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EQUIPMENT FOR MILLING CIRCULAR ARC TOOTH TRACE CYLINDRICAL GEAR

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Abstract: *Compared to gears that involve straight teeth, some advantages of curved teeth are known. For a better understanding of the behavior of circular arc tooth cylindrical gear, requirements have been formulated for designing equipment adaptable to a vertical milling machine and enabling the manufacture of such gears. The possibility of using a milling head and rotating the cylindrical workpiece by taking a movement from the feed screw of the milling machine table was considered. The principle scheme of the equipment, adaptable to a horizontal milling machine and which will allow obtaining the circular arc trace cylindrical gears by intermittent dividing, has been designed.*
Key words: *cylindrical gear, circular arc tooth trace, milling head, rolling machining, intermittent dividing.*

1. INTRODUCTION

The use of gears in the structure of various mechanisms and devices in machine manufacturing is well known. The great diversity of gears led to different processes for obtaining their teeth. On the other hand, the need to meet certain accuracy requirements has imposed a sharp development of tools, devices, and testers usable in manufacturing gears.

Particular attention must be paid to how the toothing is generated by considering the so-called directrix and generatrix curves, respectively. For most gears used in machine manufacturing, the shape of the tooth profile (of the generatrix curve) is that of an involute.

In the case of cylindrical gears, the directrix curve can be along a line segment (straight teeth), a helical curve (inclined teeth), and, respectively, a certain another curve, such as the epicycloid, the hypocycloid, the circle, the involute, etc. [1].

Compared to straight teeth, inclined and curved teeth have some advantages related to a quieter and less noisy operation, a greater length of the contact line, the possibility of transmitting higher torques, etc.

If the curved conical teeth were more intensively researched and used, the curved cylindrical teeth would be less often the object of in-depth research.

Zhang et al. established the tooth surface equations and meshing line equation specific to the circular arc tooth trace cylindrical gears and proposed using a device that includes one or several gear pairs that can be included in the tool composition [2].

Machining techniques by milling circular arc tooth trace cylindrical gears were analyzed or proposed by Stănășel et al. [3-15].

Obtaining cylindrical teeth with circular arc directrix on CNC milling machines was the objective of some of Cioara's concerns [12-14].

The processes that develop in the contact area between the teeth in the case of a circular arc tooth trace cylindrical gear were investigated by Ma et al. [16] and Wei et al. [7].

The equipment presented in the accessed specialized literature usually requires specialized machine tools, which are quite expensive and more difficult or inaccessible to small mechanical workshops when the problem arises of making circular arc tooth trace cylindrical gear. Of course, specialized equipment ensures high precision of the teeth processed on them.

This work was considered to present a technology for obtaining circular arc tooth trace cylindrical gears on a milling machine with a main shaft placed in a vertical position. A device placed on the table of such a machine provides conditions for simulating the rolling of the wheel being processed on a rack whose single tooth materialized by rotating a milling head, whose cutting teeth have a rectilinear profile in a cross-section. The device proposed in the paper can be used in the case of small mechanical workshops when it is desired to obtain circular arc tooth trace cylindrical gears. However, in contrast to the case of specialized equipment for processing cylindrical gears with circular arc tooth traces, the device does not provide high precision.

2. INITIAL CONSIDERATIONS ON MACHINING CIRCULAR ARC TOOTH TRACE CYLINDRICAL GEAR

One of the first curved teeth used mainly in bevel gears was the teeth in which the direction along which the profile of a tooth moves was a circular arc. As previously mentioned, it is accepted that the main advantages of curved teeth to straight teeth relate to a smoother transmission of movement, a greater degree of coverage, and, therefore, a reduction in the mechanical stress borne by a tooth, a more advantageous tooth bending stress, the possibility of achieving higher gear ratios, a less noisy operation of the gear, etc. [1, 17]. A disadvantage of gears with circular arc teeth is the need to use specialized machine tools or devices that can be adapted to some of the universal machine tools.

In principle, the teeth of cylindrical gears can be obtained by processes involving the removal of material from the workpiece in the form of chips, the plastic deformation of the superficial layer of the workpiece, and, in recent decades, the successive addition of material until the finalization of the toothed area.

Suppose it refers strictly to the processes for obtaining the teeth of cylindrical wheels by cutting. In that case, it can see the use of some tools that work on the principle of copying (the profile of the tool in a certain section corresponds to the profile of the gap between the teeth of the machined wheel) and, respectively, of some processes based on rolling or gearing. In this last case, the gearing of the gear to be obtained is simulated with another toothed element (cylindrical gear wheel, worm wheel, rack) acting as a cutting tool. To be able to contribute to the removal of material from the workpiece gear wheel on the toothed surfaces of the second toothed element (cylindrical gear wheel, worm wheel, or rack), certain surfaces were processed, which allowed the generation of flanks and rake surfaces and respectively some angles with convenient values from the point of view of the initiation and development of a cutting process.

The analysis of how to obtain cylindrical toothing by cutting still highlights the possibilities of continuous dividing, in which the dividing takes place continuously, and, respectively, of some toothing processes by intermittent dividing. In the latter case, it is necessary to use specialized equipment capable of ensuring the accurate periodic rotation of the workpiece with a certain angle, as is, for example, the dividing head.

If, in the case of straight toothing, the gear wheel and the tooth are characterized by values corresponding to the maximum diameter, the minimum diameter, the number of teeth, the module, the height of the tooth, etc., in the case of the curved toothing, it is still necessary to take into account some dimensions that will refer to the directrix line of the gear tooth to be obtained. For example, in the case of circular arc tooth trace cylindrical gears, it is necessary to have some information regarding the outer and inner diameter of the circles that define the circular arc, etc.

3. TECHNIQUES FOR OBTAINING CIRCULAR ARC TOOTH TRACE CYLINDRICAL GEAR

Consistent with the previously mentioned, cylindrical teeth were obtained by cutting or plastic deformation until a few decades ago. The processes that allowed the generation of cylindrical teeth by cutting involved using tools from disc cutters, finger cutters, front cutters, or planing-mortising tools.

The last decades have shown a continuous expansion of additive manufacturing processes, and such a trend of evolution of manufacturing processes has become applicable in gear manufacturing.

The expansion of the use of additive manufacturing processes has been favored by some advantages of such manufacturing technologies, namely the use of more easily accessible manufacturing equipment, the removal of the need to design and manufacture cutting tools, and the existence of certain kinematic chains of the machine tools used in manufacturing processes, etc.

At the same time, additive manufacturing requires the development (currently computer-assisted) of a numerical control program that determines the successive deposition of material, the existence of specialized equipment and materials necessary for the additive manufacturing of gears, etc. It is also necessary to mention that, at present, parts made of plastic materials or composite materials with a polymer matrix are mainly made by additive manufacturing.

However, equipment ensures the production of some parts and, therefore, some gears made of metal materials through additive manufacturing technologies. A disadvantage of manufacturing parts using additive technologies is the relatively long duration of the manufacturing processes.

In this work, it was proposed to manufacture some circular arc tooth trace cylindrical gear cutting. An intermittent dividing process was preferred, in which case the gearing of the workpiece wheel with a rack would be simulated. A milling head would materialize a curved rack tooth (fig. 1).

Analysis of the graphic representation in figure 1 highlights the need for the milling head to perform a rotational movement. At the same time, the rolling of the workpiece on the rack implies the realization of a rotation of the workpiece, correlated with a rectilinear movement of the axis of symmetry of the workpiece, to fulfill the condition of rolling on the imaginary rack of the wheel whose teeth it is necessary to obtain.

4. MACHINING OF CIRCULAR ARC TOOTH TRACE CYLINDRICAL GEAR USING A MILLING HEAD

Consideration was given to designing equipment allowing a circular arc cylindrical gear to be obtained on a vertical milling machine.

As shown in figure 1, the milling head will be mounted in the conical bore in the main shaft of the milling machine. In contrast, the various components of the equipment, intended to materialize a rolling movement of the workpiece and, respectively, a dividing movement required for the transition from machining a gap between two teeth to machining another gap, will be mounted on the table of the vertical milling machine.

The possibility of materializing a tooth of the rack with which the wheel machined by the teeth of a Gleason milling head will mesh was considered. Such circular arc tooth bevel milling heads are currently used on machines proposed initially by the American company Gleason.

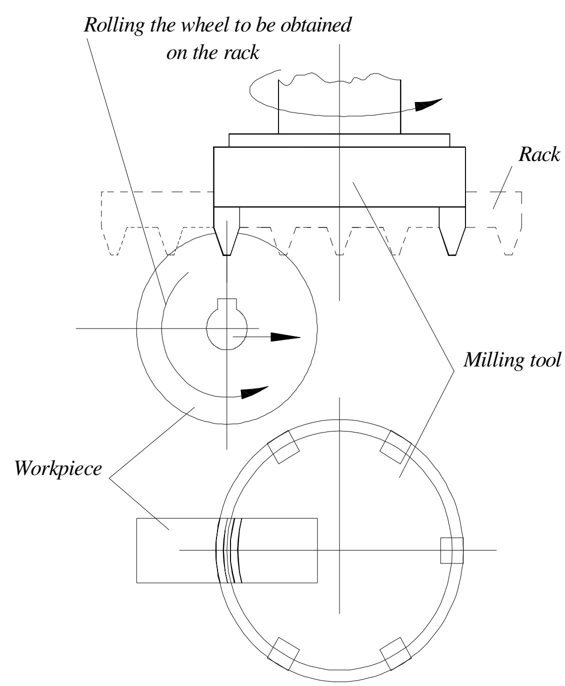

Fig.1. Generating a gap between two teeth of the spur gear by simulating the workpiece rolling on a rack whose single tooth is generated by rotating the milling head

Since the rack has a fixed position, it is also necessary to ensure the conditions for rolling the workpiece on the rack. It is thus necessary for the workpiece to make a rotational movement, while the table of the milling machine will make a feed movement along a direction parallel to the direction of the rack. At the same time, a rotation of the workpiece with a rotation speed correlated with the feed movement of the machine table is required.

5. THE PROPOSED CONSTRUCTIVE SOLUTION

Schematic representations of how the conditions for rolling the workpiece on the rack whose curved tooth is materialized by rotating the milling head are provided in figures 2 and 3.

A locating and clamping of the workpiece of a cylindrical gear in a universal chuck with jaws were thus considered. Suppose the simple locating and clamping of the workpiece in the universal chuck does not ensure a sufficiently rigid clamping. In that case, additional support can be resorted to using a suitable device.

The table of the milling machine can be driven in a feed movement along a rectilinear direction either manually by turning the hand wheel and, respectively, by mechanically turning the screw to achieve the feed movement of the table using a bevel gear, which receives a movement of rotation from the feed gearbox of the machine tool.

A rotary motion is taken from the screw, using a gear train, to affect the feed motion of the table. This movement is transmitted to the input shaft of the dividing head.

Since conventional dividing heads incorporate an output shaft parallel to the input shaft in the dividing head, it was necessary to include in the kinematic chain of rotating the workpiece a subassembly that includes a bevel gear so that the output shaft of this subassembly has an axis perpendicular to the output axis of the shaft from the dividing head. A device for locating and clamping the workpiece is mounted on the output shaft of the subassembly for changing the direction of the output shaft axis of the dividing head. In the case of the considered solution, a universal chuck was considered.

In the previously described way, conditions are provided for the rotation of the workpiece around its axis and, at the same time, for the rectilinear movement of the workpiece together with the table of the milling machine on the upper surface of which the equipment components are mounted.

To highlight some aspects related to the correlation that must exist between the pitch p_E of the curved teeth of the rack and the pitch p_s of the feed screw of the milling machine table, the schematic representation in figure 4 can be considered, in which it can also be observed how some elements of the dividing head are involved in the kinematic chain to obtain some movements for simulating the engagement conditions of the cylindrical gear wheel to be obtained with the tooth materialized by rotating the milling head.

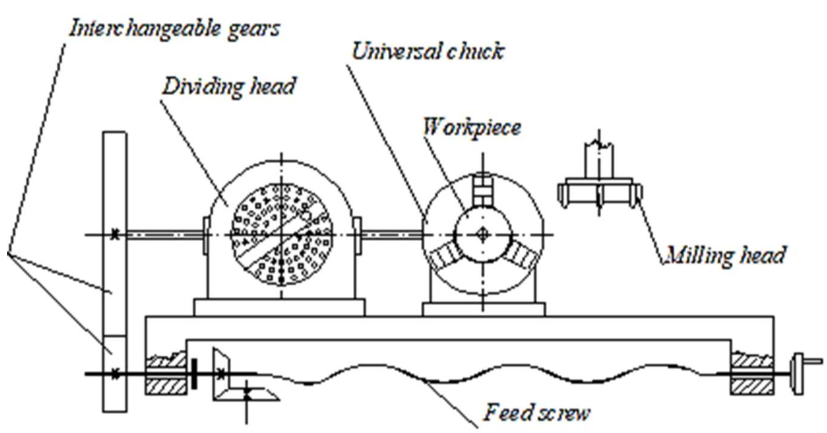

Fig.2. Front view of the device for machining circular arc tooth trace cylindrical gear

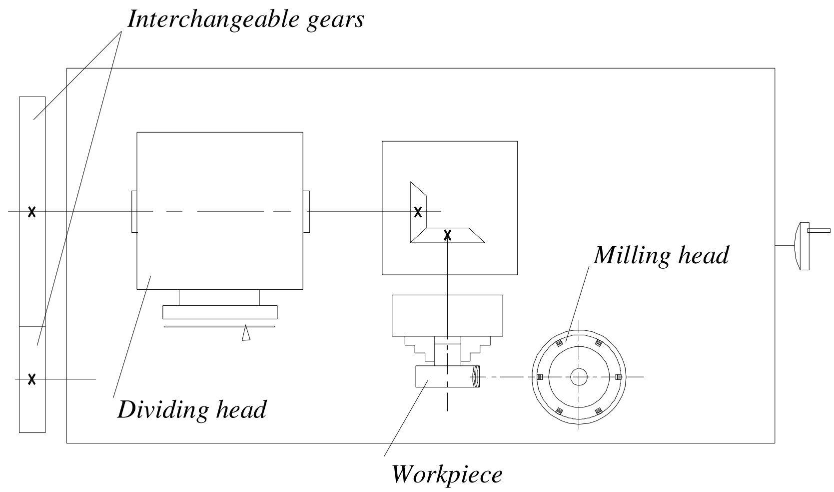

Fig.3. Top view of the device generating circular arc cylindrical teeth on a milling machine with a vertical main shaft

It is thus found that if a gear train with four movable gears is used, having the numbers of teeth z_A, z_B, z_C, and z_D, a transmission ratio k characterizes the dividing head, and the bevel gear provides a unity transmission ratio, the relationship corresponding to the operation of this kinematic chain is as follows:

$$\frac{z_A}{z_B} \cdot \frac{z_C}{z_D} = k \cdot \frac{p_s}{p_E}, \qquad (1)$$

where p_s is the pitch of the screw for making the feed movement of the milling machine table, and p_E is the pitch of the simulated rack that engages the workpiece wheel.

If the gear train comprises only two cylindrical gears, with tooth numbers z_A and z_B, then the previous relation has the form:

$$\frac{z_A}{z_B} = k \cdot \frac{p_s}{p_E}. \qquad (2)$$

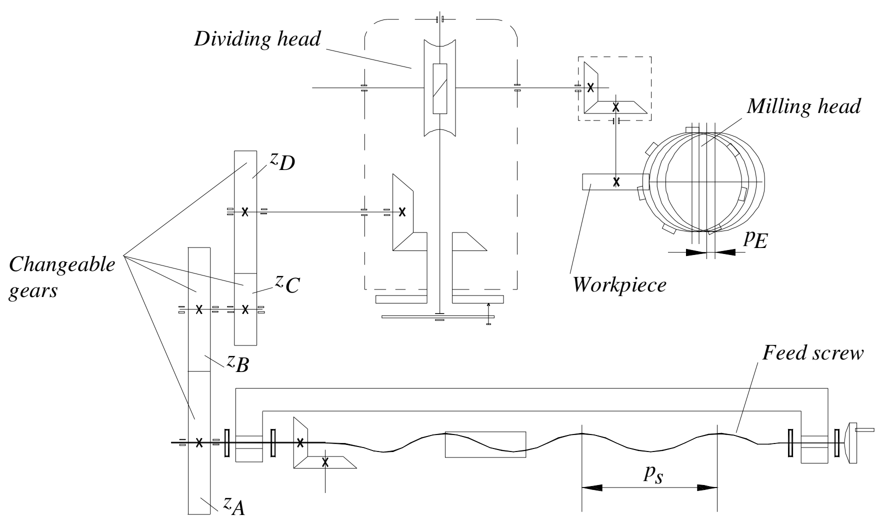

Fig.4. The use of the dividing head to transmit the rotational movement to the workpiece and to achieve an intermittent division

The previously mentioned conditions ensure the rolling of the toothed wheel on the rack whose tooth is materialized by the cutting teeth of the milling head, which is in rotational motion. After machining a gap between two teeth, the dividing head is used to bring the workpiece into the position corresponding to the machining of the next gap between the teeth of the machined wheel.

6. MILLING HEAD

A milling head is used to process the teeth, on the periphery of which cutting teeth are mounted. The tool's solution is similar to that of the Gleason's head, frequently used in machining conical gears with circular arc teeth. The circular arrangement of the cutting teeth on its periphery ensures the circular trajectory of the guide curve.

The geometry of the active areas of the cutting teeth must take into account the geometry of the reference rack, which allows the realization of teeth that respect the principle of gearing: "all gears that have the same reference rack or are made with the same generating rack (the negative of the reference rack) mesh between them" [18].

According to the provisions of the standards, the rectilinear profile of the reference rack is symmetrical about the line on which the thickness of the teeth is equal to the gap between the teeth, that is, on the reference line.

Therefore, the milling head will be equipped with cutting teeth with straight cutting edges, inclined to the axis of rotation at an angle equal to the engagement angle of $\alpha = 20^o$ (fig. 5).

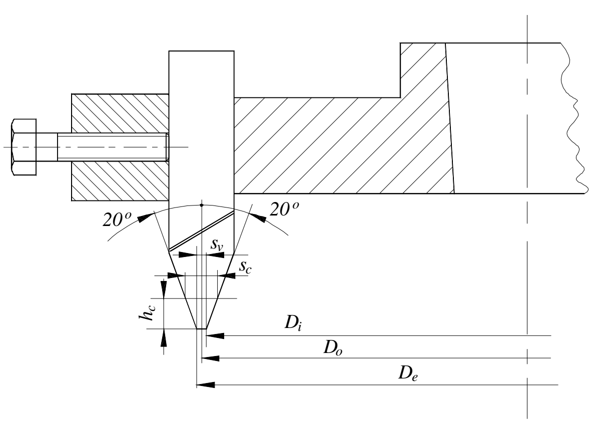

Fig.5. Specific dimensions of the active area of the tool on the milling head

Some of the cutting teeth will machine the concave flank of the gear tooth (the cutting teeth on a circumference of diameter D_e, and the others, on a circumference of diameter D_i, will generate the convex flanks of the teeth). The arrangement of the two categories of cutting teeth is done alternately.

Ensuring the accuracy of the arrangement of the tool's cutting edges takes place by simultaneously sharpening the same types of cutting tools.

To find out the dimensions of the width S_v of the tips of the cutting teeth, the mathematical relationship can be used:

$$S_v = S_c - 2h_c/2.74, \qquad (3)$$

where S_c represents the length of the constant chord of the tooth in the plane normal to the tooth, and h_c is the height at the constant chord of the tooth. The values of the two parameters can be calculated using known mathematical relations [19]:

$$S_c = m\left(\frac{\pi}{2}cos^2 \propto_0 + x\ sin2\alpha_0\right), \qquad (4)$$

$$h_c = 0.5(d_{ac} - d - S_c \text{ tg } \alpha), \qquad (5)$$

where m is the modulus of the toothing, α_0 is the pressure angle of the toothing (20°), x - the specific displacement of the profile (for undisplaced teeth, x=0), if d_{ac} is the diameter of the head circle of the wheel, d – the diameter of the rolling circle.

The connection radii from the tips of the cutting teeth must have values of 0.5-3 mm; these values correspond with the values of the connection radii at the bottom of the gap between two teeth.

To reduce the cutting force, especially on gears with a modulus higher than 1-1.5 mm, roughing toothing heads can be used, which will cut simultaneously with all three cutting edges. Such a situation requires that three cutting tools be mounted on the toothing head.

The middle cutting tool being higher with size a (fig. 6) and entering the cutting first will ensure favorable conditions for the side cutting

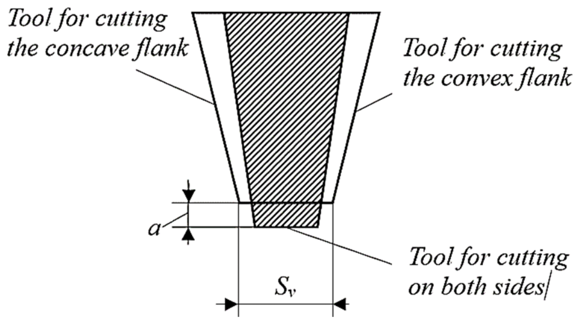

Fig.6. Milling head for tooth roughening

tools, which finalize the flanks of the teeth. The dimension *a* of the overhang is dependent on the width S_v of the tools tips, dependent, in turn, on the modulus of the teeth, and can be between 1 and 3 mm.

7. CONCLUSIONS

The existence of some advantages of circular arc tooth trace cylindrical gear has led to an increase in researchers' interest in establishing specific manufacturing technologies for such gears. In the case of research, the results of which are presented in this paper, it was decided to design equipment that would allow the processing of teeth in a circular arc on milling machines that have a vertical main shaft. The equipment components provide conditions for simulating the meshing between the machined gear and a flat rack. A rack tooth is materialized using a milling head driven in rotational motion. A dividing head is used to change from milling a gap between two teeth to milling another gap. The solution proposed for machining the circular arc tooth trace cylindrical gears is simpler and cheaper than the one that involves specialized machining equipment. Such specialized equipment, however, ensures a higher machining accuracy of the teeth produced. It is planned to make cylindrical gears that will be subjected to tests specific to these categories of machine parts.

8. REFERENCES

[1] Grămescu, T., Slătineanu, L., Pruteanu, O., Marin, A., *Gear teething technologies. Design manual* (in Romanian). Publishing House Universitas, Chișinău, 1993.

[2] Zhang, Q., Hou, L., Tang, R., Wen, G., *Method of processing and an analysis of meshing and contact of circular arc tooth trace cylindrical gears*, Transaction of FAMENA, XL-4, 2026, https://hrcak.srce.hr/file/257869.

[3] Stănăşel, I., Cherman, I., Mihăilă, I., Pantea, I., *Considerații privind bombarea danturii curbe cicloidale* Annals of the Oradea University, Fascicle of Management and Technological Engineering, 2005, https://imt.uoradea.ro/auo.fmte/files-2005/TCM_files/Iulian%20Stanasel%201.pdf.

[4] Fu, Q., Tang, R., *An improved processing method of circular-arc-tooth-trace cylindrical gear*, DEStech Transactions on Engineering and Technology Research, 2017, DOI: 10.12783/dtetr/icamm2016/7393.

[5] *Circular arc tooth profile circular arc tooth line cylindrical gear*, 2022, https://www.zhygear.com/circular-arc-tooth-profile-circular-arc-tooth-line-cylindrical-gear/.

[6] Su, J., Yan, Z., Feng, S., Wu, W., *Machining method of circular-arc-tooth-trace gear,* Patent application China no. CN109773279B**,** 2019, https://worldwide.espacenet.com/patent/search/family/066491043/publication/CN109773279A?q=pn%3DCN109773279A.

[7] Wei, Y., Guo, R., Liu, Y., Dong, C., Li, D., Wan, A., Zhao, G., *Stress of cylindrical gear with variable hyperbolic circular-arc-tooth-trace*, Symmetry, 12, 2020, 1318; doi:10.3390/sym12081318.

[8] Guo, R., Wei, Y., Liu, Y., Li, D., Yang, D., Zhao, G., *Analytical solution to contact characteristics for a variable hyperbolic circular-arc-tooth-trace cylindrical gear*, Mechanical Sciences, 12, 923–932, 2021, https://doi.org/10.5194/ms-12-923-2021.

[9] Andrei, L., Epureanu, A., Andrei, G., *Equipment for processing gears, cylindrical, with teeth in circular arc, variable in height*, Patent application Romania, no. ROA201001069A, 2010, https://patentimages.storage.googleapis.com/94/8a/9c/6322e2fd4210de/RO127384A2.pdf.

[10] Stănăşel, I., *Technique for generation of cylindrical tooth cycloidal curves*, Patent application Romania, no. RO 128815 A2, 2012, https://patentimages.storage.googleapis.com/84/7b/ba/e15b39883c9144/RO128815A2.pdf.

[11] Cioară, G.R., Pisarciuc, C., Iordăchel, B.L., *Cylindrical gear with circular arc guides and cylindrical gear milling machine with circular arc guides* (in Romanian). Patent application Romania, RO 125758 A2, 2009, https://patentimages.storage.googleapis.com/d6/25/a2/8e612c97852d20/RO125758A2.pdf.

[12] Cioară, R.G., *Cylindrical teeth milling machine with circular arc guides, with numerical control (2)* (in Romanian), Patent application Romania, no. RO133825A0, 2019, https://patentimages.storage.googleapis.com/3c/25/9f/26eae44f17fe04/RO133825A0.pdf.

[13] Cioară, R.G., *Cylindrical teeth milling machine with circular arc guides, with numerical control (3)* (in Romanian), Patent application Romania no. RO133826A0, 2019, https://patentimages.storage.googleapis.com/9c/75/d2/646beded669b6f/RO133826A0.pdf.

[14] Cioară, R.G., *Cylindrical teeth milling machine with circular arc guides, with numerical control (1)* (in Romanian), Patent application Romania no. RO 133824 AO, 2019, https://patentimages.storage.googleapis.com/5a/d0/09/dab0faf9a3e14e/RO133824A0.pdf.

[15] *Cylindrical gear wheel with circular arc guides and cylindrical gear milling machine with circular arc guides* (in Romanian), https://www.qdidactic.com/stiinta-tehnica/index.php.

[16] Ma, D., Liu, Y., Ye, Z., Wei, Y., Li, D.,1 Zhang, X., *Meshing contact impact properties of circular arc tooth trace cylindrical gear based on rotating knife dish milling process*, Mathematical Problems in Engineering, 8819818, 2021, https://doi.org/10.1155/2021/8819818.

[17] Picoş, C., *Machine manufacturing technology* (in Romanian), Editura Didactică și Pedagogică București, 1974.

[18] Rădulescu, G., Miloiu, G., Gheorghiu, N., Muntean, C., Vișa, F., Ionescu, N., Popovici, V., Dobre, G., Rașeev, M., *Design Guide in Machine Construction* (in Romanian), vol III, Technical Publishing House, București, 1986.

[19] Jula, A., Velicu, D., Dudiță, F., Moldovean, G., Chișu, E., Diaconescu, D.V., Vișa, I., *The design of involute gears* (in Romanian), Publishing House Scrisul Românesc, Craiova, 1989.

Echipament pentru frezarea danturii în arc de cerc a roților dințate cilindrice

În comparație cu roțile dințate care implică dinți drepți, sunt cunoscute unele avantaje ale dinților curbați. Pentru o mai bună cunoaștere a comportamentului roților dințate cilindrice cu dinți în arc cerc, au fost formulate cerințe pentru proiectarea unui echipament adaptabil pe o mașină de frezat orizontală și care să permită fabricarea unor astfel de roți dințate. S-a luat în considerare posibilitatea utilizării unui cap frezat și rotirea semifabricatului prin preluarea unei mișcări de la șurubul de avans al mesei mașinii de frezat. A fost concepută schema de principiu a dispozitivului, adaptabilă pe o mașină de frezat cu arbore principal vertical și care va permite prelucrarea roților cilindrice cu dinți în arc de cerc, prin divizare intermitentă.

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