



TECHNICAL UNIVERSITY OF CLUJ-NAPOCA

ACTA TECHNICA NAPOCENSIS

Series: Applied Mathematics, Mechanics, and Engineering  
Vol. 67, Issue Special II, April, 2024

## COST OPTIMISATION FOR ASSIGNED TOLERANCE TO A 2D DIMENSIONAL CHAIN USING GeoGebra SOFTWARE

Aurel TULCAN, Felicia Veronica BANCIU, Ion GROZAV

**Abstract:** *The analysis and allocation of dimensional tolerances is still an important subject because the closing dimensions of dimensional chains (LDs) have to meet the design requirements, respectively fall within the desired tolerance range in order to support the products quality requirements. This paper presents an initial analysis of a chain of 2D dimensions, next the tolerance fields of the 2D chain dimensions will be reallocated according to a transfer coefficient in relation to the closing dimension of 2D chain while minimizing the cost of these tolerance fields. To minimize the costs, the variation curves of the tolerance costs were used and finally, the assigned tolerances are verified using the Monte-Carlo method.*

**Key words:** *tolerance field, closing dimension, tolerance allocation, cost optimization, Monte-Carlo method.*

### 1. INTRODUCTION

The efforts of manufacturing specialists are directed towards achieving products at a lower cost, high quality and a shorter time to market. Optimizing the cost of tolerances is a topic widely debated in the specialized literature, being a common topic both for industry and for academia and research because maintaining the costs attached to the manufacture of products as lower as possible, almost without exception, it also addresses this area of part's allocated tolerances. Thus, the identification of a valid set of tolerance values creating a balance between quality and cost is a challenging task, because cost, quality, operation and behaviour depend on them [1, 2].

Too small tolerances can cause a longer manufacturing process, more manufacturing operations, very well-trained operators or high precision machine tools and measuring devices. If tolerances are set too loose the manufacturing costs decrease but the surface and dimensional, positioning quality characteristics will decrease. In this note, of lowering the costs attached to the product development process and decreasing the time required for conception, embodiment design and layout design, manufacturing it is

important that the tolerances attached to the part's dimensions in a dimensional chain to be analysed and optimized from the early design stages. Tolerance-cost optimization is developed following a linear or non-linear regression analysis based on the data of the cost-tolerance experiment and to derive the correlation curve between the two. The concerns in this direction of the minimum cost of tolerances are presented in a multitude of specialized papers e.g. works that present an integrated method for tolerance analysis and cost evaluation [3, 4], the cost-tolerance functions used in minimum-cost tolerance allocation on mechanical assemblies [5], optimal tolerance allocation using least-cost model by applying optimization techniques (GA) [6], neural networks [7], production cost-tolerance models and a hybrid-model tolerance optimization formulation for tolerance synthesis are presented in [8], a model minimizes the combination of quality loss and manufacturing cost simultaneously in a single objective function by setting both process tolerances and design tolerances simultaneously [9], a holistic overview of tolerance-cost optimization; a comprehensive mind map covering all relevant aspects of tolerance-cost optimization was presented and discussed in detail in [10].



### 3. DIMENSIONAL CHAIN ANALYSIS

DC from figure 2 is composed of two distinct kinds of the component dimensions:

- DC1={D1, ...,D8};
- Dp on the inclined plane.

DC1 by their dimensions and tolerances influences the position of the endpoint Pf, and the dimension Dp has the coordinates influenced by deviations from the inclination of the inclined plane.

It can be assumed that the Dp dimension induces on the direction of the closing dimension C a tolerance field  $T_{pC}$ :

$$T_{pC} = 2 \cdot mDp \cdot \sin(Tup/2) \quad (2)$$

in which:

-  $mDp$  is the modulus of the size vector Dp, which is calculated as the distance between the origin of O (0,0) and the point of intersection  $O_{LD}$  of normal line to the average position of the inclined plane with the inclined plane (see figure 2);

-  $Tup$  - the tolerance field of the angle of the inclined plane ( $Tup = 2^\circ$ ).

For analysing the dimensional chain tolerances presented, the software module from Solidworks [11] can be used, but this has an inconvenience. One inconvenience is that the percentage of transfer of the variations in the component dimensions is presented, but it is not indicated how much they are modified in order to obtain the desired value of the tolerance field at the closing dimension C. The designer must modify the tolerance fields of the component dimensions until the Solidworks software will display a tolerance to closing dimension C that satisfies the initial requirements. Another drawback is the fact that the software in Solidworks calculates the tolerances using two statistical methods of calculation:

- The worst-case method
- SSR Method (Sum Square Root).

One possible method that is closer to real situations is the Monte-Carlo method. In this method randomly assigns values in the tolerance field of the component dimensions and is checked by calculating the closing dimension C. Thus, it can be determined the more real dispersion field of the closing dimension C and

thus checks if it satisfies the desired tolerance field for this closing dimension C.

### 4. INITIAL TOLERANCES ANALYSIS

The designer initially has the tolerance fields for the component dimensions of the dimensional chain DC. It is desired to check whether they induce at the closing dimension C a dispersion field smaller than a desired tolerance field  $Td=0.7$  mm. Geogebra software [12] will be used to analyse the tolerances. In this software, a calculation program will be developed that allows the dimensional chain visualization, the verification of the initial tolerances, the allocation and verification of other tolerances, which will satisfy the tolerance field allowed at the closing dimension of dimensional chain.

The initial data for DC shall be entered into matrix m1 (see figure 3a).

DatelnitialeLD1 m u Tm Tu AccesSchimbare = 11

$$m1 = \begin{pmatrix} 40 & 70 & 25 & 8.04 & 7.99 & 22 & 4.02 & 3.98 \\ 0^\circ & 90^\circ & 180^\circ & 0^\circ & 180^\circ & 270^\circ & 40^\circ & 220^\circ \\ 0.1 & 0.1 & 0.16 & 0.04 & 0.02 & 0.02 & 0.02 & 0.02 \\ 0.1^\circ & 0.1^\circ & 0.1^\circ & 0.1^\circ & 0.1^\circ & 0.1^\circ & 0.1^\circ & 0.1^\circ \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

a)

Td = 0.7  
 Cp = 1.33  
 CofK = {3, 15, 23, 25, 15, 9, 12, 22, 13}  
 ExpK = {-0.8, -0.7, -0.75, -0.6, -0.5, -0.2, -0.1, -0.3, -0.2}  
 Δk = 1.2  
 TstLim = 0.7  
 up = 130°  
 Tup = 2°  
 nD1 = 8

b)

Fig.3. Initial data

The matrix m1 comprises in rows:

- Modules of dimensions D1, ..., D8;
- Angles of dimensions D1, ..., D8 with Ox axis;
- The Tk tolerance fields of the D1, ..., D8 dimension modules;
- The tolerance fields of the angles of the D1, ..., D8 dimensions vectors;
- Acceptance / non-acceptance coefficient (1/0) for changing the tolerances of dimensions D1, ..., D8.

The dimensional vectors of the DC are shown in figure 2. DC also includes dimensions that

assure J1 and J2 clearance. These clearances must be maintained at the same value, regarding constructive considerations, therefore the tolerance fields of the dimensions that make these clearances, cannot be modified (the case of D4, D5, D7, D8 dimensions). Figure 3b also shows other initial data required in the calculation program. Using the initial data, the average dimension values of DC1 were calculated and the vectors of these component dimensions were plotted. The modulus of the closing dimension  $mC=42.4228$ , and the angle of the closing dimensions C is  $uC=40^\circ$  (figure 4).

```

Pk0 = {(0, 0), (40, 0), (40, 70), (15, 70), (23.04, 70), (15.05, 70), (15.05, 48), (18.1295, 50.5
Pf = (15.0806, 48.0257)
mC = 42.4228
dc: 19.2836x - 22.9813y = -812.8854
Pp = (-17.4171, 20.7569)
Pk = {(-17.4171, 20.7569), (0, 0), (40, 0), (40, 70), (15, 70), (23.04, 70), (15.05, 70), (15.05,
Vk = {(17.4171, -20.7569), (40, 0), (0, 70), (-25, 0), (8.04, 0), (-7.99, 0), (
OrigineLDtot=21
mDp = 27.0962
Tpc = 0.9458
DimensiInchidereC = 22
vC = (42.4228; 40°)
uC = 40°
    
```

Fig.4. Initial dimensional chain data

The perpendicular line from the endpoint Pf on the plane pl (see figure 5) will intersect pl at the point Pp, which relative to the origin Op (0,0) will define the segment mDp. This mDp segment, taking into account the inclination tolerance of the plane pl ( $Tup=2^\circ$ ), will define the tolerance field  $Tpc=0.9458$  (see relation (2) and figure 4). This  $Tpc$  value will fill in row 3 of the matrix m1, it is inserted on position 1. Thus, the vector of the initial tolerances Target will be:

$$Tint = \begin{Bmatrix} 0.9458, 0.1, 0.1, 0.16, 0.04, \\ 0.02, 0.02, 0.02, 0.02 \end{Bmatrix} \quad (3)$$

The vectors of the DC dimensions will make with the direction of the closing vector vC the angles gk:

$$gk = \begin{Bmatrix} 90^\circ, 40^\circ, 310^\circ, 220^\circ, 40^\circ, \\ 220^\circ, 130^\circ, 0^\circ, 180^\circ \end{Bmatrix} \quad (4)$$

The transfer coefficients of the variations in the component dimensions shall be:

$$ct = \cos(gk) = \begin{Bmatrix} 0, 0.766, 0.6428, \\ -0.766, 0.766, -0.766, \\ -0.6428, 1, -1 \end{Bmatrix} \quad (5)$$

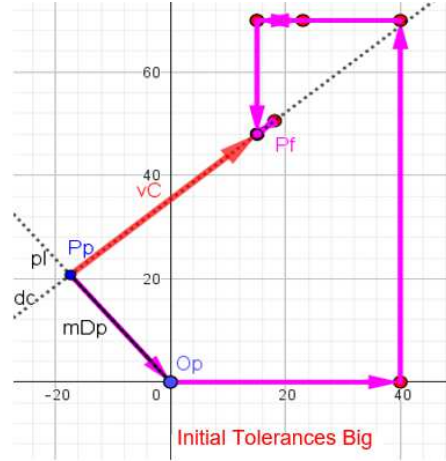


Fig.5. Initial Tolerances Big

The first Dimension Dp of the DC, however, does not transfer changes to the closing dimension C by changing its module, but rather by changing the position of the plane pl, because of the tolerance  $Tup$ , this being  $Tpc$ , previously calculated, but which propagates in the direction of the vector vC, so the first term of the vector (5) will be 1 and not 0. The vector of the modified transfer coefficients (ctt) will be:

$$ctt = \begin{Bmatrix} 1, 0.766, 0.6428, -0.766, 0.766, \\ -0.766, -0.6428, 1, -1 \end{Bmatrix} \quad (6)$$

The Mote-Carlo method will be applied, constituting a matrix series sm of nc values for the dimension modules in CD1, which will be projected on the Ox and Oy axes constituting the matrixes series of values of the projections of the dimensions Dk, k=1, ..., 8, obtaining the matrices sDkxc and sDkyc (see figure 6). These series of projections will define the  $x_{Pf}$  and  $y_{Pf}$  coordinate series of the endpoint Pf respectively. Also it can be defined an mdPf medium endpoint (see figure 7).

In order to obtain series of values of the closing dimension C, a series of sUp values of the variation of the angle of inclination  $up = 130^\circ$  of the plane pl in its tolerance field  $Tup = 2^\circ$  is defined. This series will constitute a series of values of the scfm angular coefficient, which is used to calculate the series of values of the smC closing dimension modules.

The series of values of the smC closing dimension module allows to check the initial tolerances. The statistical analysis of the smC values is shown in figure 8.





By graphically representing some points that have the coordinates (t, K) it is possible to choose an exponential function of the form which will approximate the dependence of the cost on the tolerance chosen for the D<sub>k</sub> dimension.

$$y = CofK \cdot t^{ExpK} \quad (7)$$

With the expression of the costs of tolerances, the expression of cost derivatives is easily obtained (see figure 10).

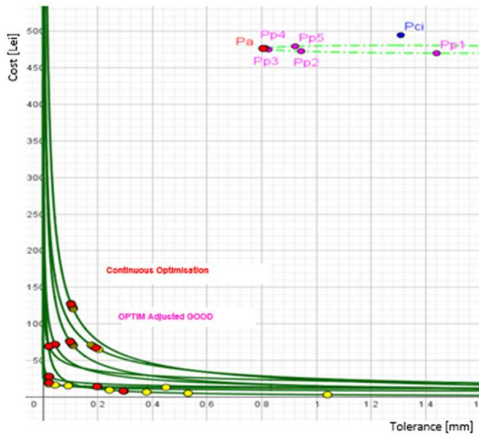


Fig.10. Tolerances cost curves, their derivatives and optimisation steps

Next comes the optimization of the values of the dimension tolerances D<sub>k</sub> depending on the transfer coefficient ct and the achievement price of the tolerances. This optimization is done starting from a value of the tolerances (preferably their initial value (or a fraction of them) see pci point in figure 10) and creating optimization steps.

It will be considered a step of increase in the cost ΔK. The cost derivatives for each D<sub>k</sub> dimension are known, on each axis of the graphical representation of the area K=f(tk) for k=1,...,5 a diagram will appear as in the figure 11. Depending on the growth step of the ΔK cost, can calculate the decrease step of the Δtpk tolerance, which is the tolerance of the D<sub>k</sub> dimension projected in the direction of the closing dimension C. The calculation will be made for each D<sub>k</sub> dimension and after that the T tolerance of the closing dimension C for this iterative step will be calculated.

$$\Delta t_{pk} = \frac{\Delta K}{\tan \alpha_k} \quad (8)$$

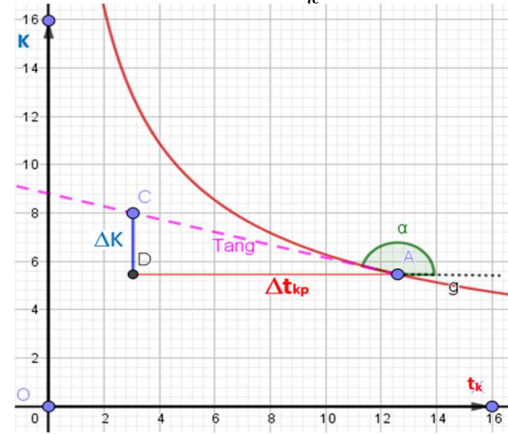


Fig.11. Calculation of the decrease in tolerances depending on the step ΔK

The summation of the tolerance projections Δtpk will be done according to the worst-case method.

$$T = \sum_{k=1}^{nD} \Delta t_{pk} \quad (9)$$

They were considered 5 iterative steps, following that by adjusting the sizes mT1 and ΔK, at the 5th step a tolerance to the closing dimension T ≤ Tdc will be obtained, in which case "Optimally Achieved" will be displayed, or if this case has not been reached, "Continue Optimization" will be displayed, (see figure 10). In figure 10 can observe that starting approximately with the point obtained in step 4, Pp4, the field of tolerance to the closing dimension cannot be obtained less, it increases (see Figure 10-point Pp5).

All intermediary values of the D<sub>k</sub> dimension tolerances will be stored in the mT5 matrix. These intermediate and final tolerances will allow the calculation of their costs and the graphical representation of the cost curves K according to the value of the tolerances K=f(T<sub>k</sub>), (see figure 8). Also calculated the total cost for all steps Kp1, Kp2, ..., Kp5. Pp coordinate points Pp (Dim. Close tolerance, Cost of tolerances), for all optimization steps (see figure 10 and figure 12) will describe a conic cA (ellipse passing through all points Pp<sub>i</sub>).

Recalculating the individual costs of all component dimensions for all 5 optimization

steps, the cost points on each cost curve were represented (see the yellow dots in figure 10 and figure 12). On the approximation curve you can define a point Pa, which is placed at the lowest value of the tolerance. The coordinates of this point Pa, constitute the Adjusted Optimum for the considered DC case.

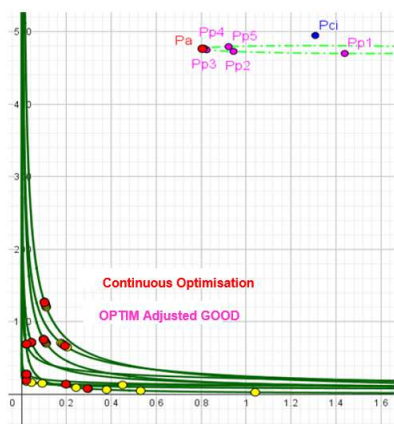


Fig.12. The points of the optimisation steps on the cost curves

### 6. ANALYSIS OF ALLOCATED TOLERANCES

This adjusted optimum must be verified by the Monte-Carlo method. Similar to the initial tolerances, the adjusted optimum will be checked by the Monte-Carlo method. Since the first coordinate of the Pa point indicates only the estimated tolerance field of the closing dimension, it is necessary to define which tolerance fields are assigned to the component dimensions of the DC. For this purpose, the point Pp<sub>i</sub> of the optimization steps that provided a minimum for the tolerance field of the closing dimension will be used (point Pp<sub>4</sub> in this case, see figure 12). So Pp<sub>4</sub> has the component dimension tolerances (in row 4) of the matrices mT<sub>4</sub> and mT<sub>5</sub>, respectively (not presented in this paper). All this point Pp<sub>4</sub> resulted by calculating in the case of the worst the value of the closing size tolerance StTpr<sub>4</sub>=0.812. The correction coefficients ka will be calculated:

$$ka = \{8.0017, 75.3652, 126.6547, 66.9066, 1.8226, 719.3427, 14.1138, 69.316, 27.9394\} \quad (10)$$

With this correction coefficient ka, the tolerance fields for the adjusted optimal dimensions will be defined (11) and shown in figure 12.

$$Tka = \frac{x(Pa)}{ka} = \begin{Bmatrix} 0.2934, 0.0996, 0.1028, \\ 0.1938, 0.0436, 0.0218, \\ 0.1974, 0.0218, 0.0218 \end{Bmatrix} \quad (11)$$

These tolerance fields will be checked by the Monte-Carlo method, similar to the initial tolerances. The verification revealed that the capability indices of the adjusted tolerances is only Cpa=1.09 (Cpa<1.33), but is estimated to be only 0.105% scraps.

In example the initial tolerances were:

$$Initial Tol. = \begin{Bmatrix} 2^\circ, 0.1, 0.1, 0.16, 0.04, \\ 0.02, 0.02, 0.02, 0.02 \end{Bmatrix} \quad (12)$$

They have an initial cost of Initial\_Cost = 494.72 lei and providing a tolerance field of Tinit = 1,622 at the closing dimension, higher than the desired one Td = 0.7.

Through allocation of tolerances, one shall consider those tolerances that are optimal (13):

$$Optimal Tol. = \begin{Bmatrix} 0.6204^\circ, 0.0996, 0.1028, \\ 0.1938, 0.0436, 0.0218, \\ 0.1974, 0.0218, 0.0218 \end{Bmatrix} \quad (13)$$

They have an Optimal\_Cost=479.46 lei and provide a tolerance field of Topt=0.641 at the closing dimension, lower than the desired one Td=0.7. If the difference between the initial cost and allocation cost are small (e.g., 3.08%) that means that the first designer has a very good experience in tolerance allocation - a desired situation. For many practitioners this approach and mathematical formulation are useful, using GeoGebra software it is easy to apply in conditions of solving problems related to the assignment of tolerances depending on the transfer coefficient and the cost of tolerance fields.

### 7. CONCLUSIONS

The objective of this paper was to minimize manufacturing costs, taking into account compliance with the requirements related to the values of the tolerance fields in a 2D dimensional chain. Potential applications and practical benefits of the research are that allows a quick and easy way to verify if tolerances of different Dimensional Chains are satisfying or not. Observing the cost of tolerances in production, it can be improved the form of costs curves used for tolerance allocation and in the same time could offer a solid way to recompute

some tolerance, by optimizing the production costs. A limitation is given by the fact that more information about the costs of different tolerances obtained by different production operations should be known. A future research direction will be to find a better way to sum all the tolerances (modules, directions, forms and asperity) to have a single tolerance objective to optimize.

## 8. REFERENCES

- [1] Seong, H. J., Pauline, K., Nguyen K.V.T., Sangmun S.H., *Optimal Tolerance Design and Optimization for a Pharmaceutical Quality Characteristic Publishing Corporation*, Math. Probl. in Engineering Volume, Article ID 706962, 17 pages, Hindawi Publishing Corporation, 2013, <http://dx.doi.org/10.1155/2013/706962>
- [2] Shin, S., Govindaluri, M. S., Cho, B. R., *Integrating the Lambert W function to a tolerance optimization problem. Quality and Rel Eng. Int*, 21(8), 795-808, 2005, <https://doi.org/10.1002/qre.687>
- [3] Di Stefano, P., *Tolerances analysis and cost evaluation for product life cycle*, International Journal of Production Research, 44, 1943-1961, 10.1080/00207540500465832, 2006
- [4] Paul J. D. Jr., *Dimensioning and Tolerancing Hb.*, Mc Graw Hill, 1999.
- [5] Armillotta, A., *Selection of parameters in cost-tolerance functions: review and approach*, Int. J. of Adv. Man. Tech., 108. 10.1007/s00170-020-05400-z, 2020
- [6] Prabhakaran, G., Asokan P., Ramesh P., Rajendran S., *Genetic-algorithm-based optimal tolerance allocation using a least-cost model*, Int J Adv Manuf Tech. 24: 647–660 DOI 10.1007/s00170-003-1606-1, 2004
- [7] Lin Z.C., Chang D., *Cost-tolerance anal. model based on a neural network meth*, Int J Prod Res 40(6):1429–1452, 2002, <http://doi.org/10.1080/00207540110116282>
- [8] Dong Z., Hu W., Xue D., *New Production Cost-Tolerance Models for Tolerance Synthesis*, Journal of Engineering for Industry, Vol. 116/199, ASME, 1994
- [9] Ye B., Salustri F.A., *Simultaneous tolerance synthesis for manufacturing and quality*, Res Eng Des 14(2):98–106, 2003
- [10] Hallmann, M., Schleich B., Wartzack S., *From tolerance allocation to tolerance-cost optimization: a comprehensive literature review*, International Journal of Advanced Manufacturing Technology 107:4859–4912, <https://doi.org/10.1007/s00170-020-05254-5>, 2020
- [11] \*\*\*, SolidWorks software/tolerances analysis,
- [12] \*\*\*, GeoGebra software 5.0.284.0 for Windows - Filehippo.com
- [13] Tulcan, A., Banciu, V., F., Grozav, I., *Analysis and Allocation of Dimensional Tolerances using GeoGebra Software*, ATN Applied Mechanics, Mathematics and Engineering 65(4), pp 1409-1416, 2022

### Optimizarea costului toleranțelor alocate unui lanț de dimensiuni 2D utilizând softul GeoGebra

Analiza și alocarea toleranțelor dimensionale ale lanțurilor de dimensiuni sunt analizate în literatura de specialitate prin diferite metode. În această lucrare se prezintă o analiză inițială a unui lanț de dimensiuni 2D iar în cazul în care câmpurile de toleranță nu satisfac cerințele impuse, câmpurile de toleranță ale dimensiunilor lanțului 2D vor fi realocate în funcție de un coeficient de transfer în raport cu dimensiunea de închidere a lanțului de dimensiuni totodată minimizând costul acestor câmpuri de toleranță. Pentru minimizarea costurilor au fost folosite curbele de variație a costurilor toleranțelor și nu coeficienți (constanți) de pondere relativă a costurilor toleranțelor. În final toleranțele alocate sunt verificate folosind metoda Monte-Carlo.

**TULCAN Aurel**, PhD Associate Professor, Politehnica University of Timișoara, Materials and Manufacturing Engineering Department, Str. Timiș, No.12, Ap. 34, 300650, Timișoara, Romania, [aurel.tulcan@upt.ro](mailto:aurel.tulcan@upt.ro)

**BANCIU Felicia Veronica**, PhD, Lecturer, Politehnica University of Timișoara, Materials and Manufacturing Engineering Department, Bd. Gen. Ion Dragalina No.33, Ap. 34, 300166, Timișoara, Romania, [felicia.banciu@upt.ro](mailto:felicia.banciu@upt.ro)

**GROZAV Ion**, PhD, Associate Professor, Politehnica University of Timișoara, Materials and Manufacturing Engineering Department, Str. 1 Decembrie, No. 90/B, Ap. 4, 300586, Timișoara, Romania, [ion.grozav@upt.ro](mailto:ion.grozav@upt.ro)