor caracteristici materialelor celulare, materialelor țesute tridimensionale și ale altor materiale de tip sandwich.

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