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RAIL DAMPER PARAMETERS' EFFECT ON THE FREQUENCY RESPONSE FUNCTION OF THE RAIL

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Abstract: *The paper presents the influence of the viscoelastic features of a rail damper upon the frequency response function of the rail in terms of receptance. To this aim, a simplified model of the rail is used consisting of single-track span: rail and half support at each end of the rail, including rail pad, sleeper and ballast. The rail model is a free-free Timoshenko beam, the rail pad and ballast are modelled by a Kelvin-Voigt system for each, and the sleeper is modelled as a rigid body in vertical translation. Rail receptance is calculated with the help of the frequency-domain Green's function. Effect of the eigenfrequencies of the rail damper as well as the distance between the sleepers on the rail receptance is outlined and analyzed.*
Key words: *track; rail damper; rail receptance; frequency-domain Green function; pinned-pinned resonance.*

1. INTRODUCTION

Although rail transport is recognized as environmentally friendly, the movement of trains generate noise pollution in the railway zones. The noise of railway vehicles has several mechanisms of generation, and the noise produced by the wheel-rail vibration has the largest contribution to the total noise level accompanying the movement of trains.

Three important mechanisms by which wheel-rail vibrations are excited have been identified, namely: the overlapping of rolling surface roughness, the wheel/rail impact due to the discontinuities of the rolling surfaces of the rail or local defects of the wheel such as the flat spot and the phenomenon of lateral stick-slip that occurs when driving in tight curves when the adhesion limit is exceeded.

The vibrations generated at the wheel/rail interface propagate through the rail and the wheel body in the form of bending waves, and thus the two bodies become acoustic radiators. The acoustic waves produced in this way are transmitted by solid and air to the interior of the carriages, affecting the comfort of the passengers, and propagate in the railway zone producing its noise pollution.

Fig. 1. Rail damper [2].

Rolling noise mitigation is based on lowering the source power and preventing the propagation of noise on the transmission paths. One of the solutions that is gaining more and more ground is the rail dampers [1].

Figure 1 shows a rail damper from UUDEN Rail Products B. V. [2].

Rail damper is oscillating mechanical system that works on the dynamic absorber principle with one or two tuning frequencies. As a rule, the rail dampers are mounted on both sides of the rail web in contact with the upper part of the rail foot and they have the role of taking the energy of the bending waves propagating along the rail and dissipating it into heat. In this way, the bending waves generated by the wheel-rail interaction propagate over a shorter distance, which causes a significant reduction in the ability of the rail to produce acoustic waves [3].

Several research are focused on assessment of the decay rate of the bending waves in rail which represents the most important parameter describing the quality of the rail dampers [4-5]. It is about of the track decay rate (TDR) which is used by researchers to evaluate the performance of rail dampers [6-7].

Many new design concepts of rail dampers have recently been presented, with improved damping properties both through technological changes and improving design [8-9].

This paper is focused on the effect of the rail damper features upon the frequency response function of the rail in terms of the rail receptance is investigated taken as example the case of light type of rail, namely UIC 49 rail. Starting from the tuning frequencies of the rail damper, the rail receptance is calculated depending to the sleeper spacing according to the sleeper number per km, as provided by the track construction regulations. This aspect is important because the frequency response function of the rail is a crucial characteristic for the dynamic contact force that generates the bending waves propagating along the track. As author knowledge this matter has not previously investigated.

2. TRACK MODEL

Figure 2 displays the simplified mechanical model of the ballasted track featured with rail dampers under the action of the harmonic vertical force, $Q(t) = Q\cos pt$, where Q is the amplitude, p is the angular frequency and t stands for time, which acts at the distance x_o from the reference frame Oxz.

Model consists of an infinite uniform Timoshenko beam discretely supported on elastic supports made up of two elastic elements between which there is a rigid body. The Timoshenko beam corresponds to the rail, the two elastic elements of the support model the railpad and the ballast, and the rigid body has the inertial property of a half sleeper.

The rail damper has two tuning frequencies and is modelled using an oscillator with two degrees of freedom. Each rail damper is related by the rail, at the middle of the distance between the sleepers.

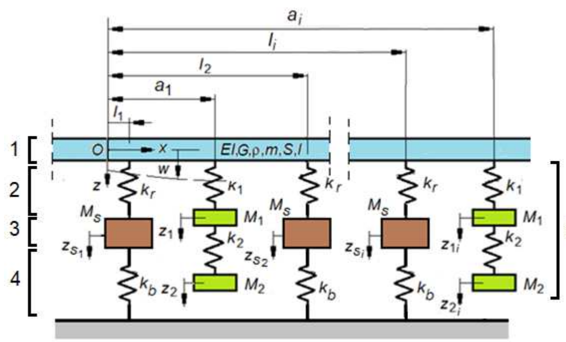

Fig. 2. Track model: 1. rail; 2. railpad; 3. sleeper; 4. ballast; 5. rail damper.

The rail damper has two tuning frequencies and is modelled using an oscillator with two degrees of freedom. Each rail damper is related by the rail, at the middle of the distance between the sleepers.

Track parameters are rail mass per unit length, m, the bending stiffness of the rail, EI, where E is Young's modulus of the rail material and I is the second moment of area, the shear stiffness of the rail, $GA\kappa$, where G is the shear modulus of the rail material, A is the area of the cross section and κ is the shear coefficient, the mass of half sleeper, M_s, the railpad stiffness, k_r, and the ballast stiffness, k_b. Parameters for the rail damper are the masses M_1 and M_2, and the stiffnesses k_1 and k_2. Damping is introduced in the track and rail damper models via the loss factor.

Equations of motion of the rail read

$$GS\kappa\left[\frac{\partial^2 w}{\partial x^2}-\frac{\partial\theta}{\partial x}\right]-m\frac{\partial^2 w}{\partial t^2}=-Q(t)\delta\left(x-x_0\right)+ k_r\sum_{i=-\infty}^{\infty}\left[\left(w_{l_i}-u_i\right)\delta\left(x-l_i\right)\right]+ k_1\sum_{i=-\infty}^{\infty}\left[\left(w_{a_i}-z_{i1}\right)\delta\left(x-a_i\right)\right] \quad (1)$$

$$EI\frac{\partial^2\theta}{\partial x^2}+GS\kappa\left(\frac{\partial w}{\partial x}-\theta\right)-\rho I\frac{\partial^2\theta}{\partial t^2}=0 \quad (2)$$

where $w = w(x, t)$ is the rail displacement and $\theta = \theta(x, t)$ – the rotation of the rail section of the x longitudinal coordinate, $u_i = u_i(t)$ is the displacement of the semi sleeper i, $z_{i1} = z_{i1}(t)$ is the displacement of the first rigid body of the rail damper within the sleeper spacing i, and

$$w_{l_i} = w(l_i, t)$$
$$w_{a_i} = w(a_i, t) \tag{3}$$

where l_i and a_i fix the position of the half sleeper i and the corresponding rail damper.

The following boundary conditions must be associated to the rail

$$\lim_{|x|\to\infty} w = 0 \tag{4}$$

Equation of motion for the i sleeper reads

$$M_s \frac{\mathrm{d}^2 u_i}{\mathrm{d}t^2} + (k_r + k_b)u_i - k_r w_{l_i} = 0. \tag{5}$$

Rail damper equations of motion for the i rail damper are

$$M_1 \frac{\mathrm{d}^2 z_{i1}}{\mathrm{d}t^2} + (k_1 + k_2) z_{i1} - k_1 w_{a_i} - k_2 z_{i2} = 0$$
$$M_2 \frac{\mathrm{d}^2 z_{i2}}{\mathrm{d}t^2} + k_2 z_{i2} - k_2 z_{i1} = 0, \tag{6}$$

where $z_{i2} = z_{i2}(t)$ is the displacement of the second rigid body of the rail damper within the sleeper spacing i.

Next, the steady-state harmonic behavior is considered, so that the variables are

$$w(x,t) = W(x)\cos\left(pt + \gamma_w(x)\right)$$
$$\theta(x,t) = \Theta(x)\cos\left(pt + \gamma_\theta(x)\right)$$
$$u_i(t) = U_i \cos\left(pt + \gamma_{u_i}\right) \tag{7}$$
$$z_{i1,2}(t) = Z_{i1,2} \cos\left(pt + \gamma_{z_{i1,2}}\right)$$

where $W(x)$ and $\Theta(x)$ are the amplitudes and $\gamma_w(x)$ and $\gamma_\theta(x)$ are the initial phases in the x section of the rail, U_i is the amplitude of the displacement of the sleeper i and $Z_{i1,2}$ and $\gamma_{zi1,2}$ are the amplitudes and initial phases corresponding to the rail damper i.

The following complex variables are associated to the real ones

$$\bar{w}(x,t) = \bar{W}(x)e^{\mathrm{i}pt} \quad \bar{\theta}(x,t) = \bar{\Theta}(x)e^{\mathrm{i}pt}$$
$$\bar{u}_i(x,t) = \bar{U}_i e^{\mathrm{i}pt} \quad \bar{z}_{i1,2}(t) = \bar{Z}_{i1,2} e^{\mathrm{i}pt} \tag{8}$$
$$\bar{Q}(t) = \bar{Q}e^{\mathrm{i}pt}$$

where $\mathrm{i}^2 = -1$ and

$$\bar{W}(x) = W(x)e^{\mathrm{i}\gamma_w(x)} \quad \bar{\Theta}(x) = \Theta(x)e^{\mathrm{i}\gamma_\theta(x)}$$
$$\bar{U}_i = U_i e^{\mathrm{i}\gamma_{ui}} \quad \bar{Z}_{i1,2} = Z_{i1,2} e^{\mathrm{i}\gamma_{zi1,2}} \tag{9}$$
$$\bar{Q} = Qe^{\mathrm{i}\cdot 0}$$

so that

$$w(x,t) = \mathrm{Re}\,\bar{w}(x,t) \quad \theta(x,t) = \mathrm{Re}\,\bar{\theta}(x,t)$$
$$u_i(t) = \mathrm{Re}\,\bar{u}_{i1,2}(t) \quad z_{i1,2}(t) = \mathrm{Re}\,\bar{z}_{i1,2}(t) \tag{10}$$
$$Q(t) = \mathrm{Re}\,\bar{Q}(t)$$

and

$$W(x) = \left|\bar{W}(x)\right| \quad \Theta(x) = \left|\bar{\Theta}(x)\right|$$
$$U_i = \left|\bar{U}_i\right| \quad Z_{i1,2} = \left|\bar{Z}_{i1,2}\right| \tag{11}$$
$$Q = \left|\bar{Q}\right|.$$

New complex variables verify the equations of motion of the rail

$$GS\kappa\left[\frac{\mathrm{d}^2\bar{W}(x)}{\mathrm{d}x^2} - \frac{\mathrm{d}\bar{\Theta}(x)}{\mathrm{d}x}\right] + \omega^2 m\bar{W}(x) =$$
$$-\bar{Q}\delta\left(x - x_0\right) + \bar{k}_r \sum_{i=-\infty}^{\infty}\left[\left(\bar{W}(l_i) - \bar{U}_i\right)\delta\left(x - l_i\right)\right] \tag{12}$$
$$+\bar{k}_1 \sum_{i=1}^{10}\left[\left(\bar{W}(a_i) - \bar{Z}_{i1}\right)\delta\left(x - a_i\right)\right]$$

$$EI\frac{\mathrm{d}^2\bar{\Theta}(x)}{\mathrm{d}x^2} + GS\kappa\left[\frac{\mathrm{d}\bar{W}(x)}{\mathrm{d}x} - \bar{\Theta}(x)\right] + \tag{13}$$
$$\omega^2\rho I\bar{\Theta}(x) = 0$$

and those of the sleeper and rail damper

$$\left(-\omega^2 M_s + \bar{k}_r + \bar{k}_b\right)\bar{U}_i - \bar{k}_r\bar{W}(l_i) = 0. \tag{14}$$

$$\left(-\omega^2 M_1 + \bar{k}_1 + \bar{k}_2\right)\bar{Z}_{i1} - \bar{k}_1\bar{W}(a_i) - \bar{k}_2\bar{Z}_{i2} = 0$$
$$\left(-\omega^2 M_2 + \bar{k}_2\right)\bar{Z}_{i2} - \bar{k}_2\bar{Z}_{i1} = 0, \tag{15}$$

where the stiffness of the elastic elements of the railpad, ballast and rail damper are complex numbers, including the loss factor

$$\begin{aligned}\bar{k}_{r,b} &= k_{r,b}\left(1+i\eta_{r,b}\right)\\ \bar{k}_{1,2} &= k_{1,2}\left(1+i\eta_{1,2}\right)\end{aligned}. \tag{16}$$

Also, the boundary conditions (4) become

$$\lim_{|x|\to\infty}\bar{W}=0. \tag{17}$$

Equations (12–15) must be reduced to a single differential equation.

By processing equations (12) and (13), the rotation angle of the rail sections is eliminated

$$\begin{aligned}&EI\frac{\mathrm{d}^4\bar{W}(x)}{\mathrm{d}x^4}+\omega^2\rho Is\frac{\mathrm{d}^2\bar{W}(x)}{\mathrm{d}x^2}+\\&\omega^2 mp\bar{W}(x)=\\&p\begin{bmatrix}-\bar{Q}\delta\left(x-x_0\right)+\\ \bar{k}_r\sum\limits_{i=-\infty}^{\infty}\left(\bar{W}(l_i)-\bar{U}_i\right)\delta\left(x-l_i\right)+\\ \bar{k}_1\sum\limits_{i=-\infty}^{-\infty}\left(\bar{W}(a_i)-\bar{Z}_{i1}\right)\delta\left(x-a_i\right)\end{bmatrix}+\\&q\begin{bmatrix}-\bar{Q}\delta''\left(x-x_0\right)+\\ \bar{k}_r\sum\limits_{i=-\infty}^{\infty}\left(\bar{W}(l_i)-\bar{U}_i\right)\delta''\left(x-l_i\right)+\\ \bar{k}_1\sum\limits_{i=-\infty}^{-\infty}\left(\bar{W}(a_i)-\bar{Z}_{i1}\right)\delta''\left(x-a_i\right)\end{bmatrix},\end{aligned} \tag{18}$$

$$p=\frac{\omega^2\rho I}{GS\kappa}-1,\quad q=\frac{EI}{GS\kappa},\quad s=1+\frac{E}{G\kappa}. \tag{19}$$

From equations (14) and (15) results successively

$$\bar{U}_i=\frac{\bar{k}_r}{-\omega^2 M+\bar{k}_r+\bar{k}_b}\bar{W}(l_i) \tag{20}$$

$$\bar{Z}_{i2}=\frac{\bar{k}_2}{-\omega^2 M_2+\bar{k}_2}\bar{Z}_{i1},\quad \bar{Z}_{i1}=\frac{K_1}{K_2}\bar{W}(a_i) \tag{21}$$

where

$$\begin{aligned}&K_1=\bar{k}_1\left(-\omega^2 M_2+\bar{k}_2\right)\\&K_2=\omega^4 M_1M_2-\omega^2\left[\bar{k}_2M_1+\left(\bar{k}_1+\bar{k}_2\right)M_2\right]\\&+\bar{k}_1\bar{k}_2.\end{aligned} \tag{22}$$

From equations (18), (20) – (22), it results

$$\begin{aligned}&EI\frac{\mathrm{d}^4\bar{W}(x)}{\mathrm{d}x^4}+\omega^2\rho Is\frac{\mathrm{d}^2\bar{W}(x)}{\mathrm{d}x^2}+\\&\omega^2 mp\bar{W}(x)=pf_1+qf_2,\end{aligned} \tag{23}$$

where

$$\begin{aligned}&f_1=-\bar{Q}\delta\left(x-x_0\right)+\bar{k}_t\sum_{i=-\infty}^{\infty}\bar{W}(l_i)\delta\left(x-l_i\right)\\&+\bar{k}_d\sum_{i=-\infty}^{-\infty}\bar{W}(a_i)\delta\left(x-a_i\right)\\&f_2=-\bar{Q}\delta''\left(x-x_0\right)+\bar{k}_t\sum_{i=-\infty}^{\infty}\bar{W}(l_i)\delta''\left(x-l_i\right)\\&+\bar{k}_d\sum_{i=-\infty}^{-\infty}\bar{W}(a_i)\delta''\left(x-a_i\right)\end{aligned} \tag{24}$$

where

$$\begin{aligned}\bar{k}_t&=\bar{k}_r\left(1-\frac{\bar{k}_r}{-\omega^2 M_s+\bar{k}_r+\bar{k}_b}\right)\\ \bar{k}_d&=\bar{k}_1\left(1-\frac{K_1}{K_2}\right)\end{aligned} \tag{25}$$

To solve equation (23) and the boundary condition (17), Green function method could be applied [10]. One can start from the following equation

$$D_x w(x)=F(x), \tag{26}$$

where D_x is the Timoshenko differential operator

$$D_x=EI\frac{\mathrm{d}^4}{\mathrm{d}x^4}+\omega^2\rho Is\frac{\mathrm{d}^2}{\mathrm{d}x^2}+\omega^2 mp \tag{27}$$

and

$$F(x)=pF\delta(x-a)+qF\delta''(x-a). \tag{28}$$

Equation solution is

$$\bar{W}(x)=\int_{-\infty}^{-\infty} g(x,\xi)F(\xi)\mathrm{d}\xi \tag{29}$$

where $g(x, \xi)$ is the Green function associated to the D_x operator [10]

$$g(x,\xi)=-\frac{\beta_2 e^{-\beta_1|x-\xi|}+i\beta_1 e^{-i\beta_2|x-\xi|}}{2\beta_1\beta_2\left(\beta_1^2+\beta_2^2\right)EI}, \tag{30}$$

where

$$\beta_{1,2}=\sqrt{\sqrt{\frac{\rho^2\omega^4}{4E^2}\left(\frac{E}{G\kappa}-1\right)^2+\frac{m\omega^2}{EI}}\mp\frac{\rho\omega^2}{2E}\left(\frac{E}{G\kappa}+1\right)}.$$

After performing the integral from the equation (29), it results

$$\bar{W}(x)=Fg_T(x,a) \tag{31}$$

where

$$g_T(x,\xi)=pg(x,\xi)+q\frac{\mathrm{d}^2 g(x,\xi)}{\mathrm{d}\xi^2}. \tag{32}$$

Applying above method to the equation (23), it follows

$$\begin{aligned}&\bar{W}(x,x_o,\omega)=-\bar{Q}g_T(x,x_o)+\\&k_t\sum_{i=-\infty}^{\infty}\bar{W}(l_i)g_T(x,l_i)+k_d\sum_{i=-\infty}^{\infty}\bar{W}(a_i)g_T(x,a_i),\end{aligned} \tag{33}$$

where the arguments of the complex amplitude of the rail show the dependence by both the passive and active section and the angular frequency.

Considering a truncated track and keeping a sufficiently large number of supports, equation (33) can be approximated by

$$\begin{aligned}&\bar{W}(x,x_o,\omega)=-\bar{Q}g_T(x,x_o)+\\&k_t\sum_{i=1}^{N}\bar{W}(l_i)g_T(x,l_i)+k_d\sum_{i=1}^{N-1}\bar{W}(a_i)g_T(x,a_i)\end{aligned} \tag{34}$$

where N is number of supports and N–1 is the number of sleepers spacing.

Considering $x = l_j$, with $j = 1$ to N, and then, $x = a_k$ with $k = 1$ to N–1, the following matrix equation reads

$$\begin{bmatrix}\mathbf{A}_{11} & \mathbf{A}_{12}\\ \mathbf{A}_{21} & \mathbf{A}_{22}\end{bmatrix}\begin{bmatrix}\mathbf{X}_1\\ \mathbf{X}_2\end{bmatrix}=\begin{bmatrix}\mathbf{B}_1\\ \mathbf{B}_2\end{bmatrix} \tag{35}$$

where

$$\begin{aligned}&\mathbf{A}_{11}=k_t\left(g_T(l_i,l_j)\right)_{1\le i,j\le N}-\mathbf{I}_N\\&\mathbf{A}_{12}=k_d\left(g_T(l_i,a_j)\right)_{1\le i\le N,\ 1\le j\le N-1}\\&\mathbf{A}_{21}=k_t\left(g_T(a_i,l_j)\right)_{1\le i\le N-1,1\le j\le N}\\&\mathbf{A}_{22}=k_d\left(g_T(a_i,a_j)\right)_{1\le i,j\le N-1}-\mathbf{I}_{N-1}\end{aligned}$$

and

$$\begin{aligned}&\mathbf{X}_1=\left[\bar{W}(l_1)\quad \bar{W}(l_2)\quad \ldots\quad \bar{W}(l_N)\right]^T\\&\mathbf{X}_2=\left[\bar{W}(a_1)\quad \bar{W}(a_2)\quad \ldots\quad \bar{W}(a_{N-1})\right]^T\\&\mathbf{B}_1=\bar{Q}\left[g_t(l_1,x_o)\quad g_t(l_2,x_o)\quad \ldots\quad g_t(l_N,x_o)\right]^T\\&\mathbf{B}_2=\bar{Q}\left[g_t(a_1,x_o)\quad g_t(a_2,x_o)\quad \ldots\quad g_t(a_{N-1},x_o)\right]^T,\end{aligned}$$

where $\mathbf{I}_N$ is the identity matrix of size N.

Equation (35) can be solved using a numerical method and then, by applying equation (33), the vertical rail displacement in the x section and the rail receptance result

$$\alpha(x,x_o,\omega)=\bar{W}(x,x_o,\omega)/\bar{Q}. \tag{36}$$

Regarding the eigenfrequencies of the rail damper, these may be obtained following formula

$$\nu_{l,h}=\sqrt{\frac{1}{2}\left[\mu\nu_1^2+\nu_2^2\mp\sqrt{\left(\mu\nu_1^2+\nu_2^2\right)^2-4\nu_1^2\nu_2^2}\right]}, \tag{37}$$

where $\nu_{l,h}$ is the low/high natural frequency of the rail damper, and

$$\nu_{1,2}=\frac{1}{2\pi}\sqrt{\frac{k_{1,2}}{M_{1,2}}},\quad \mu=1+\frac{M_1}{M_2}. \tag{38}$$

By imposing the natural frequencies of the rail damper, its decoupled frequencies result

$$\nu_1^2 = \frac{1}{2\mu}\left(\nu_l^2 + \nu_h^2 + \sqrt{\left(\nu_l^2 + \nu_h^2\right)^2 - 4\mu\nu_l^2\nu_h^2}\right)$$
$$\nu_2^2 = \frac{1}{2}\left(\nu_l^2 + \nu_h^2 - \sqrt{\left(\nu_l^2 + \nu_h^2\right)^2 - 4\mu\nu_l^2\nu_h^2}\right) \quad (39)$$

and then, the stiffness $k_{1,2}$ can be calculated knowing the mass $M_{1,2}$.

3. NUMERICAL APPLICATION

Frequency response function of a ballasted track featured with 49 UIC rail, concrete sleepers and rail dampers, is analyzed in this section using the MATLAB environmental.

Parameter values of the track are: m = 49.3 kg/m, E = 210 GPa, G = 80.8 GPa, κ = 0.4, k_r = 120 MN/m, η_r = 0.3, k_b = 60 MN/m, η_b = 0.6, m_s = 131 kg. The 49 UIC rail is featured with rail dampers specially designed.

Parameter values of the rail damper are experimentally determined [11]: M_1 = 3.65 kg, M_2 = 3.514 kg, k_1 = 50.63 MN/m, η_1 = 0.35, k_2 = 5.617 MN/m and η_2 = 0.25 (the mass and the stiffness are for two rail dampers).

The sleeper spacing is depending on the number of sleepers per km, between 1680 and 1840 sleepers/km, corresponding to the sleeper spacing of 595 mm and 544 mm [12].

Truncated track length corresponds to 200 sleepers.

Figure 3 presents the rail receptance calculated in the harmonic force section at middle of the sleeper spacing for both sleeper spacing values of 544 mm and 595 mm.

In the range of medium frequencies, the track components vibrate like a two-degree-of-freedom oscillator, which explains the existence of the two peaks from approx. 94 Hz and 379 Hz. When the sleeper spacing is higher, the railpad stiffness per unit length lows, while the rail linear mass is unaffected. On the other hand, both sleeper linear mass and the ballast stiffness per unit length decreases in similar way. The two above aspects explain why the two peaks are registered at lower frequencies.

At high frequencies, the rail vibrates like a beam attached to the sleepers and this type of vibration shows the pinned-pinned resonance (PPR). The frequency of this resonance depends on the sleeper spacing, shorter sleeper spacing, higher PPR frequency. In this case, the rail exhibits PPR at 982 Hz for d = 595 mm and 1136 Hz for d = 544 mm.

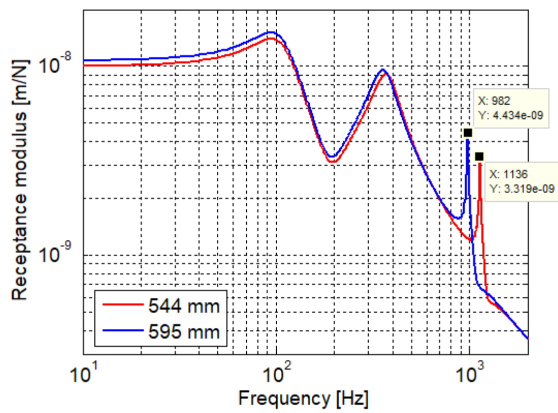

Fig. 3. Rail receptance (without rail dampers).

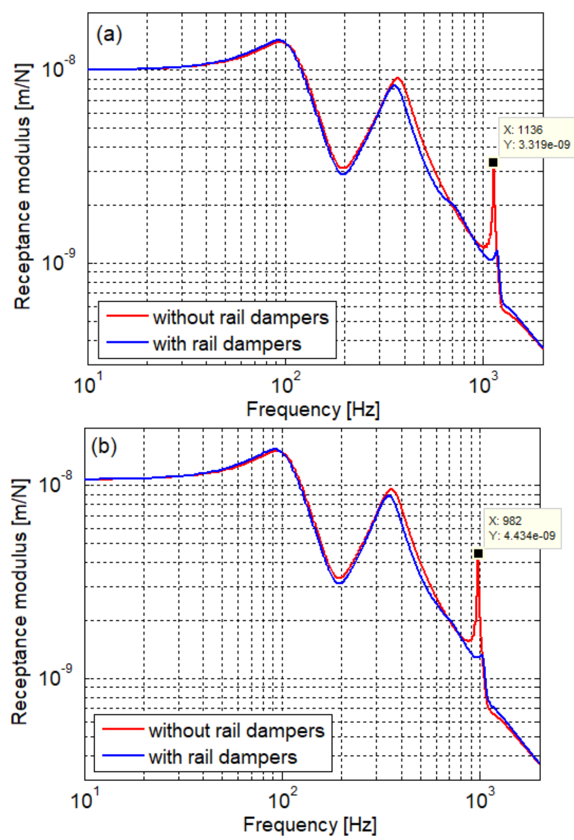

Fig. 4. Rail receptance (without and with rail dampers): (a) sleeper spacing of 544 mm; (b) sleeper spacing of 595 mm.

Figure 4 presents the effect of the rail dampers on the rail response. According to the numerical data of the rail damper, its low frequency is 139 Hz and the high frequency one is 859 Hz. Rail receptance decreases due to the rail dampers at middle and high frequencies, mainly around the pinned-pinned frequency.

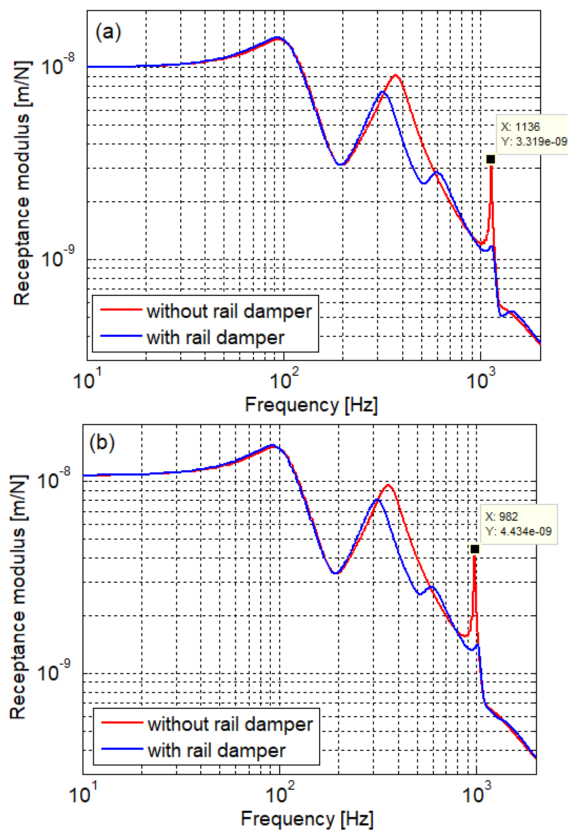

Fig. 5. Rail receptance (without and with rail dampers with ν_l = 500 Hz and ν_h = 1300 Hz): (a) sleeper spacing of 544 mm; (b) sleeper spacing of 595 mm.

Figure 5 shows what happens when the two elastic elements of the rail damper have the stiffness k_1 = 88.49 MN/m and k_2 = 95.44 MN/m respectively. The eigenfrequencies of the rail damper are at 500 Hz and 1300 Hz, the highest being higher than the PPR frequency.

The two diagrams show that the effectiveness of the rail dampers is improved covering a larger frequency range.

4. CONCLUSION

Rail damper represents a promising solution to reduce the rolling noise from the rail.

The effect of the rail dampers upon the frequency response function of the rail is investigated in the present paper, considering the practical values of the sleeper spacing applied in the track construction according to the regulation.

Track model consists of an infinite uniform Timoshenko beam resting on discretely supports. These model the rail pad, the semi sleeper and the ballast.

Rail damper model is a two-degrees-of-freedom oscillator related by the rail, at the middle distance between the semi sleepers.

The currently applied values for the sleeper spacing influence mainly the rail receptance in the range of the PPR frequency. This frequency increases cca. 150 Hz when the sleeper spacing decreases from the maximum value of 595 mm to the lowest one of 544 mm.

In the case of damped rail, a decrease in rail response at mid and high frequencies was observed for both values of sleeper considered, which clearly shows the effectiveness of the rail dampers.

To reduce the rolling noise and the rail corrugation wear, it is important that the rail dampers flatten the peak of the PPR. From this point of view, it is recommended that the high frequency of the rail damper be higher than the PPR frequency, but this recommendation is conditioned by the practical realization of the corresponding stiffening of the elastic elements of the rail damper.

The presented results show that the variation of the distance between the sleepers according to the construction regulations of the track does not affect the effectiveness of the rail shock absorbers.

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6. REFERENCES

[1] Gramowski, C., Suppin, P., *Impact of Rail Dampers on the Mainline Rail Roughness Development. In: Anderson D. et al. (eds) Noise and Vibration Mitigation for Rail Transportation Systems.* Notes on Numerical Fluid Mechanics and Multidisciplinary Design, 139 (2018) 367-374.

[2] https://www.uudenrailproducts.nl (accessed 12 June 2023).

[3] Squicciarini, G., Toward, M. G. R., Thompson, D. J., *Experimental procedures for testing the performance of rail dampers*, Journal of Sound and Vibration 359, 2015, 21-39.

[4] Thompson, D. J., Jones, C. J. C., Waters, T. P. and Farrington, D., *A tuned damping device for reducing noise from railway track* Applied Acoustics 68 (2007) 43-57.

[5] Toward, M. G. R., Squicciarini, G., Thompson, D. J., Gao, Y., *Estimating the performance of rail dampers using laboratory methods and software prediction,* 11th International Workshop on Railway Noise 9-13 September (Uddevalla, Sweden).

[6] Michalczyk, R., Brzeziński K., Zbiciak A. *Numerical Vibration Response of Railway Track Retrofitted with Single Degree of Freedom Rail Dampers*, IOP Conf. Series: Materials Science and Engineering 661, (**2019**) 2-8.

[7] Jin, H., Zhou, X. , Sun, X., Li, Z.,*Decay rate of rail with egg fastening system using tuned rail damper*, Applied Acoustics Vol.172, (2021), 2-10.

[8] Zhao, C., Wang, P, Yi, Q., Sheng, X., Lu, J., *A detailed experimental study of the validity and applicability of slotted stand-off layer rail dampers in reducing railway vibration and noise*, Journal of Low Frequency Noise, Vibration and Active Control, Vol. 37(4) 896–91, (2018) 898-900.

[9] Li, W., Wang, A., Gao, X., Ju, L., Liu, L., *Development of multi-band tuned rail damper for rail vibration control, School of Urban Railway Transportation*, Shanghai University of Engineering Science, Shanghai,(2021) 2-7.

[10] Mazilu, T., *Wheel/rail vibrations* (in Romanian), Ed. Matrixrom, Bucharest, 2008.

[11] Mazilu, T., Fologea, D., *Rail Damper Functionality-Modelling and Experimental Determinations*, Materials Science Forum Vol. 957, (2019), 23-32.

[12] Instruction of norms and tolerances for track construction and maintenance - Normal gauge lines (Instruction no 314, 1989) the Romanian Ministry of Transport and Telecommunications.

Influența parametrilor amortizorului de șină asupra funcției de răspuns în frecvență a șinei

Lucrarea prezintă influența caracteristicilor vâscoelastice ale unui amortizor de șină asupra funcției de răspuns în frecvență a șinei în termeni de receptanță. În acest scop, se folosește un model simplificat al șinei constând din travee cu o singură cale: șină și jumătate de suport la fiecare capăt al șinei, incluzând șină, traversă și balast. Modelul de șină este o grindă Timoșenko liberă, suportul și balastul sunt modelate de un sistem Kelvin-Voigt pentru fiecare, iar traversa este modelată ca un corp rigid în translație verticală. Ecuațiile mișcării sunt rezolvate folosind metoda funcției lui Green. Se analizează impactul frecvențelor naturale ale amortizorului șinei asupra recepției șinei.

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