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RELATIVE SLIDING IN MULTIPLE PAIR CONVEX-CONCAVE CONTACT OF TEETH, WITHIN PRECESSIONAL GEARING

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Abstract: *The paper describes the study of the relative sliding in the convex-concave contact of multiple pair conjugated teeth in precessional gearing with straight* $A^{D}_{CX\text{-}CV}$ *and inclined* $A^{D,\beta}_{CX\text{-}CV}$ *teeth. The paper presents the influence of the gearing parameters defined by configuration* $[Z_g-\theta, \pm 1]$ *on the geometry of the teeth contact and the difference of the flank curves, as well as the influence of the change in the shape of the teeth by shortening their height on the reduction of the relative sliding with friction in the contacts* k_i *of the pairs of teeth simultaneously conjugated to one and same angular coordinate* ψ *of the crank shaft. The energy losses in the gears are analyzed according to the relative sliding with friction between the flanks of unitary or simultaneously multiple pair conjugated teeth, as well as the bearing capacity of the convex-concave contact of the teeth depending on the difference in the flank curves.*
The experimental researches were carried out on samples of transmissions with the central wheels and the satellite made of various tribo-couples of plastic-metal materials with the application of different lubrication regimes with plastic lubricants and liquid one. Samples of kinematic and low-power precessional transmissions are presented in the paper.
Key words: *mechanical transmissions, bearing capacity of convex-concave contact, relative sliding between flanks, CNC generation of teeth.*

NOTATIONS USED IN THE PAPER

A^B – precessional gear with bolts;
A^D – precessional gear with straight teeth;
$A^{D,\beta}$ – precessional gear with straight teeth;
$A^{B}_{CX\text{-}R}$ – precessional gearing with bolts with *convex-rectilinear* contact;
$A^{B}_{CX\text{-}CV}$ – precessional gearing with bolts with *convex-concave* contact;
$A^{D}_{CX\text{-}CX}$ – precessional toothed gearing with *convex-convex* contact;
$A^{D}_{CX\text{-}CV}$ – precessional toothed gearing with *convex-concave* contact;
$A^{D}_{CX\text{-}R}$ – precessional toothed gearing with *convex-rectilinear* contact;
$A^{D,\beta}_{CX\text{-}CV}$ – precessional toothed gearing with *convex-concave* contact of inclined teeth;
$[Z_g-\theta,-1]$ – parametric configuration of the precessional gear with the correlation of the numbers of teeth of the conjugated wheels $Z_{1(4)}=Z_{2(3)}-1$ and reducer operating mode;
$[Z_g-\theta,+1]$ – parametric configuration of the precessional gear with the correlation $Z_{1(4)}=Z_{2(3)}+1$ and multiplier operating mode (of reducer in case $\delta_{1(4)-2(3)}=0°$);
$G^{con}_{r.s}$ – generation of tooth flanks by spatial tumbling-rolling with a *cone trunk* tool shape;
$G^{cil}_{m.ax}$ – generation of straight teeth with *cylindrical* tool on multi-axis CNC machine tools;
$G^{cil,\beta}_{m.ax}$ – generation of inclined teeth with *cylindrical* tool on multi-axis CNC machine tools;
2K-H, 4K-2H, 6K-3H – kinematic structures with two, four and six central wheels, respectively;
$K_{CX\text{-}CV}$ – convex-concave contact of straight teeth;
$K^{\beta}_{CX\text{-}CV}$ – convex-concave contact of inclined teeth.

1. KINEMATICS OF 2K-H TYPE PRECESSIONAL TRANSMISSIONS

2K-H, 4K-2H, and 6K-3H precessional transmissions are synthesized based on the 2K-H kinematic structure with toothed gearings $A^{D}_{CX\text{-}CV}$ and $A^{D,\beta}_{CX\text{-}CV}$ multiple-pair convex-concave, characterized by high bearing capacities, minimal energy losses and extended kinematic possibilities [1, 2, 4].

The 2K-H type precessional transmission includes satellite-wheel *g* with two toothed crowns Z_{g_1} and Z_{g_2}, which is in gearing with the immobile *b* and mobile *c* central wheels, rigidly linked to the driven shaft *V* (fig. 1).

When the crank shaft *H* rotates with the angular velocity ω_H, the satellite *g* performs spherospatial motion with a fixed point *O* called the precession center.

The transmission ratio is determined by the relation:

$$i = -\frac{Z_{g_1} Z_c}{Z_b Z_{g_2} - Z_{g_1} Z_c}, \qquad (1)$$

where Z_{g_1}, Z_{g_2} are the numbers of teeth of the satellite-wheel and Z_c, Z_b – number of teeth of the central wheels *c* and *b*.

The analysis of relation (1) shows that the 2K-H transmissions ensure the achievement of a wide range of transmission ratios.

Depending on the tooth numbers Z_b, Z_c, Z_{g_1}, Z_{g_2} and of their correlation, in transmissions with gearings $(Z_b - Z_{g_1})$ and $(Z_c - Z_{g_2})$ if $Z_b = Z_{g_1} - 1$, $Z_c = Z_{g_2} - 1$, $Z_{b,\, g_1,\, c,\, g_2} \le 60$, $\pm 20 \le i \le \pm 3600$ transmission ratios are ensured.

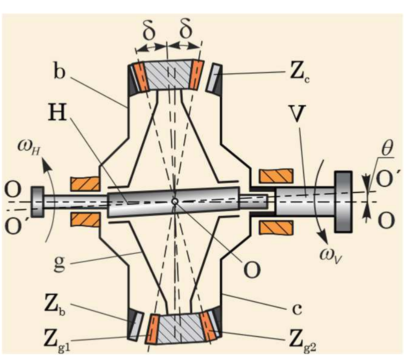


Fig.1. The kinematic structure of the 2K-H type precessional transmission [1]

In case $Z_{g_1} > Z_{g_2}$ the driving and driven shafts rotate in different directions (–), and in case $Z_{g_1} < Z_{g_2}$ in the same direction (+).

Under these conditions, if $Z_b = Z_{g_1} - 1$ and $Z_c = Z_{g_2} + 1$ the 2K-H transmission ensures transmission ratios in the range $+8{,}3 \le i \le +30{,}3$, and if $Z_b = Z_{g_1} + 1$ and $Z_c = Z_{g_2} - 1$ the 2K-H transmission ensures transmission ratios $-7{,}3 \le i \le -29{,}3$.

2. HYPOTHESES OF CONVEX-CONCAVE CONTACT OF TEETH IN THE GEARINGS $A^{D}_{CX\text{-}CV}$ AND $A^{D,\,\beta}_{CX\text{-}CV}$

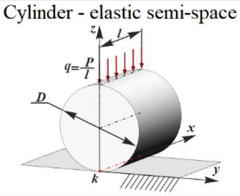
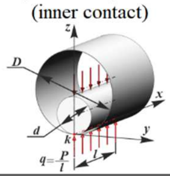
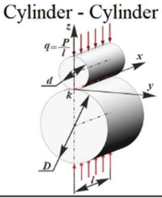
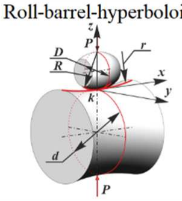
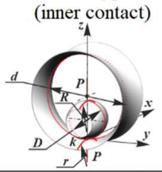
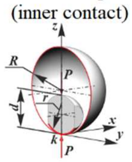
The creation of precessional gears comes down to ensuring the transformation of motion according to the principles of the fundamental law of gearing and load transmission through surfaces with punctiform or linear contact of the teeth of the toothed wheels with concurrent axes.

The classical theory of approximate contact between deformable bodies under contact conditions of tooth-tooth kinematic couples is based on the assumptions:

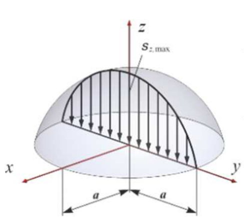
- Contact surfaces are perfectly continuous and smooth.
- Bodies in contact are homogeneous, isotropic and linearly elastic.
- Initially the contact is punctiform or linear, and the dimensions of the contact footprints after loading with a load are small in relation to the dimensions of the contracting bodies.
- Stress distribution in the contact zone follows from the Boussinesq elastic half-space theory and therefore the tangential stresses are zero.

In figure 2 (a), p. 1 – 3, examples of kinematic couplings with linear contact are presented, and in p. 4 – 7 with punctiform contact, characteristic for the teeth of the wheels of precessional gearings [5].

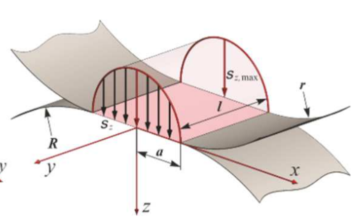
The linear contact between conjugated teeth in precessional gearing depending on the precession angle ψ transforms from contact with convex-concave geometry with a small difference in curvatures to contact with convex-rectilinear geometry and later, with convex-convex geometry.

Contact geometry in precession gears	Semi-axes of the footprint a and b, the stress of contact $\sigma_{z\,max}$ and the parameter of curvatures difference θ
Cylinder - elastic semi-space 	$b=\frac{2.35}{10^3}\left(\frac{DP}{l}\right)^{1/2}$ $\sigma_{z\,max}=271\left(\frac{P}{lD}\right)^{1/2}$ $\theta=0$
Cylinder - Cylinder (inner contact) 	$b=\frac{2.35}{10^3}\left[\frac{P}{l}\,\frac{Dd}{(D-d)}\right]^{1/2}$ $\sigma_{z\,max}=271\left[\frac{P}{l}\,\frac{(D-d)}{Dd}\right]^{1/2}$ $\theta=0$
Cylinder - Cylinder 	$b=\frac{2.35}{10^3}\left[\frac{P}{l}\,\frac{Dd}{(D+d)}\right]^{1/2}$ $\sigma_{z\,max}=271\left[\frac{P}{l}\,\frac{(D+d)}{Dd}\right]^{1/2}$ $\theta=0$
Roll-barrel-hyperboloid 	$a(b)=0.0235n_{a(b)}\frac{P^{1/3}}{\left(\frac{2}{D}+\frac{1}{R}-\frac{2}{d}-\frac{1}{r}\right)^{1/3}}$ $\sigma_{z\,max}=861n_{\sigma}P^{1/3}\left(\frac{2}{D}+\frac{1}{R}-\frac{2}{d}-\frac{1}{r}\right)$ $\theta=\frac{\frac{2}{D}-\frac{1}{R}-\frac{2}{d}+\frac{1}{r}}{\frac{2}{D}+\frac{1}{R}-\frac{2}{d}-\frac{1}{r}}$
Roll-barrel-hyperboloid (inner contact) 	$a(b)=0.0235n_{a(b)}\frac{P^{1/3}}{\left(\frac{2}{D}+\frac{1}{R}+\frac{2}{d}-\frac{1}{r}\right)^{1/3}}$ $\sigma_{z\,max}=861n_{\sigma}P^{1/3}\left(\frac{2}{D}+\frac{1}{R}+\frac{2}{d}-\frac{1}{r}\right)$ $\theta=\frac{\frac{2}{D}-\frac{1}{R}+\frac{2}{d}+\frac{1}{r}}{\frac{2}{D}+\frac{1}{R}+\frac{2}{d}-\frac{1}{r}}$
Roll-barrel-sphere (inner contact) 	$a(b)=0.0235n_{a(b)}\frac{P^{1/3}}{\left(\frac{2}{d}+\frac{1}{r}-\frac{2}{R}\right)^{1/3}}$ $\sigma_{z\,max}=861n_{\sigma}P^{1/3}\left(\frac{2}{d}+\frac{1}{r}-\frac{2}{R}\right)^{2/3}$ $\theta=\frac{\frac{1}{r}-\frac{2}{d}}{\frac{2}{d}+\frac{1}{r}-\frac{2}{R}}$

a)



b)



c)

Fig.2. Punctiform and linear contact geometry in precessional gearing (a), punctiform contact footprint (b) and stresses distribution in linear teeth contact (c) [2]

In the upper class kinematic couplings, the initial punctiform or linear contact, after the application of the bearing load, turns into an imprint with the geometry of an ellipse with the semi-axes „a" and „b" (see fig. 2 a, b, c).

The contact σ_z and deformable stresses depend on the load P, the elastic properties of the material and the geometric shape of the flank surfaces of the teeth, and are determined the classical Hertzian contact theory [5].

The linear-elastic properties of tooth contact are characterized by the Poison coefficients ν_1 and ν_2 and the Young's moduli E_1 and E_2 of the tooth materials. The generalizing shape parameters of the teeth are the equivalent radius of curvature ρ_e and the parameter of the difference of flank curvatures $(\rho_{k_i}-r)$ conjugated in the contacts k_i depending on the precession angle ψ.

3. LINEAR VELOCITY OF THE CONTACT POINT E_1 ON THE TEETH PROFILE OF THE CENTRAL WHEEL

The interaction of conjugated teeth in precessional gearing takes place with the presence of relative sliding between the flanks.

It was found that both the geometric shape of the teeth contact and the kinematics of the contact point depend on the value configuration of the geometric parameters of the gear, noted by $[Z_g-\theta, \pm 1]$, among which Z_1, Z_2, δ, θ, β and the correlation of teeth $Z_1=Z_2-1$ or $Z_1=Z_2+1$.

The position vector of the contact point E_1 of the conjugated teeth, which belongs to the profile of the central wheel tooth (fig. 3), is identified through the vector equation:

$$\mathbf{r}_{E_1}=\mathbf{r}_{G_2}=\mathbf{G}_2\mathbf{E}_1=\mathbf{r}_{G_2}+\mathbf{r}, \qquad (2)$$

where $\mathbf{G}_2\mathbf{E}_1=\mathbf{r}=R\sin\beta\frac{\mathbf{v}_{G_2}\times\mathbf{r}_{G_2}}{\left|\mathbf{v}_{G_2}\times\mathbf{r}_{G_2}\right|}$ and $\mathbf{r}_{G_2}=\mathbf{r}_G\cos\beta$, where $\mathbf{r}$ is the position vector of the point E_1 relative to the point G_2 and has the modulus equal to the radius of curvature of the profile of the satellite teeth with a circle arc profile, $\mathbf{r}_G$ and $\mathbf{r}_{G_2}$ are the position vectors of the origin of the radius of curvature of the teeth in the circle arc of the satellite on the sphere and on the direction GO (fig. 3).

The position vector of the origin of the radius of curvature $\mathbf{r}_{G_2}$ can be expressed by the coordinates x_{G_2}, y_{G_2} and z_{G_2}:

$$\mathbf{r}_{G_2} = x_{G_2}\mathbf{i} + y_{G_2}\mathbf{j} + z_{G_2}\mathbf{k}, \quad (3)$$

and the velocity vector $\mathbf{V}_{G_2}$ of the point G_2 are $V_{G_2} = \dot{r}_{G_2} = \dot{x}_{G_2} i + \dot{y}_{G_2} \mathrm{j} + \dot{z}_{G_2} k$.

The trajectory of the motion on the sphere with radius R of the origin of the radius of curvature r of the profile of the teeth G (X_G, Y_G, Z_G) of the satellite according to the angle ψ, in the fixed coordinate system $OXYZ$:

$$\begin{aligned} X_G &= R\cos\delta\left(-\cos\psi\sin\varphi_\psi + \sin\psi\cos\varphi_\psi\right) \\ &\quad - R\sin\delta\sin\psi\sin\theta; \\ Y_G &= -R\cos\delta\left(\sin\psi\sin\varphi_\psi + \cos\psi\cos\varphi_\psi\cos\theta\right) \\ &\quad + R\sin\delta\sin\psi\sin\theta; \\ Z_G &= -R\cos\delta\cos\varphi_\psi\sin\theta - R\sin\delta\cos\theta, \end{aligned} \quad (4)$$

where R is the radius of the sphere originating in the center of the coordinate system O located in the precession center, δ is the angle of the conical axoid, θ – the nutation angle, ψ denotes the positioning angle of the crankshaft (of precession) and φ_ψ is the rotation angle of the satellite around its own axis.

To determine the position vector of the contact point E_1 (fig. 3), which belongs to the profile of the tooth of the central wheel, we note on the axis OG the point G_2, located in its normal section, and we identify the trajectory of its motion in the fixed coordinate system $OXYZ$ expressed by coordinates X_{G_2}, Y_{G_2}, Z_{G_2} depending on ψ:

$$\begin{aligned} X_{G_2} &= R\cos\beta\cos\delta\left(-\cos\psi\sin\varphi_\psi \right.\\ &\quad \left. + \sin\psi\cos\varphi_\psi\cos\theta\right) \\ &\quad - R\cos\beta\sin\delta\sin\theta\sin\psi; \\ Y_{G_2} &= -R\cos\beta\cos\delta\left(\sin\psi\sin\varphi_\psi \right.\\ &\quad \left. + \cos\psi\cos\varphi_\psi\cos\theta\right) \\ &\quad + R\cos\beta\sin\delta\sin\theta\cos\psi; \\ Z_{G_2} &= -R\cos\beta\cos\delta\sin\theta\cos\varphi_\psi \\ &\quad - R\cos\beta\sin\delta\cos\theta, \end{aligned} \quad (5)$$

where β is the tip angle of the satellite tooth.

The velocity vector projections $\mathbf{V}_{G_2}$ are:

$$\begin{aligned} V_{G_2X} &= R\cos\beta\,\omega\left\{\cos\delta\left[\sin\psi\sin\varphi_\psi\left(1 - \frac{Z_1}{Z_2}\cos\theta\right) - \cos\psi\cos\varphi_\psi\left(\frac{Z_1}{Z_2} - \cos\theta\right)\right] - \sin\delta\sin\theta\cos\psi\right\}; \\ V_{G_2Y} &= -R\cos\beta\,\omega\left\{\cos\delta\left[\cos\psi\sin\varphi_\psi\left(1 - \frac{Z_1}{Z_2}\cos\theta\right) + \sin\psi\cos\varphi_\psi\left(\frac{Z_1}{Z_2} - \cos\theta\right)\right] + \sin\delta\sin\theta\cos\psi\right\}; \\ V_{G_2Z} &= R\cos\beta\,\omega\frac{Z_1}{Z_2}\cos\delta\sin\theta\sin\varphi_\psi, \end{aligned} \quad (6)$$

where Z_1 is the number of teeth of the central wheel; Z_2 – the number of teeth of the satellite; ω – the angular velocity of the crankshaft, which for its rotation frequency n=3000 rpm will be $\omega = 2\pi\, n/60\mathrm{s}^{-1}$.

The projections of the vector $\mathbf{V}_{E_2}$ on the axes X, Y, Z are determined:

$$\begin{aligned} V_{E_1X} &= \dot{x}_{G_2} + \frac{R\sin\beta}{\left(\sqrt{a_x^2 + a_y^2 + a_z^2}\right)^3}\left[\left(a_x^2 + a_y^2 + a_z^2\right)\dot{a}_x - \left(a_x\dot{a}_x + a_y\dot{a}_y + a_z\dot{a}_z\right)a_x\right]; \\ V_{E_1Y} &= \dot{y}_{G_2} + \frac{R\sin\beta}{\left(\sqrt{a_x^2 + a_y^2 + a_z^2}\right)^3}\left[\left(a_x^2 + a_y^2 + a_z^2\right)\dot{a}_y - \left(a_x\dot{a}_x + a_y\dot{a}_y + a_z\dot{a}_z\right)a_y\right]; \\ V_{E_1Z} &= \dot{z}_{G_2} + \frac{R\sin\beta}{\left(\sqrt{a_x^2 + a_y^2 + a_z^2}\right)^3}\left[\left(a_x^2 + a_y^2 + a_z^2\right)\dot{a}_z - \left(a_x\dot{a}_x + a_y\dot{a}_y + a_z\dot{a}_z\right)a_z\right]. \end{aligned} \quad (7)$$

The modulus of the velocity of the contact point E_1 is determined from:

$$V_{E_1} = \sqrt{V_{E_1X}^2 + V_{E_1Y}^2 + V_{E_1Z}^2}. \quad (8)$$

The relative sliding between the teeth flanks in the contact E is determined by the difference between the linear velocities of the contact points on the profiles of the central wheel tooth V_{E_1} and the satellite wheel tooth V_{E_2}, and is defined by the relation:

$$V_{al(\psi)} = V_{E_1(\psi)} - V_{E_2(\psi)}. \quad (9)$$

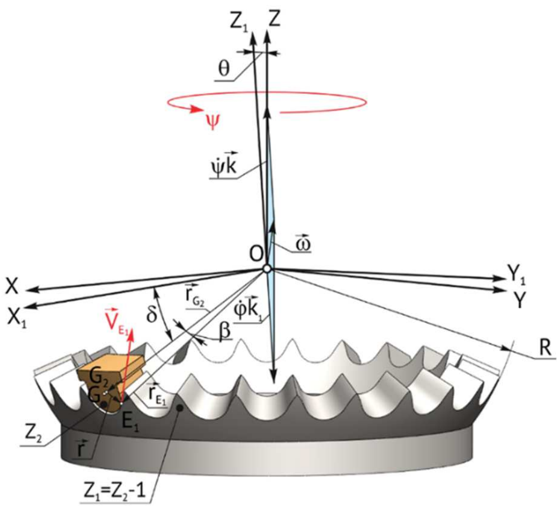


Fig.3. The position of the velocity vector of the contact point $\mathbf{V}_{E_1}$. [2]

4. LINEAR VELOCITY OF THE CONTACT POINT E_2 ON THE TEETH PROFILE OF THE SATELLITE WHEEL

To determine the relative sliding between the conjugated teeth and the assessment of energy losses in the toothed gear, the linear velocity of the point on the profile of the satellite wheel teeth V_{E_2} depending the same positioning angle of the crankshaft ψ is determined (see fig. 4) [3].

$$\varphi = -Z_1\psi/Z_2 = -\psi_1. \qquad (10)$$

The coordinates of the contact point E_2 on the profile of the teeth of the satellite wheel are determined according to the expressions:

$$\begin{aligned} x_{1E_2} &= x_{E_1}\left(\cos\psi\cos\psi_1 + \sin\psi\sin\psi_1\cos\theta\right) \\ &+ y_{E_1}\left(\sin\psi\cos\psi_1 - \cos\psi\sin\psi_1\cos\theta\right) \\ &- z_{E_1}\sin\theta\sin\psi_1; \\ y_{1E_2} &= x_{E_1}\left(\cos\psi\sin\psi_1 - \sin\psi\cos\psi_1\cos\theta\right) \\ &+ y_{E_1}\left(\sin\psi\sin\psi_1 + \cos\psi\sin\psi_1\cos\theta\right) \\ &+ z_{E_1}\sin\theta\sin\psi_1; \\ z_{1E_2} &= x_{E_1}\sin\theta\sin\psi - y_{E_1}\sin\theta\cos\psi \\ &+ z_{E_1}\cos\theta. \end{aligned} \qquad (11)$$

The projections of the velocity vector of the point E_2 on the profile of the satellite tooth are calculated by deriving in relation to time the coordinates x_{E_2}, y_{E_2} and z_{E_2}, which are dependent on the coordinates x_{E_1}, y_{E_1}, z_{E_1} and Euler's angles.

Thus, we obtain the expressions for the projections of the velocity of the point E_2 on the coordinate axes X_1, Y_1, Z_1.

$$\begin{aligned} V_{E_2X_1} &= \dot{x}_{E_1}(\cos\psi\cos\psi_1 + \sin\psi\sin\psi_1\cos\theta) \\ &+ \dot{y}_{E_1}(\sin\psi\cos\psi_1 - \cos\psi\sin\psi_1\cos\theta) \\ &- \dot{z}_{E_1}\sin\theta\sin\psi_1 + \dot{\psi}\left\{x_{E_1}\left[-\left(1 - \frac{Z_1}{Z_2}\cos\theta\right)\sin\psi\cos\psi_1 + \left(-\frac{Z_1}{Z_2} + \cos\theta\right)\cos\psi\sin\psi_1\right]\right. \\ &+ y_{E_1}\left[\left(1 - \frac{Z_1}{Z_2}\cos\theta\right)\cos\psi\cos\psi_1 + \left(-\frac{Z_1}{Z_2} + \cos\theta\right)\sin\psi\sin\psi_1\right] \left. - \dot{z}_{E_1}\frac{Z_1}{Z_2}\sin\theta\sin\psi_1\right\}; \end{aligned}$$

$$\begin{aligned} V_{E_2Y_1} = \dot{y}_{E_2} &= \dot{x}_{E_1}(\cos\psi\sin\psi_1 - \sin\psi\sin\psi_1\cos\theta) \\ &+ \dot{y}_{E_1}(\sin\psi\sin\psi_1 + \cos\psi\cos\psi_1\cos\theta) \\ &- \dot{z}_{E_1}\sin\theta\cos\psi_1 \\ &+ \dot{\psi}\left\{x_{E_1}\left[-\left(1 - \frac{Z_1}{Z_2}\cos\theta\right)\sin\psi\sin\psi_1 - \left(-\frac{Z_1}{Z_2} + \cos\theta\right)\cos\psi\cos\psi_1\right]\right. \\ &+ y_{E_1}\left[\left(1 - \frac{Z_1}{Z_2}\cos\theta\right)\cos\psi\sin\psi_1 - \left(-\frac{Z_1}{Z_2} + \cos\theta\right)\sin\psi\cos\psi_1\right] \left. - \dot{z}_{E_1}\frac{Z_1}{Z_2}\sin\theta\sin\psi_1\right\}; \\ V_{E_2Z_1} = \dot{z}_{E_2} &= \dot{x}_{E_1}\sin\theta\sin\psi - \dot{y}_{E_1}\sin\theta\cos\psi + \dot{z}_{E_1}\cos\theta \\ &+ \dot{\psi}(x_{E_1}\sin\theta\cos\psi + y_{E_1}\sin\theta\sin\psi). \end{aligned} \qquad (12)$$

Using the relations (10) for the calculation of the projections of the velocity of the contact point E_2, we determine the modulus of the velocity of the contact point E_2 on the profile of the satellite tooth according to the relation:

$$V_{E_2} = \sqrt{\dot{x}_{1E_2}^2 + \dot{y}_{1E_2}^2 + \dot{z}_{1E_2}^2}, \qquad (13)$$

where

$$\begin{aligned} \dot{X}_{1E_2} &= V_{E_2X_1} = V_{G_2X_1} + \frac{R\sin\beta}{\left(\sqrt{a_x^2 + a_y^2 + a_z^2}\right)^3}\left\{(a_x^2 + a_y^2 + a_z^2)\dot{a}_x - (a_x\dot{a}_x + a_y\dot{a}_y + a_z\dot{a}_z)a_x\right\}; \\ \dot{Y}_{1E_2} &= V_{E_2Y_1} = V_{G_2Y_1} + \frac{R\sin\beta}{\left(\sqrt{a_x^2 + a_y^2 + a_z^2}\right)^3}\left\{(a_x^2 + a_y^2 + a_z^2)\dot{a}_y - (a_x\dot{a}_x + a_y\dot{a}_y + a_z\dot{a}_z)a_y\right\}; \end{aligned} \qquad (14)$$

$$\dot{Z}_{1E_2} = V_{E_2Z_1} = V_{G_2Z_1} + \frac{R\sin\beta}{\left(\sqrt{a_x^2+a_y^2+a_z^2}\right)^3}\left\{\left(a_x^2+a_y^2+a_z^2\right)\dot{a}_z - \left(a_x\dot{a}_x + a_y\dot{a}_y + a_z\dot{a}_z\right)a_z\right\}.$$

The direction of the velocity vector $\mathbf{V}_{E_2}$ is calculated using the functions of the directional cosines:

$$\cos\left(\mathbf{V}_{E_2},\mathbf{i}_1\right) = \frac{V_{E_2X_1}}{V_{E_2}};\quad \cos\left(\mathbf{V}_{E_2},\mathbf{j}_1\right) = \frac{V_{E_2Y_1}}{V_{E_2}};\quad \cos\left(\mathbf{V}_{E_2},\mathbf{k}_1\right) = \frac{V_{E_2Z_1}}{V_{E_2}}. \tag{15}$$

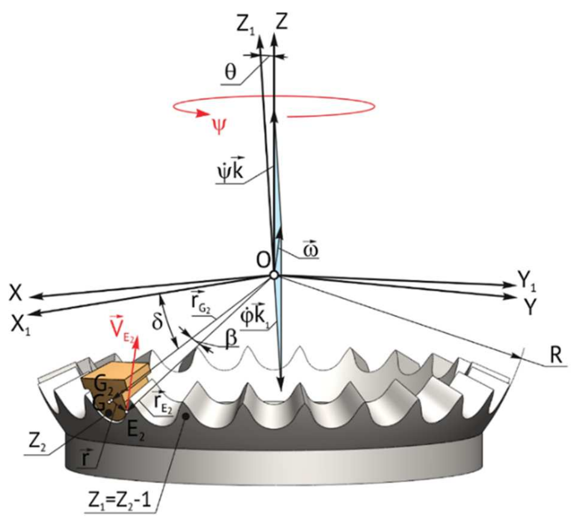


Fig.4. The position of the velocity vector of the contact point $\mathbf{V}_{E_2}$

5. RELATIVE SLIDING BETWEEN THE TEETH FLANKS MODIFIED BY SHORTENING THE HEIGHTS

The relative sliding with friction between the flanks of the conjugated teeth in the precessional gearing $A^D_{CX\text{-}CV}$ varies within a precessional cycle depending on the angular coordinate of the contact point ψ_{k_i} of each pair of simultaneously conjugated teeth, which, in turn, depends on the precession angle ψ of the crankshaft and is determined from relation (13).

Figure 5 (a) shows the variation of the relative sliding velocity V_{al} depending on the precession angle ψ for the precessional gearing $A^D_{CX\text{-}CV}$ with the tooth correlation $Z_1=Z_2-1$ and the angle of the conical axoid $\delta=22{,}5^\circ$, figure 5 (b) – with the correlation $Z_1=Z_2+1$ and $\delta=0^\circ$, and figure 5 (c) – with the correlation $Z_1=Z_2+1$ and $\delta=22{,}5^\circ$.

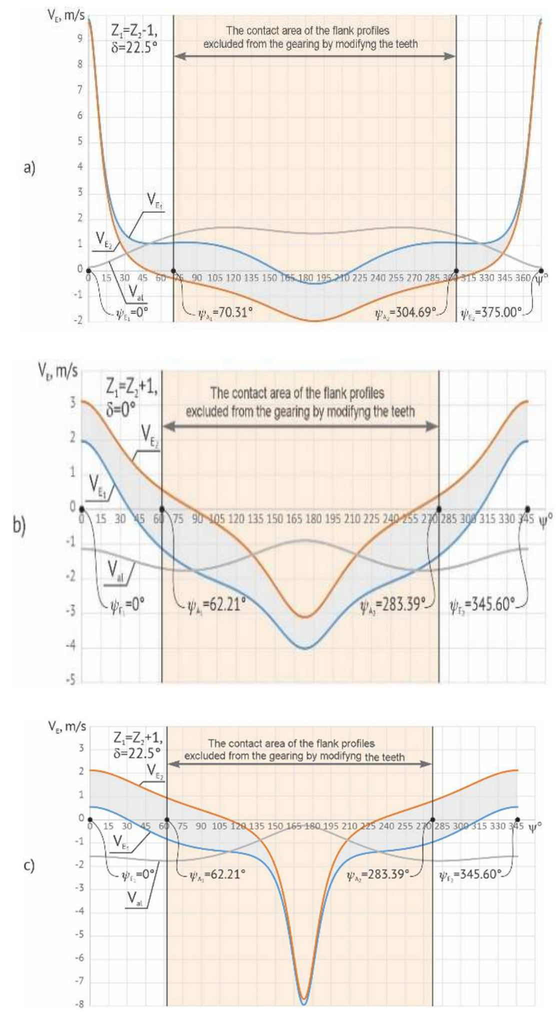


Fig.5. The linear velocities in contact V_{E_2}, V_{E_2} and the relative sliding velocity V_{al} between the teeth in the precessional gearing $A^D_{CX\text{-}CV}$ with the correlation of the teeth $Z_1=Z_2-1$, $Z_1=24$ and $\delta=22{,}5^\circ$ (a); $Z_1=Z_2+1$, $Z_1=25$ and $\delta=0^\circ$ (b); $Z_1=Z_2+1$, $Z_1=25$ and $\delta=22{,}5^\circ$ (c)

It should be noted that in the toothed precessional gearing $A^D_{CX\text{-}CV}$ the reference frontal multiplicity ε_a is up to 100% pairs of simultaneously conjugated teeth.

For one and the same positioning of the crankshaft experienced through the precession angle ψ it is necessary for the gearing to contain a certain number of pairs of teeth at the same time, spaced from each other by the angular pitch $\psi=360Z_2/Z_1^2$.

From the analysis of the variation of the relative velocities between the flanks of the conjugated teeth shown in figure 5 (a), (b), (c) it is concluded that the minimum relative sliding dominates in the contacts of the conjugation of the profiles of the teeth of the central wheels in the foot area with the top profiles of the teeth of the satellite-wheel.

For these reasons, in order to reduce the energy losses in the toothed gearings $A^{D}_{CX\text{-}CV}$ it is proposed to modify the teeth by shortening their height by cutting the tips of the central wheel teeth [2, 6]. Thus, shortening the height of the teeth by cutting off their tips ensures that a sufficient number of pairs of teeth determined from the calculation of the resistance of the teeth to the contact pressure σ_H are kept in gearing.

For example, for the gearing with correlation $Z_1=Z_2-1$ and $\delta=22{,}5^{\circ}$ four pairs of teeth with angular pitch $\psi_{k_i}=360Z_2/Z_1^2$ extend to the crankshaft precession angle $\psi=62{,}4^{\circ}$ for the contacts on the active profiles of the teeth, and for gearings with $Z_1=Z_2+1$ and $\delta=0^{\circ}$ or $\delta=22{,}5^{\circ}$ four pairs of teeth extend to the precession angle of the crankshaft $\psi=55{,}2^{\circ}$.

Figure 6 (a) shows the variation of the velocity modules V_{E_1} and V_{E_2} at the contact points E_1 and E_2 on the tooth profile of the satellite-wheel and, respectively, on the profile of the central wheel tooth and the sliding velocity V_{al} with friction at point E depending on the precession angle ψ for the gearing $Z_1=Z_2-1$ and $\delta_{(1\text{-}2)}=22{,}5^{\circ}$ ($Z_1=24$, $Z_2=25$, $\theta=3{,}5^{\circ}$, $r=6{,}27$ mm, $R=75$ mm).

For the case where the crankshaft rotates with the frequency of revolutions $n_1=3000$ min^{-1}, we obtain the linear V_{E_1} and V_{E_2}, and sliding V_{al} velocities shown in figure 6 (a). The mechanical efficiency of the gear is the expression of the energy losses generated by the sliding friction forces between the conjugated flanks, and the bearing capacity of the convex-concave contact results from the size and the difference in their radii of curvature.

For these reasons, the influence on the contact kinematics and geometry of different parametric configurations $[Z_g-\theta,\pm1]$ is examined, distinguished from each other only by the correlation $Z_1=Z_2\pm1$ and by the angle of the conical axoid $\delta\geq0^{\circ}$.

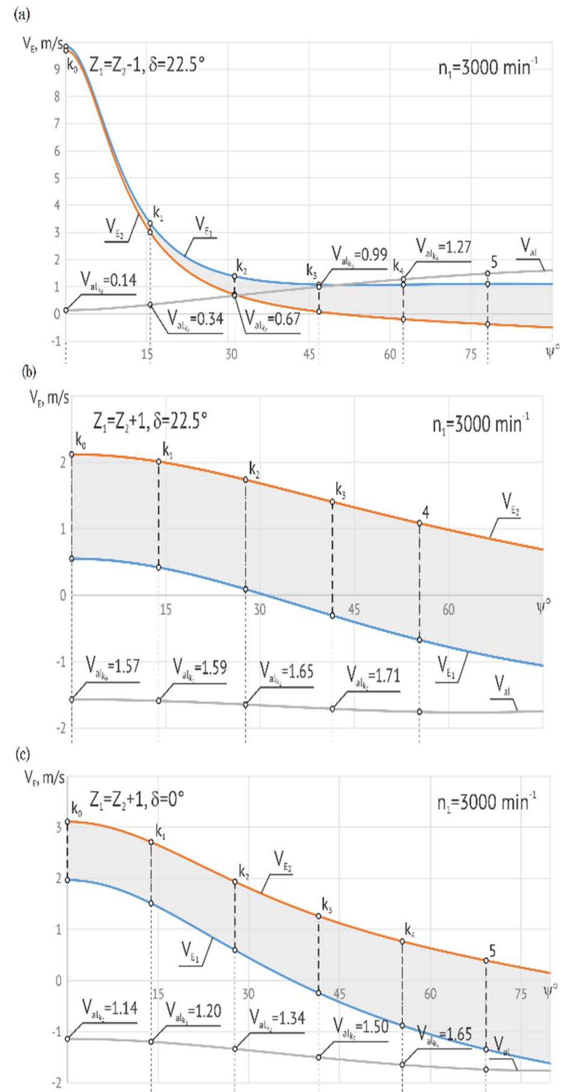


Fig.6. The linear V_{E_2}, V_{E_2} and sliding V_{al} velocities in the contact k_i depending on ψ for: $Z_1=Z_2-1$ and $\delta=22{,}5^{\circ}$ (a); $Z_1=Z_2+1$ and $\delta=0^{\circ}$ (b); $Z_1=Z_2+1$ and $\delta=22{,}5^{\circ}$ (c)

From the analysis of the contact kinematics of the teeth in the gearings shown in figure 6 (a), (b), (c) it is found that in the gearing corresponding to the parametric configuration $[Z_g-\theta,-1]$ with the correlation of the number of teeth $Z_1=Z_2-1$ and the angle of the conical axoid $\delta=22{,}5^{\circ}$ (see fig. 6, a), the relative sliding velocity V_{al}, in contacts k_0, k_1, k_2 and k_3 is relatively lower, which implies the opportunity to use this gearing for the design of precessional gears with of reducer operation regime. At the same time, it is worth noting that in the gearing with the configuration $[Z_g-\theta,+1]$ and the angle of the conical axoid $\delta>0^{\circ}$ (fig. 6, b) it is recommended to be used in transmissions with a multiplier operation regime and with the angle of the conical axoid $\delta=0^{\circ}$ – of reducer with small transmission ratios contained in the ranges $-7{,}3\leq i\leq -29{,}3$ for $\delta_{(1\text{-}2)}=0^{\circ}$, $Z_1=Z_2+1$ and in the range $+8{,}3\leq i\leq +30{,}3$ for $\delta_{(3\text{-}4)}=0^{\circ}$ $Z_3=Z_4+1$.

6. VARIATION OF THE RELATIVE SLIDING BETWEEN THE CONJUGATED FLANKS OF THE TEETH

The geometry of the contact and the kinematics of the contact point of the conjugated teeth in the gear (Z_1–Z_2) or (Z_3–Z_4) are dependent only on the configuration $[Z_g–\theta,\pm1]$ of the respective gear, i.e. the relative sliding between the teeth in gearing (Z_1–Z_2) have constant values for any another gearing (Z_3–Z_4) with which it conjugates and vice versa.

Figure 7 shows the influence of the configuration parameters $[Z_g–\theta,\pm1]$ on the relative sliding speed from the precessional transmissions, with the transmission ratios $i=-124$, $i=-164$, $i=+15$ and $i=-14$. Also, for the more complex comparative analysis, the relative slips in the gears $A^{D}_{CX\text{-}CV}$ with the ratios $i=-360$, $i=-1080$, $i=-2115$ are presented.

The analysis of figure 7 demonstrates that the evolution of the relative sliding V_{al}, depends on the correlation $Z_{1(4)}=Z_{2(3)}-1$ or $Z_{1(4)}=Z_{2(3)}+1$ of the numbers of teeth of the geared wheels. For the correlation of the number of teeth $Z_{1(4)}=Z_{2(3)}-1$, the relative sliding velocities are presented by the curves $V_{al_1}\ldots V_{al_5}$ and $V_{al_8}\ldots V_{al_{14}}$, which increase from the minimum in the pair of teeth conjugated in contact k_0 (fig. 7) to the maximum in the pair of teeth conjugated in contact k_4.

For the correlation $Z_{1(4)}=Z_{2(3)}+1$, the relative sliding velocities are presented by the curves V_{al_6} and V_{al_7}, from which it can be seen that in the pairs of teeth with the angular coordinate up to $\psi_k=40^{\circ}$ the relative sliding is higher than in the case of the correlation $Z_{1(4)}=Z_{2(3)}-1$.

The variation of the sliding velocities modules V_{al_6} and V_{al_7} depending on ψ_k is shown by dotted curves, which facilitates the analysis of the sliding velocities in the gears $A^{D}_{CX\text{-}CV}$ with the correlation $Z_{1(4)}=Z_{2(3)}+1$ compared to the sliding velocities for the correlation $Z_{1(4)}=Z_{2(3)}-1$.

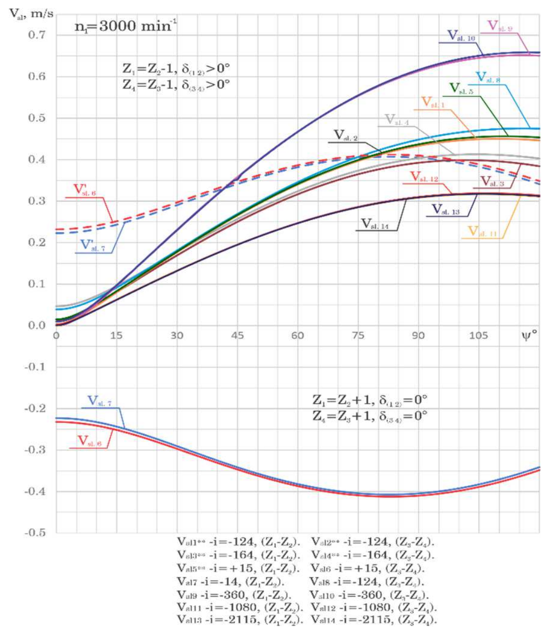


Fig.7. The influence of configuration parameters $[Z_g–\theta,\pm1]$ on the relative sliding velocity between conjugated flanks

7. TECHNOLOGICAL AND APPLICATION ASPECTS

The following procedures were developed for the manufacture of the wheels of precessional gearings $A^{D}_{CX\text{-}CV}$ [3]:

- generation of tooth flanks by spatial tumbling-rolling with truncated cone-shaped tool $G^{con}_{r.s}$, with peripherally profiled disk-shaped tool $G^{disc}_{r.s}$ and with cylindrical tool on multi-axis machine tools $G^{cil}_{m.ax}$;
- on CNC numerically controlled machine tools;
- plastic injection under pressure;
- sintering with metal powder pressing.

The precessional gears A^{D} with straight toothing with multiple pair convex-concave contact are recommended to be used at peripheral speeds up to 5 m/s.

To reduce the forces in the gear, respectively on the shafts and bearings, the satellite wheel is mounted axially floating, and the pressure angle between the flanks α_w must be reduced as much as possible by the rational selection of the configuration parameters $[Z_g–\theta,\pm1]$.

The technologies of execution of conical wheels with straight teeth or inclined are more demanding and sensitive to the precision of execution and/or assembly deviations.

In order to reduce the elastic deformations of the crankshaft, they are recommended to be oversized, not due to resistance conditions, but due to stiffness conditions. To reduce the dynamic loads and additional stresses on the bearings, it is appropriate to create constructive possibilities of axial floating of the satellite wheel and to ensure the coincidence of the precession center with the top of the conical axoids $\delta_{(1\text{-}2)}$ and $\delta_{(3\text{-}4)}$.

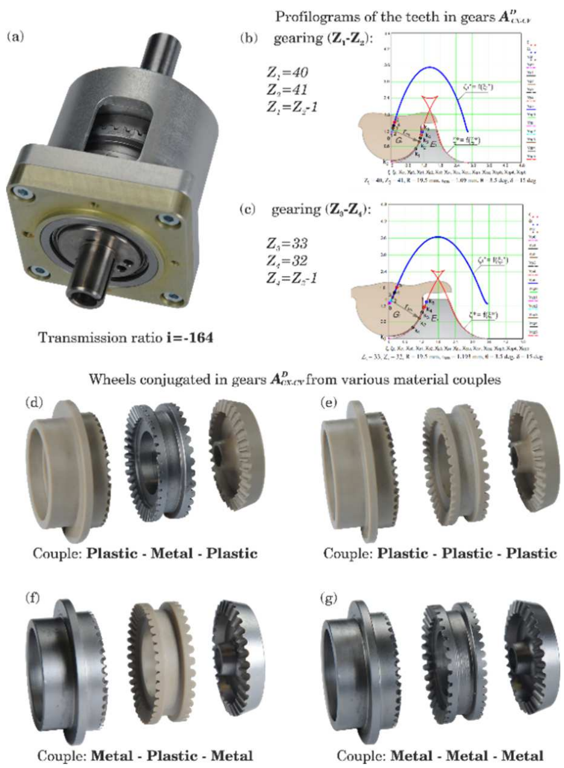
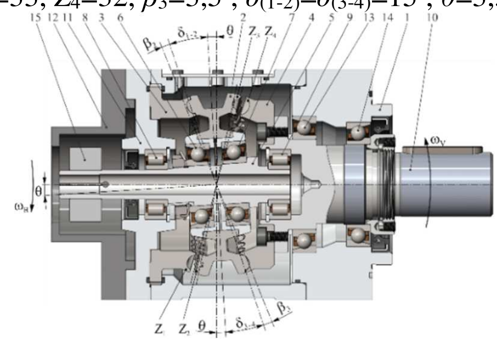


Fig.8. 2K-H precessional transmission, transmission ratio i= –164 with gears $A^D_{CX\text{-}CV}$ (Z_1=40, Z_2=41, Z_3=33, Z_4=32, $\delta_{(1\text{-}2)}$=$\delta_{(3\text{-}4)}$=15°, β_2=3,2°, β_3=3,5°, θ=3,5° D_e=42 mm): (a) – general view; (b), (c) – profilograms of teeth (Z_1–Z_2) and (Z_3–Z_4), respectively; (d), (e), (f), (g) – toothe wheels in axial unfolding of paired materials, steel 4140 and plastic *Glass-Filled PEEK*, manufactured on CNC machine tools (physical component display – manufactured)

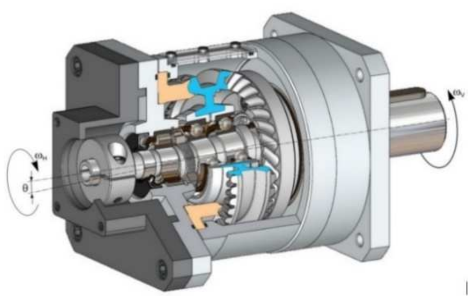
Figure 8 shows the precessional transmission with i= –164, achieved in gears $A^D_{CX\text{-}CV}$ with straight teeth with Z_1=40, Z_2=41, $\delta_{(1\text{-}2)}$=15°, β_2=3,2°, Z_3=33, Z_4=32, $\delta_{(3\text{-}4)}$=15°, β_3=3,5°, θ=3,5° and D_e = 42 mm.

In the figure 8 (a) the general view of the transmission is shown, in (b), (c) – the profilograms of the conjugated teeth in the gears (Z_1–Z_2) and (Z_3–Z_4), respectively, in (d), (e), (f) and (g) – the toothed wheels in axial unfolding in various pairs of materials, steel 4140 and plastic *Glass-Filled PEEK*.

The 2K-H precession transmission shown in figure 9 has the transmission ratio i= –164 also achieved in gears with inclined teeth in circle arcs with parameters Z_1=40, Z_2=41, β_2=3,2°, Z_3=33, Z_4=32, β_3=3,5°, $\delta_{(1\text{-}2)}$=$\delta_{(3\text{-}4)}$=15°, θ=3,5°.



(a)



(b)

Fig 9. 2K-H i= – 164 precessional transmission with gears $A^{D,\,\beta}_{CX\text{-}CV}$: general drawing (a); 3D overview (b)

8. CONCLUSIONS AND RECOMMENDATIONS

1. To reduce the relative sliding V_{al} in contact, the shape of the teeth is changed by shortening their height by cutting the tips, so that we keep in the convex-concave contact only a sufficient number of pairs of teeth determined from the contact pressure resistance calculation σ_H of the teeth only at points $k_0 – k_3$ or k_1–k_3 (k_4).
2. The toothed precessional gear A^D with the correlation Z_1=Z_2–1 and Z_4=Z_3–1, and the angle of the conical axoid δ>0° is recommended to be used in precessional transmissions with a reducer operation regime, due to the geometry of the convex-

concave contact with the small difference in the radii of curvature and due to the low relative sliding velocity with friction between the conjugated flanks.

3. The precessional gear A^D with the correlation Z_4=Z_3+1 (or Z_1=Z_2+1), the angle of the conical axoid $\delta \geq 0^{\circ}$ and the profile angle of the teeth of the central wheel higher than $\alpha_w > 45^{\circ}$ is recommended to be used in transmissions with a multiplier operation regime, due to the kinetostatics favorable to the transformation of the rotational motion of the central wheel into spherospatial motion of the satellite-wheel with the frequency of precession cycles n_s=Z_4.

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Alunecare relativă în angrenările precesionale multiple cu contact convex-concav al dinților

Lucrarea este consacrată studiului alunecării relative în contactul convex-concav al dinților conjugați multipar în angrenare precesională dinți drepți $A^{D}_{CX\text{-}CV}$ și înclinați $A^{D,\beta}_{CX\text{-}CV}$. În lucrare se prezintă influența parametrilor angrenării definiți de configurația [Z_g – θ, ±1] asupra geometriei contactului dinților și a diferenței curburilor de flanc, precum și influența modificării formei dinților prin scurtarea înălțimii lor asupra diminuării alunecării relative cu frecare în contactele k_i a perechilor de dinți simultan conjugate la una și aceiași coordonată unghiulară ψ a arborelui-manivelă. Sunt analizate pierderile energetice în angrenări în funcție de alunecarea relativă cu frecare între flancurile dinților conjugați unitar sau simultan multipar, precum și capacitatea portantă a contactului convex-concav al dinților în funcție de diferența curburilor de flanc.

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