TECHNICAL UNIVERSITY OF CLUJ-NAPOCA

ACTA TECHNICA NAPOCENSIS

Series: Applied Mathematics, Mechanics, and Engineering
Vol. 67, Issue Special II, April, 2024

DYNAMIC BEHAVIOUR OF A FREIGHT WAGON WITH WHEEL FLAT WHEN RUNNING ON THE TRACK

Vlăduț-Marian DINU, Traian MAZILU, Ana-Maria DINU

Abstract: *The flat is a serious damage of the rolling surface of the railway wheel. This defect occurs more frequently at the freight wagons when the wheel is blocked due to a braking system malfunction. In this paper, the dynamic behavior of a freight wagon with wheel flat when running along the track is examined to outline the influence of the dry friction from the wagon suspension upon the wheel/rail contact force and the carbody acceleration. To this aim, the wagon model is reduced to a two-mass oscillator with dry friction and the track is modelled using an infinite Euler-Bernoulli beam resting on two-layer viscoelastic foundation. Equations of motion are solved via the Green function method. It is shown the stick-slip vibration of the carbody excited by the wheel flat. Influence of the speed upon the carbody acceleration and the wheel/rail contact force is pointed out.*
Key words: *freight wagon; track; 2-DOF oscillator; dry friction; wheel flat; time-domain Green function; wheel-rail contact force; carbody acceleration.*

1. INTRODUCTION

Wheel flat is a serious defect of the rolling surface of the running gear of a railway vehicle (Fig. 1).

Fig.1. Wheel flat [1]

When a railway vehicle is running in braking behavior on wet rails it is possible that one or more wheelsets to be locked caused by a disfunction in the braking system, resulting in wheelset/s with wheel flat defect. This defect is more frequently at freight wagons where the maintenance standards are more permissive compared to passenger cars and locomotives due to economic reasons.

Effect of the wheel flat upon the dynamics of the railway vehicles has been studied from many aspects. For instance, detection of the vehicles with wheel flat defects using the measurements of the vertical acceleration at the level of axle-box is treated in ref. [2]. Other detection methods use either the time-domain images and deep learning [3] or the wavelet transform [4]. In ref. [5], the vertical vibration behavior of a passenger carbody induced by wheel flat defect has been studied depending on the primary and secondary suspension stiffness. Prediction of impact noise caused by the wheel flats is presented in many papers [6, 7].

In all previous papers, the vehicle model consists of a rigid body or multi-DOF oscillator with linear characteristics. However, the suspension of the freight wagons has nonlinear characteristic due to the dry friction and this paper contributes to simulate the dynamic behavior of the wagons with wheel flats.

In this paper, the case of a freight wagon with wheel flat running on the track is presented to point out some of the most important features of the dynamic behavior. To this end, a simplified model of the wagon consisting of a moving two-DOF oscillator with dry friction able to simulate the stick-slip phenomenon, on a beam resting on two-layer foundation – the track model – is considered. Equations of motion are numerically solved using Green's function method [8]. The effect of the suspension with dry friction upon the dynamic behaviour of the wagon in terms of the carbody acceleration and wheel/rail contact force is outlined.

2. MODEL OF THE FREIGHT WAGON-TRACK SYSTEM

2.1. Model description

Figure 2 display the simplified mechanical model of a freight wagon running with constant velocity on a straight track (Fig. 3). Both freight wagon and track are reduced to 1/2 unit of each due to the symmetry. Furthermore, neglecting the influence of the carbody pitch, the freight wagon can be assimilated with a 2-DOF oscillator to simulate the carbody bounce.

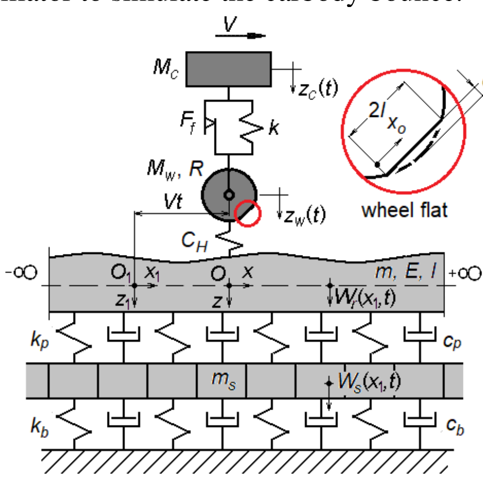

Fig.2. Model of the freight wagon-track system

Fig.3. Two-axle freight wagon [9]

Oscillator consists of the upper rigid body of M_c mass representing the 1/4 carbody corresponding to a wheel, and the lower rigid body of M_w mass which models the wheel (1/2 wheelset). Wheel has the radius R and is affected by the presence of a flat of length $2l$ and of depth e.

Wagon suspension is modeled using an ideal elastic element of stiffness k, working in parallel with a damping element with dry force friction F_f.

Track model includes an infinite uniform Euler-Bernoulli beam resting on three-layer continuous foundation. Upper and lower layer consist of uniform distributed Kelvin-Voigt system (Winkler foundation with damping) and model the viscoelastic effect of the rail pad and ballast. Middle layer is of pure inertial nature, without shear force and bending moment, and introduces the influence of the sleeper's inertia. Parameters for the rail model are as follows: m – the mass per unit length, E – the longitudinal modulus of elasticity, I – the second moment of area and the parameters for the three layers of the track model are: k_p and c_p – the stiffness and damping of the rail pad, m_s – the mass per unit length of the sleepers, k_b and c_b – the stiffness and damping of the ballast.

It should be noticed that the track can be considered as a periodic spatial structure and, by neglecting the influence of the sleeper bay, the frequency domain of applicability of the model is reduced to 6–700 Hz [10]. Such model is extensively used to analyze basic features of the interaction between the vehicle and track [11].

Vehicle and track motion is related to the fixed reference frame $O_1x_1z_1$: $z_c(t)$ and $z_w(t)$ – the carbody and wheel displacement at time t, $w_r(x_1, t)$ and $w_s(x_1, t)$ – the rail and sleeper displacement at time t in the track section x.

When the wagon is moving along the track, the wheel/rail contact force, $Q(t)$, has two components, one is constant, being given by the vehicle weight and namely the static load per wheel, Q_o, and the other one is time dependent and namely the dynamic component of the contact force, $\Delta Q(t)$, which is the effect of the interaction between the wheel flat and rail

$$Q(t) = Q_o + \Delta Q(t), \qquad (1)$$

where $Q_o = (M_c+M_w)g$ with g – the gravitational acceleration.

The static load determines the equilibrium position, meanwhile the dynamic component of the contact force excites the transitory behavior. Consequently, all displacements describing the wagon and track vertical motion have two components corresponding to the equilibrium position and the transitory behavior

$$
\begin{aligned}
z_c(t) &= z_{co} + \Delta z_c(t) \qquad z_w(t) = z_{wo} + \Delta z_w(t) \\
w_r(x_1,t) &= w_{ro}(x_1,t) + \Delta w_r(x_1,t) \\
w_s(x_1,t) &= w_{so}(x_1,t) + \Delta w_s(x_1,t),
\end{aligned} \tag{2}
$$

where z_{co}, z_{wo}, $w_{ro}(x,t)$ and $w_{so}(x,t)$ correspond to the equilibrium position, and $\Delta z_c(t)$, $\Delta z_w(t)$, $\Delta w_r(x, t)$ and $\Delta w_s(x, t)$ describe the transitory behavior. Equilibrium position of the track is time dependent in respect to the $O_1x_1z_1$ reference frame because the contact force is a moving force, but this position is unchanging in time when the Oxz moving reference frame is considered, where $x_1 = x + Vt$ and $z_1 = z$.

2.2. Equations of motion

First, the equations of motion of the transient behavior are written in respect to the fixed reference frame:

- for wagon

$$
\begin{aligned}
&M_c \frac{\mathrm{d}^2 \Delta z_c}{\mathrm{d}t^2} + k\left(\Delta z_c - \Delta z_w\right) - F_f = 0; \\
&M_w \frac{\mathrm{d}^2 \Delta z_w}{\mathrm{d}t^2} + k\left(\Delta z_w - \Delta z_c\right) + F_f = -\Delta Q,
\end{aligned} \tag{3}
$$

where F_f is the suspension dry friction force which can be modeled in the simplest way following Coulomb's low of friction

$$
\begin{aligned}
&\text{for } \Delta\dot{z}_c - \Delta\dot{z}_w \neq 0 \\
&F_f = -\mu_c\left(M_c g + k\left(\Delta z_c - \Delta z_w\right)\right)\operatorname{sgn}\left(\Delta\dot{z}_c - \Delta\dot{z}_w\right) \\
&\text{for } \Delta\dot{z}_c - \Delta\dot{z}_w = 0 \\
&\left|F_f\right| \le \mu_s\left[M_c g + k\left(\Delta z_c - \Delta z_w\right)\right],
\end{aligned} \tag{4}
$$

where μ_s and μ_c are the coefficients of static and kinetic friction;

- for track

$$
\begin{aligned}
&EI \frac{\partial^4 \Delta w_r}{\partial x_1^{\,4}} + m \frac{\partial^2 \Delta w_r}{\partial t^2} + c_p \frac{\partial\left(\Delta w_r - \Delta w_s\right)}{\partial t} + \\
&k_p\left(\Delta w_r - \Delta w_s\right) = \Delta Q \delta(x_1 - Vt)
\end{aligned} \tag{5}
$$

$$
\begin{aligned}
&m_s \frac{\partial^2 \Delta w_s}{\partial t^2} + \left(c_p + c_b\right)\frac{\partial \Delta w_s}{\partial t} + \\
&\left(k_p + k_b\right)\Delta w_s - c_p \frac{\partial \Delta w_r}{\partial t} - k_p \Delta w_r = 0,
\end{aligned} \tag{6}
$$

where $\delta(.)$ is the Dirac delta function.

The following boundary conditions should be associated to the equations of motion of the track

$$
\lim_{|x_1 - Vt| \to \infty} \Delta w_{r,s}(x_1,t) = 0 . \tag{7}
$$

Equations of motion are completed with the equation of the wheel/rail contact force

$$
\Delta Q(t) = k_H\left(\Delta z_w(t) - \Delta w_r(x_1,t) - r(t)\right), \tag{8}
$$

where $r(t)$ is the wheel/rail displacement due to the wheel flat and k_H is the contact stiffness which can be calculated using Hertz's constant, δ_o is the wheel/rail deflection under the static load

$$
k_H = \frac{Q_o}{\delta_o}, \tag{9}
$$

where δ_o is the static deflection of the wheel/rail contact

$$
\delta_o = \left(\frac{Q_o}{C_H}\right)^{2/3}, \tag{10}
$$

where C_H is Hertz's constant.

Wheel/rail contact force obeys the condition

$$
Q(t) \ge 0 . \tag{11}
$$

The following formula is recommended for the wheel/rail displacement due to the wheel flat [6]

$$
r(x_o) = \frac{e}{2}\left(1 - \cos\pi\frac{x_o}{l}\right) \text{ for } 0 \le x_o \le 2l , \tag{12}
$$

where e is the wheel flat depth, $2l$ – wheel flat length and x_o is the coordinate along the flat (Fig. 2).

This formula can be rewritten depending on the wheel coordinate along the track, and, because of $x_1 = Vt$, depending on the time t

$$r(t) = \frac{e}{2}\left(1 - \cos 2\pi^2 \frac{R}{l}\left(\{\xi(t)\} - \xi^-\right)\right) f(t) \quad (13)$$

where $\{.\}$ is the fractional-part function and

$$\xi(t) = \frac{Vt}{2\pi R}, \qquad \xi^- = \frac{1}{2}\left(1 - \frac{l}{\pi R}\right),$$
$$f(t) = \sigma\left(\{\xi(t)\} - \xi^-\right) - \sigma\left(\{\xi(t)\} - \xi^+\right), \quad (14)$$

where

$$\xi^+ = \frac{1}{2}\left(1 + \frac{l}{\pi R}\right). \quad (15)$$

Second, the equation of motion of the track is rewritten in respect to the moving reference frame

$$EI\frac{\partial^4 \Delta w_r}{\partial x^4} + mV^2\frac{\partial^2 \Delta w_r}{\partial x^2} - c_p V\frac{\partial(\Delta w_r - \Delta w_s)}{\partial x} -$$
$$2mV\frac{\partial^2 \Delta w_r}{\partial x \partial t} + m\frac{\partial^2 \Delta w_r}{\partial t^2} + c_p\frac{\partial(\Delta w_r - \Delta w_s)}{\partial t} +$$
$$k_p(\Delta w_r - \Delta w_s) = \Delta Q\delta(0),$$
$$m_s V^2\frac{\partial^2 \Delta w_s}{\partial x^2} - (c_p + c_b)V\frac{\partial \Delta w_s}{\partial x} + c_p V\frac{\partial \Delta w_r}{\partial x} -$$
$$2m_s V\frac{\partial^2 \Delta w_s}{\partial x \partial t} + m_s\frac{\partial^2 \Delta w_s}{\partial t^2} + (c_p + c_b)\frac{\partial \Delta w_s}{\partial t} -$$
$$c_p\frac{\partial \Delta w_r}{\partial t} + (k_p + k_b)\Delta w_s - k_p \Delta w_r = 0.$$
(16)

Boundary condition (7) becomes

$$\lim_{|x|\to\infty} \Delta w_{r,s}(x,t) = 0 \quad (17)$$

and the equation of the wheel/rail contact force takes the following shape

$$\Delta Q(t) = k_H\left(\delta_o + \Delta z_w(t) - \Delta w_r(0,t) - r(t)\right). \quad (18)$$

Equations of motion of the wagon/track can be solved using Green's function method [8].

3. NUMERICAL APPLICATION

In this section the numerical results obtained with the help of the wagon/track model are presented and analyzed.

Numerical values correspond to a two-axle freight-wagon of Esx type (Fig. 2) and a ballasted track with concrete sleepers.

Parameter values for the 1/4 2-axle freight wagon are the carbody mass m_c = 2100/9350 kg (empty/loaded wagon), wheel mass m_w = 650 kg, wheel radius R = 0.5 m, suspension stiffness k = 2570000 N/m, coefficient of friction 0.33/0.30 (kinematic/static). Parameter values for the track are the rail linear mass m = 60 kg/m, longitudinal modulus of elasticity E = 210 MPa, second moment of area I = 3055 cm^4, rail pad stiffness k_p = 119.266 MN/m^2, rail pad damping coefficient c_p = 137.6 kNs/m^2 , sleeper linear mass m_s = 230.3 kg/m, ballast stiffness k_b = 100.917 MN/m^2 and ballast damping coefficient c_b = 91.47 kNs/m^2. Hertz's constant C_H = $11.86 \cdot 10^{10}$ N/m$^{3/2}$ and the contact stiffness has the following values for empty/loaded wagon k_H = 770.34 MN/m and k_H = 1120.44 MN/m. Gravitational acceleration is 10 m/s^2.

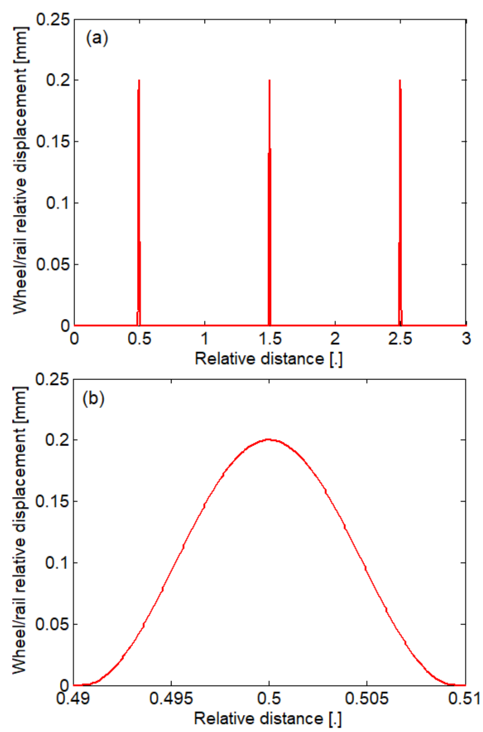

Fig.4. Wheel/rail relative displacement due to the wheel flat: (a) along three-wheel circumferences; (b) wheel flat detail

Figure 4 shows the wheel/rail displacement due to the wheel flat with length $2l = 60$ mm and depth $e = 0.2$ mm calculated with the help of Eq. (13) over a distance equal with three-wheel circumferences. According to the reference system, the positive value of the wheel-rail relative displacement shows that the wheel has downward movement at each rotation due to the presence of the wheel flat.

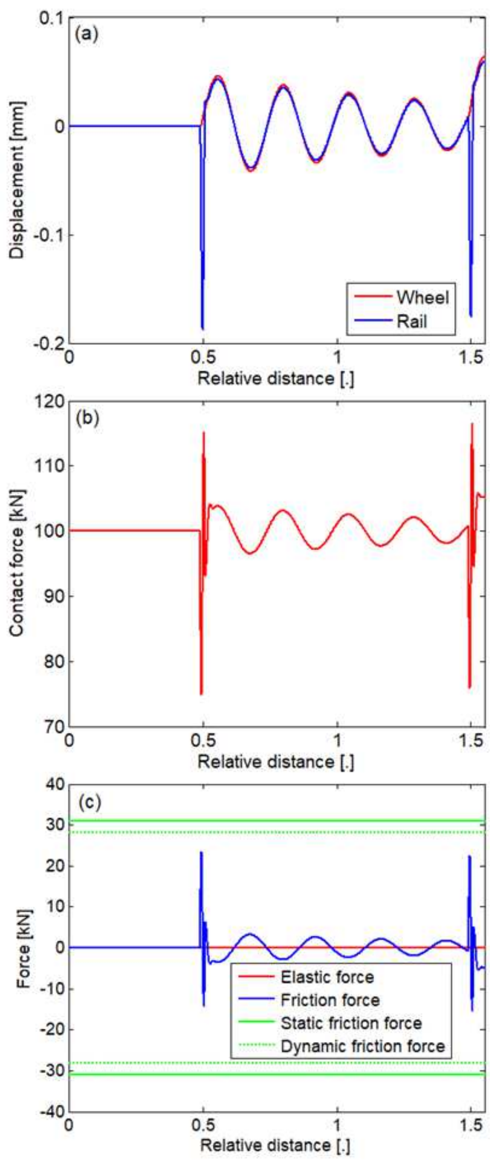

Fig.5. Wagon/rail interaction due to wheel flat at 40 km/h: (a) wheel and rail displacement; (b) wheel/rail contact force; (c) elastic and dry friction forces

Figure 5 presents what it happens during the first rotation and a half of the wheel when the loaded freight wagon has the wheel flat as described above and is moving at 40 km/h.

Passing the wheel over the wheel flat gives the center of the wheel a downward–upward trajectory due to the geometry of the wheel flat and the action of the static load on the wheel. The downward trajectory of the wheel has the effect of unloading the wheel-rail contact due to the inertia of the wheel. At the same time, the elastic reaction of the track pushes up the rail.

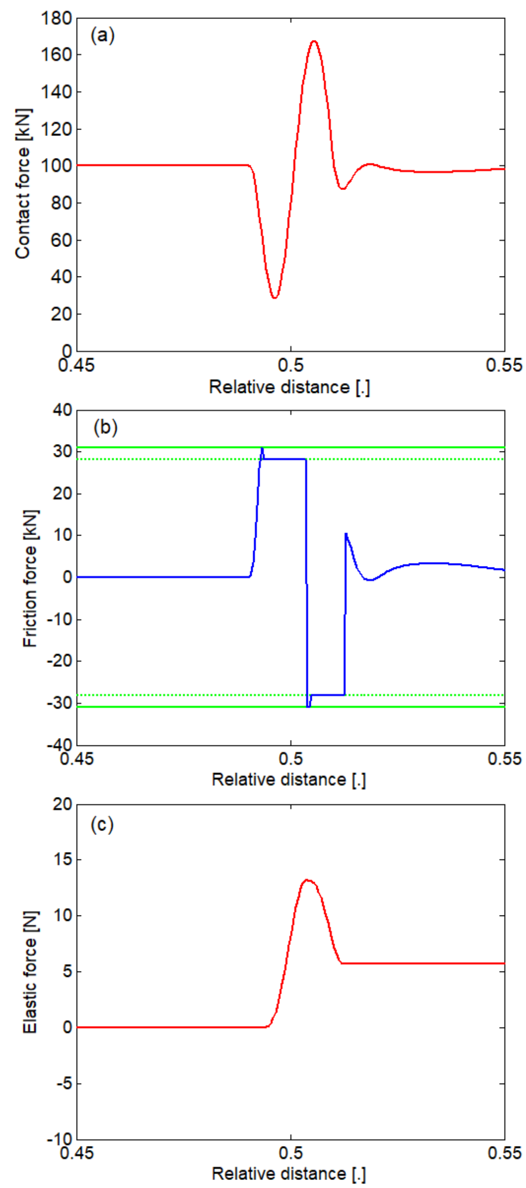

Fig.6. Wheel/rail impact force and the dry friction and elastic forces in the wagon suspension at 120 km/h: (a) contact force; (b) dry friction force; (c) elastic force

When the wheel passes the midpoint and the wheel center follows an upward trajectory, there is a sudden change in momentum that causes the impact force loading the contact. After passing the wheel flat, the motion is a damped vibration due to the damping of the track. Indeed, the contribution of the suspension damping is null as can be seen in Fig. 5 (c), where the elastic force in the wagon suspension is zero and the dry friction force is lower than the dynamic dry friction force; the vehicle suspension is locked.

At higher speeds, the impact force increases because the change in the momentum is more rapidly. Consequently, the wheel equilibrium is broken due to the fact the dry friction force is limited to the static friction force, and the difference must be counterbalanced by the elastic force resulting in the suspension unlocking. Wagon vibration takes the form of a sequence in which stick phases alternate with sliding phases – the stick-slip phenomenon.

For example, figure 6 illustrates what happens when the freight wagon with the wheel flat of identical shape as previous is running at 120 km/h. Only the time-series of the contact force and the dry friction and elastic forces are displayed. The contact force is higher, and the dry friction force reaches the static contact force. In this point, the sliding sequence begins and is lasting until the dry friction force is smaller than the dynamic dry friction force. During this sequence, the elastic force changes, after which it remains constant resulting another stick sequence.

Contact force depends on the dimensions of the wheel flat. For instance, Figure 7 presents the contact force at 120 km/h when the wheel flat deep takes three values, namely 0.2 mm – the reference case, and respectively 0.4 and 0.6mm. Length of the wheel flat is 60 mm as in the reference case. It observes that in the reference case the contact force is a continuous time function with a maximum value of 169 kN. Considering the wheel flat depth of 0.4 mm, the contact force becomes a discontinuous time-function because the contact between the wheel and rail is lost – contact force is null. Maximum impact follows at 273 kN. Finally, the wheel flat of 0.6 mm depth can cause contact force of 470 kN, almost 5 times higher than the static load per wheel.

From the ride quality viewpoint, carbody acceleration and impact force are very important. Figure 7 shows the dependence of the carbody acceleration and contact dynamic force upon the speed for the loaded and empty wagon. When is loaded, at low speed, carbody acceleration increases with the speed following the similar manner as the dynamic force because the suspension is locked. Beyond 50-60 km/h, the suspension begins to work and protects the carbody against the impact force produced by the wheel flat. Carbody acceleration remains unchanged to any speed.

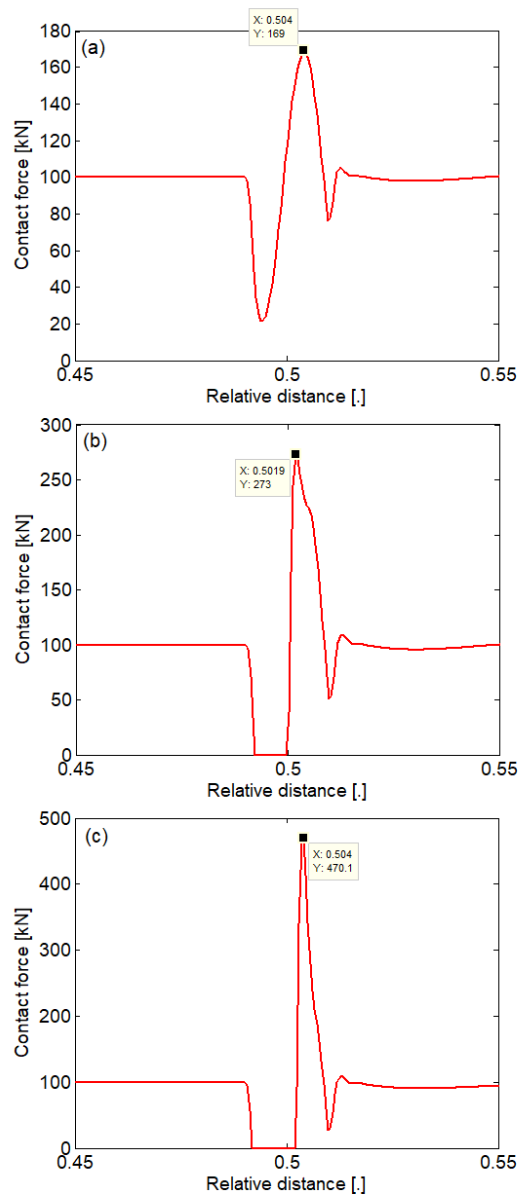

Fig.7. Contact force at 120 km/h: (a) e = 0.2 mm; (b) e = 0.4 mm; (c) e = 0.6 mm

In the empty wagon case, the suspension is unlocked within the speed range used in simulation, which explains why the carbody acceleration is practically constant to any speed.

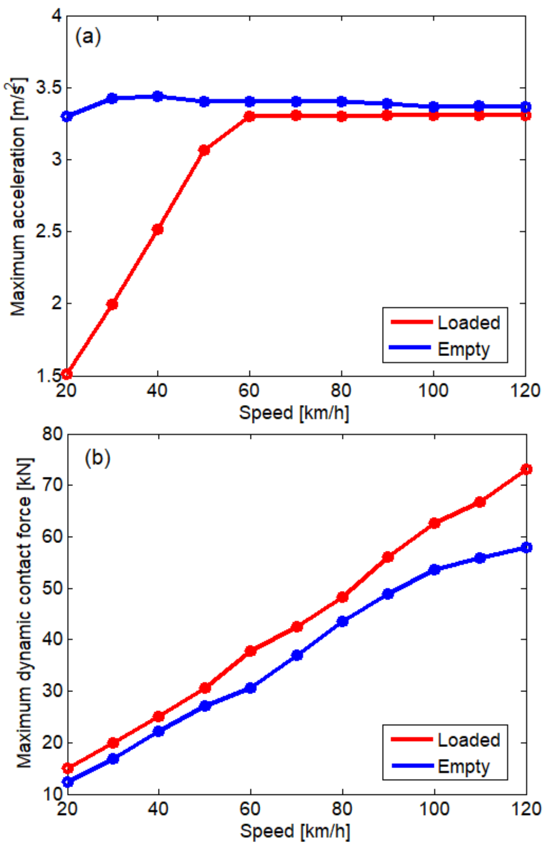

Fig.8. Carbody acceleration and wheel/rail contact dynamic force: (a) carbody acceleration; (b) contact force

Dynamic force constantly increases as the speed increases being higher when the wagon is loaded. This aspect can be explained by the fact that the contact stiffness is greater in the case of the loaded wagon.

4. CONCLUSION

Wheel flat is an undesirable defect in the rolling gear of the railway vehicles which causes high impact force affecting the ride quality and the integrity of the track.

In this paper, the model of 2-DOF oscillator with dry friction is used to analyze some basic aspects raised by the running of the freight wagons with wheel flats. Model of the track consists of a beam supported by two continuous elastic layers. To integrate the equations of motion, a numerically method based upon the theory of Green's function is adopted.

When the freight wagon with wheel flat is running along the track, there is a limit speed from which the wagon suspension starts working, and the wagon vibration behavior is characterized by the stick-slip phenomenon.

The suspension's locking limit speed is lower for the empty wagon and higher for the loaded wagon.

Maximum carbody acceleration is limited by the stick-slip phenomenon, but it doesn't influence upon the wheel/rail contact force.

Freight wagon with wheel flat running may cause the wagon to derail due to loss of wheel/rail contact loss and may cause the rail to break due to the exceptional contact forces induced.

Future research must consider a more realistic model of the 2-axle wagon taking into account the pitch movement of the carbody and the improved suspension characteristic.

Acknowledgement

This work was supported by a grant of the Ministry of Research, Innovation and Digitization, CCCDI - UEFISCDI, project number PN-III-P2-2.1-PED-2021-0601, within PNCDI III".

5. REFERENCES

[1] *Flat spot*, https://en.Wikipedia.org/wiki/Flat_spot.

[2] Bosso, N., Gugliotta, A., Zampieri, N., *Wheel flat detection algorithm for onboard diagnostic*, Measurement 123 (2018) 193-202.

[3] Ye, Y., Huang, C., Zeng, J., Zhou, Y., Li, F., Shock detection of rotating machinery based on activated time-domain images and deep learning: An application to railway wheel flat detection, Mechanical Systems and Signal Processing 186 (2023) 109856.

[4] Belotti, V., Crenna, F., Michelini, R., Rossi, G., Wheel-flat diagnostic tool via wavelet transform, Mechanical Systems and Signal Processing 20 (2006) 1953–1966.

[5] Steišūnasa, S., Ján Dižob, J., Gintautas Bureikaa, G., Žuraulisa, V., Examination of *Vertical Dynamics of Passenger Car with Wheel Flat Considering Suspension Parameters*, Procedia Engineering 187 (2017) 235-241.

[6] Wu, T. X., Thompson, D. J., A hybrid model for the noise generation due to railway wheel flats, Journal of Sound and Vibration 251 (2002) 115-139.

[7] Yang, J., Thompson, D.J., Time-domain prediction of impact noise from wheel flats based on measured profiles, Journal of Sound and Vibration 333 (2014) 3981-3995.

[8] Mazilu, T., Dinu, V.–M., *Interaction between a moving oscillator with dry friction and an infinite Euler-Bernoulli beam on viscoelastic foundation – Green's function method*, UPB Scientific Bulletin Series D: Mechanical Engineering, 83 (2021), 73-82.

[9] *Esx - 2-axle uncovered wagons,* http://www.cfrmarfa.com/index2.php?option=com_content&view=article&id=103:esx-es&catid=47.

[10] Mazilu T., Wheel-rail vibrations (in Romanian), Ed. Matrixrom, 2008, Bucharest, p. 165.

[11] Dimitrovová, Z., Two-layer model of the railway track: Analysis of the critical velocity and instability of two moving proximate masses, International Journal of Mechanical Sciences 217 (2022), 107042.

Regimul dinamic al unui vagon de marfă cu loc plan la o osie la circulația pe cale

Locul plan este un defect important al suprafeței de rulare al roții unui vehicul feroviar. Acest defect apare mai ales la vagoanele de marfă când osia este blocată din cauza unei funcționări deficitare a sistemului de frânare. În această lucrare este analizat comportamentul dinamic al unui vagon de marfă cu loc plan atunci când rulează de-a lungul căii pentru a pune în evidență influența frecării uscate a suspensiei vagonului asupra forței de contact roată-șină și a accelerației cutiei. În acest scop, modelul vagonului de marfă este redus la un oscilator cu două corpuri rigide cu frecare uscată, iar calea de rulare este modelată folosind o grindă infinită de tip Euler-Bernoulli rezemată pe o fundație vâscoelastică cu două straturi. Ecuațiile de mișcare sunt rezolvate prin metoda funcției Green.

Vlăduț-Marian DINU, PhD Student, Doctoral School of Mechanics and Mechatronics, NUST POLITEHNICA of Bucharest, Splaiul Independenței nr. 313, Bucharest, CP 060042, Romania, dinu_a_vlad@yahoo.com

Traian MAZILU, Full Professor, Department of Railway Vehicles, NUST POLITEHNICA of Bucharest, Splaiul Independenței nr. 313, Bucharest, CP 060042, Romania, traian.mazilu@upb.ro

Ana-Maria DINU, PhD Student, Doctoral School of Mechanics and Mechatronics, NUST POLITEHNICA of Bucharest, Splaiul Independenței nr. 313, Bucharest, CP 060042, Romania, dinu_ana.maria@yahoo.com