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NUMERICAL CALCULATION OF THE MULTIPARA CONVEX-CONCAVE PRECESSION GEAR

Stanislav LEALIN, Dumitru VENGHER

Abstract: *In modern methods of calculating the maximum allowable loads on precision gear reducers, computer simulations are increasingly being used. In this paper, the moments/loads limits on the precision gear reducer and the multi-pair convex-concave precessional gearing in it will be calculated. The theoretical calculation of the maximum allowable moment on the satellite with multiple gearing will be performed using the calculation formulas given in reference. At the same time, three pairs of teeth on one side of the satellite and three pairs on the opposite side are engaged in the gearing. The forces between them are equalized. The modeling will be done based on a special module for calculation and analysis of complex structures using the integrated Stress Analysis module (Autodesk Inventor Professional 2022 software). It utilizes finite element analysis based on the ANSYS software. Furthermore, it has been compared both calculations and examine the load distribution overall on all gearbox components. It has been considered the most heavily loaded areas and, if necessary, provide solutions. It has been optimized individual components and increase the size of some bearings to achieve the optimal possible load on the gearbox.*

Keywords: *Precessional Gearing, ANSYS, Stress Analysis, Inventor Software, Geometry, Kinematics and Contact Load, Numerical Calculation.*

1. INTRODUCTION

The first invention patents regarding the bolted gear A^B where: *Precessional Planetary Transmission* with Multiparous Engagement $A^B{}_{CX-R}$, registered on 30.05.1983 (SU 1020667 A) with priority date of 11.02.1981, and the Multiparous Engagement $A^B{}_{CX-CV}$, registered on 07.06.1988 (SU 1401203 A1) with priority date of 26.05.1986. The first invention with the gear engagement AD - Toothed Precessional Transmission with Multiparous Engagement $A^B{}_{CX-CV}$ was registered on 30.01.1989 (SU 1455094 A1) with priority date of 13.05.1986, authored by Ion Bostan [1].

The modification of the convex/concave profile geometry of the tooth flanks in the $A^B{}_{CX-CV}$ engagement and its dependency on the parametric configuration [Z_g-Θ, ±1, with the application of State Secret protection with the stamp "Service Use".

Simultaneously with the research and development of precessional gears A^B and A^D, manufacturing technologies for conical wheels with non-standardized flank profiles were also developed [1].

The theoretical calculation of the maximum allowable moment on the satellite with multiple gearing will be performed using the special calculation using module “Stress Analysis”.

In the course of this work, we have compared both calculations and examine the load distribution overall on all gearbox components.

It has been considered the most heavily loaded areas and, if necessary, provide solutions. It has been optimized individual components and increase the size of some bearings to achieve the optimal possible load on the gearbox.

2. BRIEF OVERVIEW OF PRECESSIONAL TRANSMISSIONS

The 2K-H precessional transmission (Figure 1) includes the satellite wheel g with two gear teeth Z_{g1} and Z_{g2}, which engage with the stationary central wheels b and the mobile wheels c, connected to the driven shaft *V*. When the crankshaft H rotates with an angular velocity ω_H, the satellite wheel g performs a spherospacial motion with a fixed-point *O* called the center of precession.

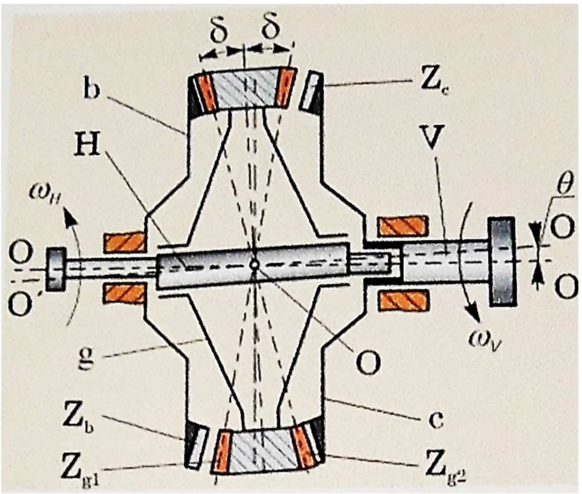


Fig.1. Kinematic Structure of the 2K-H Precessional Transmission [7]

The transmission ratio is determined by the relationship (Formula 1):

$$i = -\frac{Z_{g1}Z_c}{Z_bZ_{g2}-Z_{g2}Z_c} \qquad (1)$$

where: Z_{g1}, Z_{g2} are the numbers of teeth on the gear crowns of the satellite wheel, and Z_c, Z_b are the numbers of teeth on the central wheels *c* and *b*.

The analysis of the relationship (Formula 1) shows that 2K-H precessional transmissions provide a wide range of transmission ratios. The maximum kinematic effect is achieved for the following tooth relationships: Z_b=Z_{g2}, Z_{g1}=Z_{g2}+1, Z_c=Z_{g2}-1, pentru Z_c=Z_{g1}, Z_{g2}=Z_{g1}+1, Z_b=Z_{g1}-1.

Based on the numbers of teeth Z_b, Z_c, Z_{g1}, Z_{g2} and their ratios, in transmissions with the engagements (Z_b-Z_g) and (Z_c-Z_{g2}) where $Z_{b,g1,c,g2}$≤60, transmission ratios of ±20≤*i*≤±3600 are ensured [7,8].

Under these conditions, when Z_b=Z_{g1}-1 and Z_b=Z_{g2}+1 the 2K-H transmission ensures transmission ratios in the range of +8,3≤ *i*≤+30,3. Additionally, when Z_b=Z_{g2}+1 and Z_c=Z_{g2}-1 the 2K-H transmission ensures transmission ratios in the range of -7,3≤ *i*≤-29,3 [2,7,8].

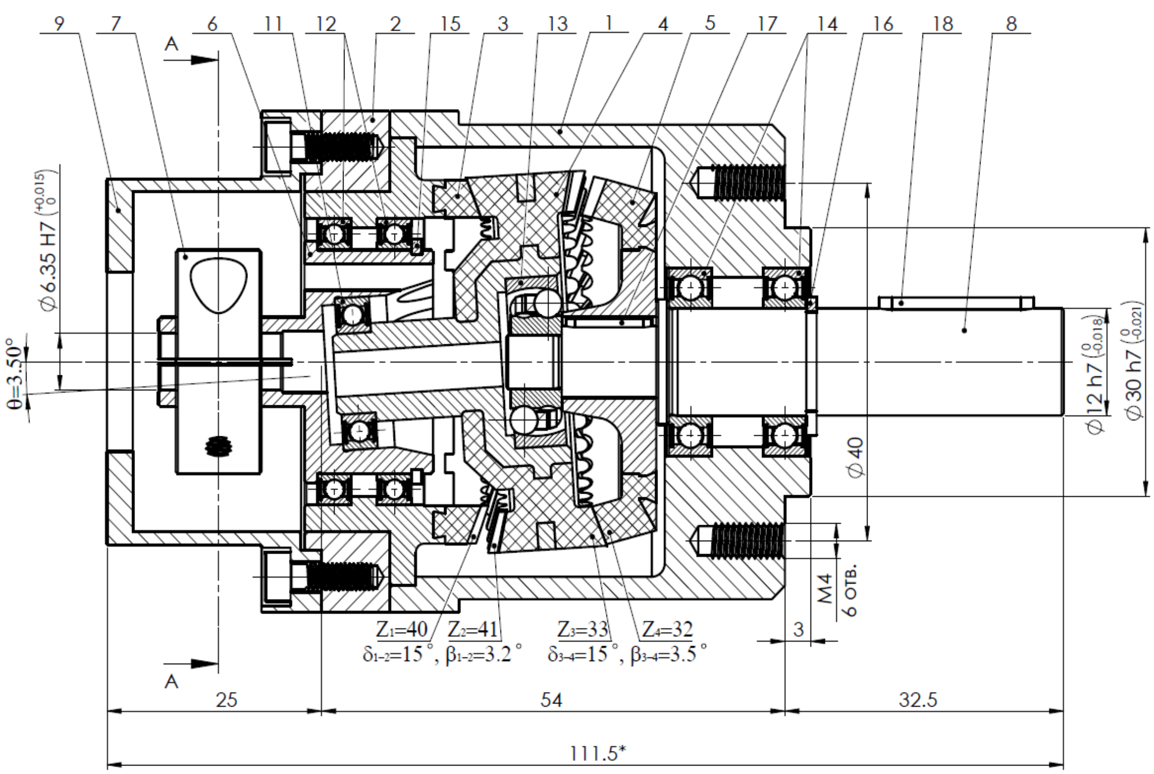


Fig.2. A precessional gear reducer with a tooth-to-tooth engagement type $A^D_{CX\text{-}CV}$

3. THEORETICAL CALCULATION OF THE LIMITING TORQUES/LOADS IN A PRECESSIONAL GEARBOX WITH ENGAGEMENT TYPE $A^{D}_{CX\text{-}CV}$

During the writing of this work, calculations were also carried out for a new, modern design of a precessional gear reducer with a tooth-to-tooth engagement type (Figure 2) [9,10,11]. In this modification, the classic rollers present in the previous generation are absent on both parts of the satellite. Additionally, the supporting axes for these rollers are also absent.

The new design is technologically advanced and consists of a reduced number of movable/rotating elements. The engagement and torque transmission occur directly between the fixed wheel (element 3 in Figure 2) and the satellite (element 4 in Figure 2) through a tooth-to-tooth connection.

List of elements in the new gearbox design: 1 - Housing; 2 - Cover; 3 - Fixed wheel; 4 - Satellite; 5 - Driven wheel; 6 - Crank; 7 - Clamp; 8 - Driven shaft; 9 - Mounting cover; 10 - Plug; 11 - Bearing 618/9-2Z; 12 - Bearing 61705 2Z; 13 - Bearing 126 TN9; 14 - Bearing 61801 2Z; 15 - Ring AE25 NBS; 16 - Ring A12; 17 - Key 2x2x10; 18 - Key 3x3x18.

This gearbox configuration has a transmission ratio of 1/164. The diameter of the satellite is d_m = 42 mm. The material used is 40 Cr steel (4140 Normal) with a hardness of HRC = 40. Based on these data and considering that there are 3 teeth simultaneously engaged on each side of the satellite, we can perform a theoretical calculation of the maximum torque T_{4max} for our engagement. The calculation of the maximum torque will be carried out using the formula provided below (Formula 2) [1].

$$T_{4max} = \frac{d_m^3 * (\sigma'_{hp})^2 * \cos\alpha_\omega * \varphi_{bd} * Z_\varepsilon * tg\beta_3 \rho_4}{53{,}3^3 * (\rho_4 - r_3) * K_{hp} * K_{h\beta} * K_{hv}} \qquad (2)$$

According to the results of the theoretical calculation, we obtain T_{4max} = 25.619 Nm. Based on this value, we will assign the recommended operating torque for this satellite as T_4 = 15 Nm.

Let's calculate the maximum input torque T1 to the shaft using formula (Formula 3) [1].

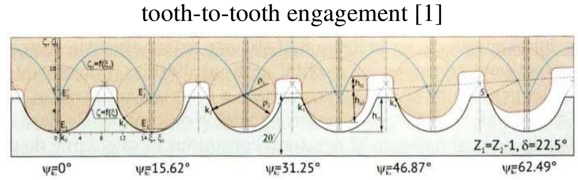
$$T_1 = \frac{T_{4max}}{i * \eta} \qquad (3)$$

where:
T_1 – moment on the input shaft;
T_{4max} - calculated maximum torque;
i – transmission ratio = 164;
η - coefficient = 0.8 [1].

The obtained value of T_1 is needed for selecting the drive element of the gearbox. In our case, T_1 = 0.195 N*m.

Additionally, it is necessary to consider that the load on the teeth engaged simultaneously is not uniform (Figure 3) [1,6,8]. For further calculations, the following load distribution among the teeth was assumed: K_1 - 30%, K_2 - 45%, K_3 - 25%. There may also be a slight contact with 4 pairs of teeth simultaneously, but it can be neglected for further simulations.

Fig.3. Multi-pair tooth contact in a gear reducer with tooth-to-tooth engagement [1]



The obtained data will be sufficient for conducting simulations and calculations of this precession gearbox. In the subsequent chapter, a highly accurate 3D model of this gearbox (Figure 4) is required for the calculations.

4. PRACTICAL CALCULATION AND RESULTS IN A PRECESSIONAL GEAR REDUCER WITH TOOTH-TO-TOOTH ENGAGEMENT $A^{D}_{CX\text{-}CV}$

In modern calculation methods for determining the maximum permissible loads on precessional gear reducers, computer simulations are increasingly being utilized. In this study, we will calculate the maximum allowable moments/loads on the precessional gear reducer (Figure 2) and the multi-pair convex-concave precessional gearing within it [12]. The theoretical calculation of the maximum permissible moment on the satellite, with multi-pair tooth engagement, will be conducted using the calculation formulas provided in the previous section. In the engagement, three pairs of teeth on one side of the satellite and three pairs on the opposite side are simultaneously involved, and the forces within them are balanced.

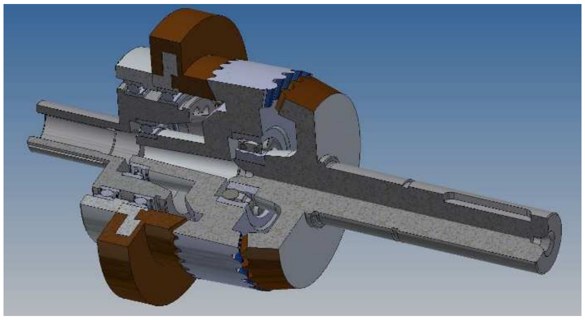


Fig.4. 3D model of precessional gear reducer with a tooth-to-tooth engagement type $A^{D}_{CX\text{-}CV}$

We will conduct the modeling based on a special module for the calculation and analysis of complex structures using the integrated Stress Analysis module (Autodesk Inventor Professional 2022 software) [5]. It employs finite element analysis based on the ANSYS program. Subsequently, we will compare both calculations and examine the overall load distribution on all gearbox components. We will consider the most heavily loaded areas and provide alternative solutions if necessary. The 3D model of the gearbox is shown below (Figure 4).

All gearbox components were described in the previous section. The distinctive feature of this transmission is the use of a self-aligning bearing, 126 TN9 (element 13 in Figure 2). It allows for free rolling of the satellite relative to the input and output shafts.

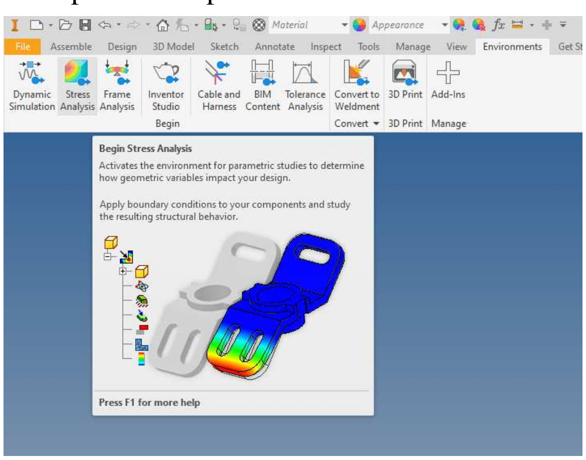


Fig.5. The specific module in the Autodesk Inventor Professional software: "Stress Analysis"

This bearing has several advantages over conventional ones but also experiences significantly higher contact loads due to its specific design. The small contact patch during the rolling process and the high contact loads can lead to unexpected results in subsequent simulations [13,16].

Next, we will input all our parameters from the theoretical calculations into the 3D model.

The specific module in the Autodesk Inventor Professional software is called Stress Analysis (Figure 5). Its working principle is described in references [5,14,15].

Furthermore, in Table 1 and Table 2, the physical properties and materials of the satellite are provided. Additionally, Table 3 presents data on the dimensions and shapes of the finite elements.

Table 1.

Physical properties of the satellite.

Mass	0.428484 kg
Area	38692.7 mm^2
Volume	54524.6 mm^3
Center of Gravity	x=-4.2924 mm y=-0.066022 mm z=-0.0124399 mm

Table 2.

Finite element settings for the satellite

Avg. Element Size (fraction of model diameter)	0.1
Min. Element Size (fraction of avg. size)	0.2
Grading Factor	1.5
Max. Turn Angle	60 deg
Create Curved Mesh Elements	No
Use part based measure for Assembly mesh	Yes

Table 3.

Materials of the satellite

Name	**Steel, High Strength, Low Alloy**	
General	Mass Density	7.85 g/cm^3
	Yield Strength	275.8 MPa
	Ultimate Tensile Strength	448 MPa
Stress	Young's Modulus	200 GPa
	Poisson's Ratio	0.287 ul
	Shear Modulus	77.7001 GPa
Part Name(s)	4. Satelit_OUT_Der.ipt 4. Satelit_IN_Der.ipt 6. Arborele manivela.ipt 3. Roata dintata nemiscata_1.ipt 3. Roata dintata nemiscata.ipt	

We set the calculated maximum moment on the output gear and shaft (Figure 6) as T_{4max} = 25.619 N*m. For the convenience of calculations, we assume that the input shaft is clamped and fully fixed. This way, we simulate the worst-case scenario where the input shaft is locked, while the maximum allowable moment is still present on the opposite side.

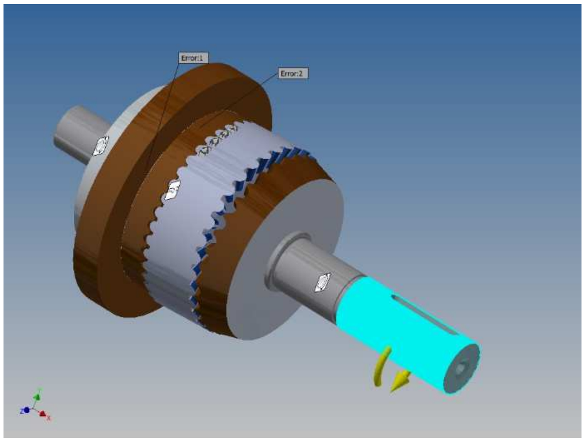


Fig.6. The applied torque

The stress simulation results of the satellite will be presented in Table 4.

Table 4.

Results of satellite stress simulations

Name	**Minimum**	**Maximum**
Volume	54527.7 mm^3	
Mass	0.428509 kg	
Von Mises Stress	0 MPa	808.893 MPa
1st Principal Stress	-494.719 MPa	1295.19 MPa
3rd Principal Stress	-1352.97 MPa	459.084 MPa
Displacement	0 mm	0.0484318 mm
Safety Factor	0.309064 ul	15 ul
Stress XX	-575.098 MPa	499.887 MPa
Stress XY	-225.252 MPa	194.166 MPa
Stress XZ	-325.267 MPa	213.684 MPa
Stress YY	-769.663 MPa	716.22 MPa
Stress YZ	-354.441 MPa	335.462 MPa
Stress ZZ	-1107.61 MPa	1081.16 MPa
X Displacement	-0.010887 mm	0.0108366 mm
Y Displacement	-0.047016 mm	0.0471736 mm
Z Displacement	-0.048430 mm	0.0456595 mm
Equivalent Strain	0 ul	0.00435139 ul
Contact Pressure X	-337.335 MPa	442.505 MPa
Contact Pressure Y	-376.816 MPa	472.25 MPa
Contact Pressure Z	-525.104 MPa	606.068 MPa

Next, we will present figures and values in graphical form.

We will consider the following critical simulation results:

1. Von Mises stress (Figure 7)
2. First principal stress (Figure 8)
3. Third principal stress (Figure 9)
4. Deformation (Figure 10)
5. Safety factor (Figure 11)
6. Contact stresses (Figure 12)

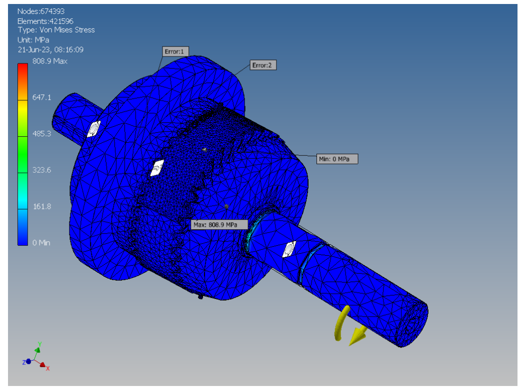


Fig.7. Equivalent stress according to Von Mises, MPa

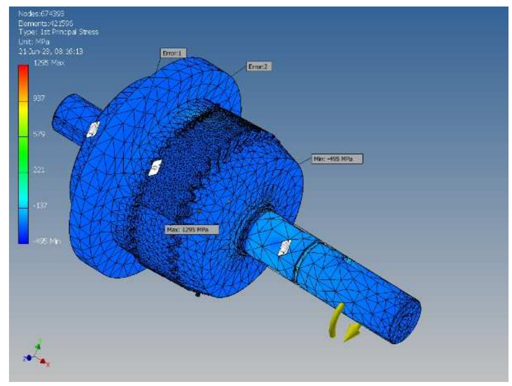


Fig.8. First principal stress, MPa

Based on the simulation results, we can confirm that our multi-pair gearing can withstand even the maximum torque values. Thus, the theoretical calculations provided earlier have been accurate.

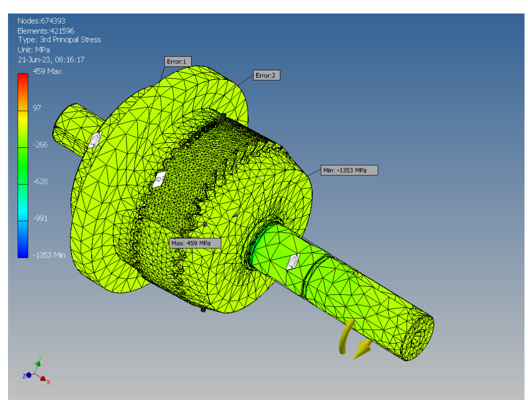


Fig.9. Third principal stress, MPa

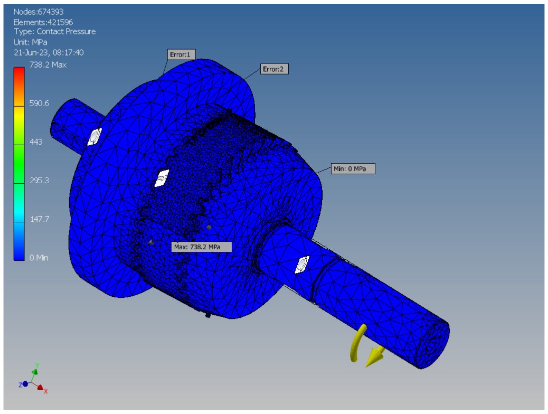


Fig.10. Linear deformation, mm

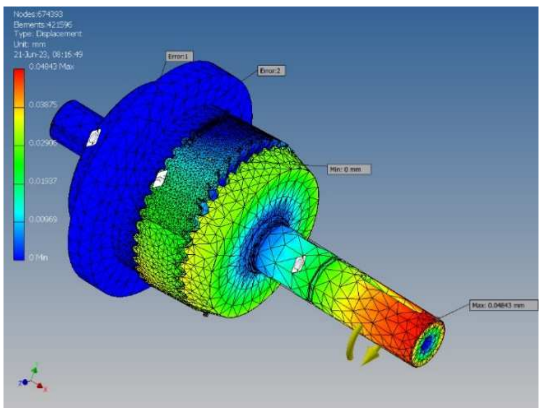


Fig.11. Safety factor

However, even though the satellite teeth can withstand the maximum allowable torque, based on the calculation results, we can identify the following problematic areas in the gearbox.

According to the safety factor values, we can see that the weakest component is the self-aligning bearing itself.

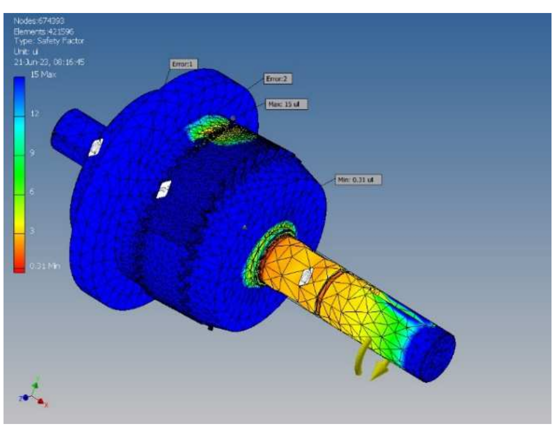


Fig.12. Contact stress, MPa.

The absolute value of the safety factor is 0.31 (Figure 11).

Simulations were conducted, which showed that the contact stresses in the multi-pair gearing are in the range of 300-350 MPa. Problems and weak points were identified in the components of the precessional gearbox, where the contact stresses reach 738 MPa, which is significantly higher than the permissible value (Figure 12).

Another weak point is the output shaft, particularly the technological channels. The safety factor values on the shaft are also unacceptable for this diameter (Figure 11).

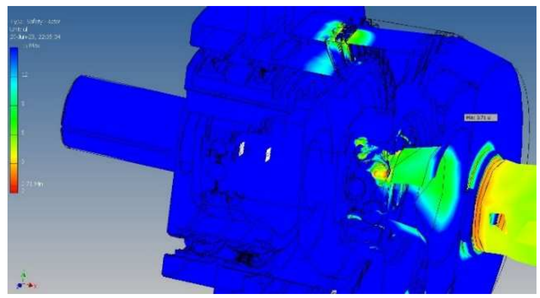


Fig.13. Self-aligning bearing. Safety factor at a working torque of 15 Nm

The next step was to perform recalculations, but with a working torque of T_4 = 15 Nm, based on our theoretical calculations. Despite the reduction in the working torque, the problem remains the same. The absolute value of the safety factor is 0.71 (Figure 13), which is still unacceptable. The load on the multi-pair gearing decreases significantly at the working torque (Figure 14).

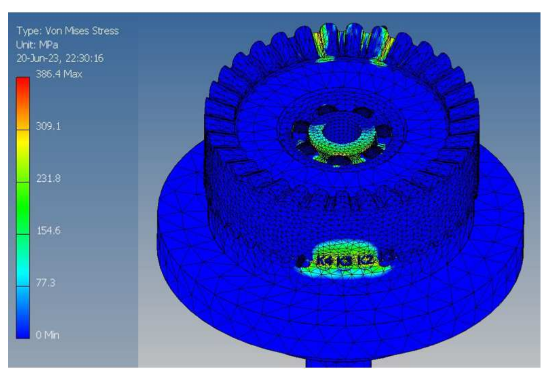


Fig.14. Multi-pair gearing. Contact stresses.

In the next section, we will propose solutions for the problematic areas of this gearbox. We will also conduct a subsequent simulation to ensure the functionality of our assembly.

5. PROPOSED SOLUTIONS AND REPEAT CALCULATIONS FOR THE WEAK POINTS OF THE GEARBOX WITH TOOTH-TO-TOOTH ENGAGEMENT $A^{D}_{CX\text{-}CV}$

After studying the theoretical calculations and conducting subsequent simulations, we have identified the weak points in the design of our precessional gearbox. We will now propose solutions to reinforce these weak components and perform another simulation. We will recalculate and verify the results.

First and foremost, we need to address the issue of elevated contact stresses in the self-aligning bearing. We have decided to explore the dimensions of bearings from the 1XX TN9 series (Figure 15). After conducting additional simulations, we have determined that the 129 TN9 bearing can handle the current load. It is approximately 1.5 times larger in size compared to the original bearing. The technical specifications of the new bearing are presented in Figure 15.

Secondly, it is necessary to increase the diameter of the output shaft from the nominal 12 mm to the proposed 15 mm.

	d [mm]	D [mm]	B [mm]	C [kN]	C_0 [kN]
135 TN9	5	19	6	2.51	0.48
126 TN9	6	19	6	2.47	0.48
127 TN9	7	22	7	2.65	0.56
108 TN9	8	22	7	2.65	0.56
129 TN9	9	26	8	3.9	0.815

Fig. 15. Technical specifications of the new bearing from the 1XX TN9 series.

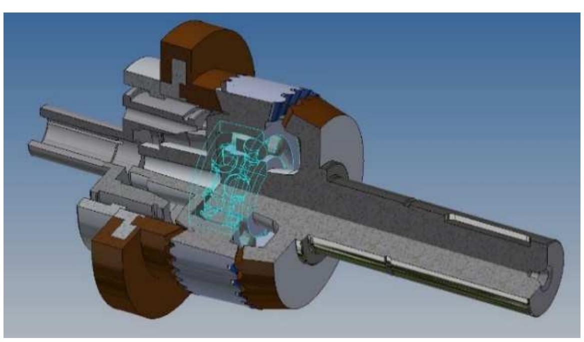


Fig. 16. New configuration of the 3D model of the gearbox

After modifying the 3D model, we obtained a new configuration (Figure 16). The internal bore inside the satellite was enlarged to accommodate the larger bearing, and the diameter of the output shaft was increased.

After the calculations, we have confirmed that our proposed modifications to the design and replacement of certain standard components were justified. The equivalent stress values according to the Von Mises was reduced from 809 MPa to the permissible limits of 245.8 MPa.

Similarly, the safety factor value was increased from 0.71 to the allowable value of 1.12 units.

6. CONCLUSIONS

We have successfully addressed all the objectives and reinforced the gearbox structure. This configuration of the gearbox can withstand even the maximum calculated torque of 15 N*m.

The equivalent stress values according to the Von Mises was reduced from 809 MPa to the permissible limits of 245.8 MPa.

Similarly, the safety factor value was increased from 0.71 to the allowable value of 1.12 units.

With the larger bearing size and increased diameter of the output shaft from the nominal 12 mm to the proposed 15 mm, the maximum stresses have shifted to the area of the multi-pair gearing.

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Calculul numeric al angrenării precesionale convex-concave multipare

Metodele moderne de calcul a sarcinilor maxime admise pentru reductoarele de viteză de precizie, simulările computerizate sunt din ce în ce mai utilizate în inginerie. În această lucrare se vor calcula momentele/limitele de sarcină ale reductorului de precizie al angrenajului precesional convex-concav multipar. Calculul teoretic al momentului maxim admisibil pe satelitul cu angrenare multipară se va efectua folosind formulele de calcul date în referința. În același timp, trei perechi de dinți pe o parte a satelitului și trei perechi pe partea opusă sunt angrenate concomitent. Forțele dintre ele sunt egalizate. Modelarea se va face pe baza unui modul special de calcul și analiză a structurilor complexe folosind modulul integrat Stress Analysis (software-ul Autodesk Inventor Professional 2022). Acesta utilizează analiza cu elemente finite bazată pe software-ul ANSYS.

Stanislav LEALIN, PhD Student, Technical University of Moldova, Department of Manufacturing Engineering, str. Studentilor, 3/1, MD-2045, Chisinau, Republic of Moldova, stanislav.lealin@gmail.com

Dumitru VENGHER, PhD Student, Assistant professor, College of Technical University of Moldova, Rep. Moldova, city Chișinău, avn. Ștefan cel Mare 168, dumitru.vengher@adm.utm.md