TECHNICAL UNIVERSITY OF CLUJ-NAPOCA

ACTA TECHNICA NAPOCENSIS

Series: Applied Mathematics, Mechanics, and Engineering
Vol. 67, Issue II, June, 2024

# ADVANCED KINEMATICS MODELING OF A 2RTR SERIAL ROBOT

**Iulia Cristina MARE, Iuliu NEGREAN**

***Abstract:*** *This paper explores and details the application of the matrix exponentials algorithm in the direct kinematic analysis of a 2RTR-type multi-body system. With a meticulous and in-depth approach, the study develops an advanced mathematical model for describing the motion of the multi-body system in space. The proposed algorithm demonstrates efficiency and precision in determining the final position and orientation of the end effector in Cartesian space, considering the specific constraints of the structure. By employing advanced matrix analysis methods and mechanics-specific techniques, the paper contributes to a deeper understanding of the kinematic behavior of the 2RTR system. The data obtained in the kinematic study are crucial for investigating the dynamic characteristics of any multi-body system*

***Key words:*** *kinematics modeling, matrix algorithm, matrix exponentials, robotics.*

## 1. INTRODUCTION

The kinematics of multi-body systems is addressed in specialized literature [1]-[9] as a discipline that focuses on studying the motion of material systems from a geometric perspective, without considering the influence of mass or the forces acting on them. Fundamental knowledge of space, time, velocity, and acceleration is essential to facilitate kinematic analysis.

As we delve into the kinematics of multi-body systems, we progressively transition from the kinematics of a material point and a system of material points to rigid bodies and systems of rigid bodies. These concepts are closely interconnected and contribute to the study of body systems. Key elements that require attention include trajectory, velocity, and acceleration for material points, and in the case of rigid bodies or systems of rigid bodies, motion equations, velocity distribution, and acceleration distribution must be considered.

To conduct kinematic analysis, it is essential to have a reference system to whom motion is related, whether it is a fixed system (for absolute motion) or a mobile system (for relative motion). Additionally, to perform kinematic studies, it is important to examine the geometry of multi-body systems before analyzing them from a kinematic perspective.

In the presented kinematic study, a 2TRT-type robot structure, specifically the Epson RS4-551 robot, has been considered (figure 1).


**Fig. 1** Epson RS4-551 robot [13]

## 2. ADVANCED KINEMATICS

**Received**: 24.04.24; **Similarities**: 29.05.24: **Reviewed**: 27.05./26.05.24: **Accepted**:25.06.24.

The kinematic structure of the 2RTR-type robot (rotation-rotation-translation-rotation) is considered in the initial configuration, denoted as $\bar{\theta}^{(0)} = [q_i = 0; i = 1 \rightarrow n]^T$. in figure 2.

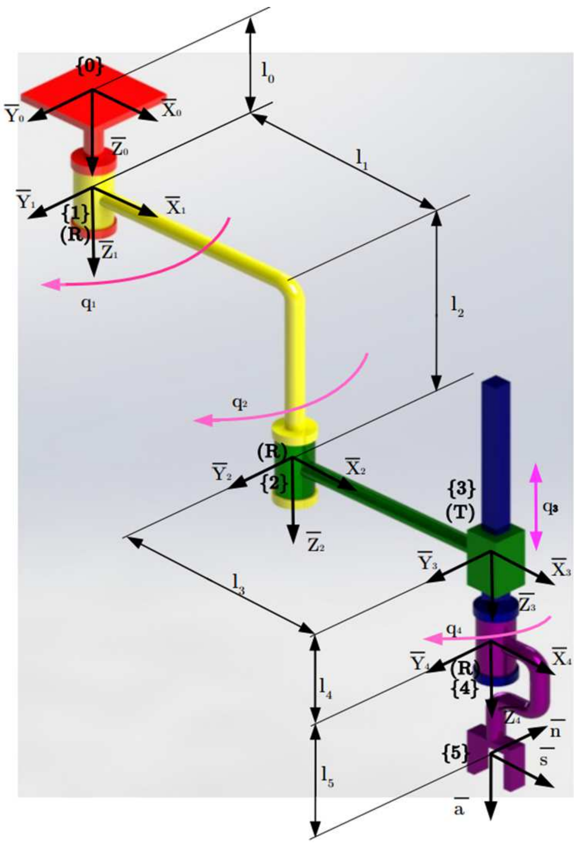


**Fig. 2** The kinematic diagram of the 2RTR structure

By applying the matrix exponentials algorithm in direct kinematics developed in [1]-[10], the Jacobian matrix (velocity transfer matrix) is determined. The Jacobian matrix consists of two components, namely the linear transfer matrix and the angular transfer matrix symbolized by ${}^0J_V\left(\bar{\theta}\right)$ and ${}^0J_\Omega\left(\bar{\theta}\right)$ respectively.

## 2.1. THE LINEAR VELOCITY TRANSFER MATRIX

In accordance with [1]-[6], it is understood that the linear velocity transfer matrix establishes the mathematical relationship between generalized velocities and operational velocities, with the latter being part of the unknowns. Furthermore, the algorithm for the matrix exponential of the Jacobian takes into account that operational velocities are the result of the first-order derivative with respect to time, applied to the last column of the pose matrix (homogeneous transformation) between the systems $\{0\} \rightarrow \{n\}$.

To determine the linear velocity transfer matrices for the 2RTR structure, in accordance with [1]-[10], an outer loop is initiated, denoted by $(i = 1 \rightarrow 4)$, for the determination of the linear velocity transfer matrices.

For the application of the second or third variant of Jacobian matrix calculation, considering [1]-[10], first, the matrices and exponentials are determined using the generalized expressions:

$$\underset{(3\times 3)}{ME}(V_{i1}) = \prod_{j=0}^{i-1} \exp\left\{\left\{\bar{k}_j^{(0)} \times\right\} q_j \cdot \Delta_j\right\} \tag{1}$$

$$\underset{(3\times 6)}{ME}(V_{i2}) = \left[\begin{array}{cc} I_3 & \Delta_i \cdot \left\{\bar{k}_i^{(0)} \times\right\} \end{array}\right] \tag{2}$$

$$\underset{\left\{6x\left[9+3\cdot(3-i)\right]\right\}}{ME}(V_{i3}) = \begin{bmatrix} I_3 & [0] & [0] \\ [0] & \left\{\begin{array}{c} \left[\begin{array}{c} \prod\limits_{j=0}^{i-1} \exp\left\{\left\{\bar{k}_m^{(0)} \times\right\} \cdot q_m \cdot \delta_m \cdot \Delta_m\right\} \\ where: \quad k = i \rightarrow 4 \end{array}\right] \\ iar \quad \delta_m = \left\{\begin{array}{l} \{0; \ m = i-1\}; \\ \{1; \ m \geq i\} \end{array}\right\} \end{array}\right\} & \sigma_1 \end{bmatrix} \tag{3}$$

Where: $\sigma_1 = \prod\limits_{k=i}^{n} \exp\left\{\left\{\bar{k}_k^{(0)} \times\right\} \cdot q_k \cdot \Delta_k\right\}$

Applying the previously mentioned expressions to the studied structure yields:

$$\underset{(3\times 3)}{ME}(V_{11}) = \prod_{j=0}^{0} \exp\left\{\left\{\bar{k}_j^{(0)} \times\right\} q_j \cdot \Delta_j\right\} \tag{4}$$

$$\underset{(3\times 3)}{ME}(V_{11}) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{5}$$

$$\underset{(3\times 6)}{ME}(V_{12}) = \left\{\left[\begin{array}{cc} I_3 & \Delta_1 \cdot \left\{\bar{k}_1^{(0)} \times\right\} \end{array}\right]\right\} \tag{6}$$

$$\underset{(3\times6)}{ME}(V_{12})=\left\{\begin{bmatrix}1&0&0&0&-1&0\\0&1&0&1&0&0\\0&0&1&0&0&0\end{bmatrix}\right\} \tag{7}$$

$$\underset{(6x15)}{ME}(V_{13})=\begin{bmatrix}I_3 & [0] & [0]\\ [0] & \left\{\begin{bmatrix}\prod\limits_{m=0}^{0}\exp\left\{\left\{\bar{k}_m^{(0)}\times\right\}q_m\cdot\delta_m\cdot\Delta_m\right\}\\ k=1\to4\end{bmatrix}\right. & \sigma_2\\ & \delta_m=\left\{\begin{matrix}\{0;\ m=i-1\};\\ \{1;\ m\geq i\}\end{matrix}\right\} & \end{bmatrix} \tag{8}$$

Where: $\sigma_2=\prod\limits_{k=1}^{3}\exp\left\{\left\{\bar{k}_k^{(0)}\times\right\}\cdot q_k\cdot\Delta_k\right\}$

$$\underset{(6x15)}{ME}(V_{13})=\begin{bmatrix}1&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\0&1&0&0&0&0&0&0&0&0&0&0&0&0&0\\0&0&1&0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&1&0&0&cq_1&-sq_1&0&cq_1&-sq_1&0&c(q_1+q_2)&-s(q_1+q_2)&0\\0&0&0&0&1&0&sq_1&cq_1&0&sq_1&cq_1&0&-s(q_1+q_2)&c(q_1+q_2)&0\\0&0&0&0&0&1&0&0&1&0&0&1&0&0&1\end{bmatrix} \tag{9}$$

Further on, the linear velocity transfer matrices will be determined for i=2:

$$\underset{(3x3)}{ME}(V_{21})=\prod_{j=0}^{1}\exp\left\{\left\{\bar{k}_j^{(0)}\times\right\}\cdot q_j\cdot\Delta_j\right\} \tag{10}$$

$$\underset{(3x3)}{ME}(V_{21})=\begin{bmatrix}cq_2&-sq_2&0\\sq_2&cq_2&0\\0&0&1\end{bmatrix} \tag{11}$$

$$\underset{(3\times6)}{ME}(V_{22})=\begin{bmatrix}I_3 & \Delta_1\cdot\left\{\bar{k}_1^{(0)}\times\right\}\end{bmatrix} \tag{12}$$

$$\underset{(3\times6)}{ME}(V_{22})=\begin{bmatrix}1&0&0&0&-1&0\\0&1&0&1&0&0\\0&0&1&0&0&0\end{bmatrix} \tag{13}$$

$$\underset{(6x12)}{ME}(V_{23})=\begin{bmatrix}I_3 & [0] & [0]\\ [0] & \left\{\begin{bmatrix}\prod\limits_{m=1}^{k-1}\exp\left\{\left\{\bar{k}_m^{(0)}\times\right\}\cdot q_m\cdot\delta_m\cdot\Delta_m\right\}\\ k=2\to4\end{bmatrix}\right\} & \sigma_3\\ & \delta_m=\left\{\begin{matrix}\{0;\ m=1\};\\ \{1;\ m\geq2\}\end{matrix}\right\} & \end{bmatrix} \tag{14}$$

Where: $\sigma_3=\prod\limits_{k=2}^{3}\exp\left\{\left\{\bar{k}_k^{(0)}\times\right\}q_k\cdot\Delta_k\right\}$

$$\underset{(6x12)}{ME}(V_{23})=\begin{bmatrix}1&0&0&0&0&0&0&0&0&0&0&0\\0&1&0&0&0&0&0&0&0&0&0&0\\0&0&1&0&0&0&0&0&0&0&0&0\\0&0&0&1&0&0&cq_2&-sq_2&0&cq_2&-sq_2&0\\0&0&0&0&1&0&sq_2&cq_2&0&sq_2&cq_2&0\\0&0&0&0&0&1&0&0&1&0&0&1\end{bmatrix} \tag{15}$$

Continuing, the linear velocity transfer matrices will be determined for i=3:

$$\underset{(3\times3)}{ME}(V_{31})=\prod_{j=0}^{2}\exp\left\{\left\{\bar{k}_j^{(0)}\times\right\}q_j\cdot\Delta_j\right\} \tag{16}$$

$$\underset{(3\times3)}{ME}(V_{31})=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix} \tag{17}$$

$$\underset{(3\times6)}{ME}(V_{32})=\begin{bmatrix}I_3 & \left\{\bar{k}_3^{(0)}\times\right\}\end{bmatrix} \tag{18}$$

$$\underset{(3\times6)}{ME}(V_{32})=\begin{bmatrix}1&0&0&0&0&0\\0&1&0&0&0&0\\0&0&1&0&0&0\end{bmatrix} \tag{19}$$

$$\underset{(6x9)}{ME}(V_{33})=\begin{bmatrix}I_3 & [0] & [0]\\ [0] & \left\{\begin{bmatrix}\prod\limits_{m=2}^{2}\exp\left\{\left\{\bar{k}_m^{(0)}\times\right\}q_m\cdot\delta_m\cdot\Delta_m\right\}\end{bmatrix}\right\} & \sigma_4\\ & \delta_m=\left\{\begin{matrix}\{0;\ m=2\};\\ \{1;\ m\geq3\}\end{matrix}\right\} & \end{bmatrix} \tag{20}$$

Where: $\sigma_4=\prod\limits_{k=3}^{3}\exp\left\{\left\{\bar{k}_k^{(0)}\times\right\}q_k\cdot\Delta_k\right\}$

$$\underset{(6x9)}{ME}(V_{33}) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \tag{21}$$

Continuing, the linear velocity transfer matrices will be determined for the last kinematic pair of the multi-body system under study (i=4):

$$\underset{(3\times3)}{ME}(V_{41}) = \prod_{j=0}^{3} \exp\left\{\left\{\bar{k}_j^{(0)} \times\right\} q_j \cdot \Delta_j\right\} \tag{22}$$

$$\underset{(3\times3)}{ME}(V_{41}) = \begin{bmatrix} cq_4 & -sq_4 & 0 \\ sq_4 & cq_4 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{23}$$

$$\underset{(3\times6)}{ME}(V_{42}) = \begin{bmatrix} I_3 & \left\{\bar{k}_4^{(0)} \times\right\} \end{bmatrix} \tag{24}$$

$$\underset{(3\times6)}{ME}(V_{42}) = \begin{bmatrix} 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \tag{25}$$

$$\underset{(6x9)}{ME}(V_{43}) = \begin{bmatrix} I_3 & [0] & [0] \\ [0] & \left\{\begin{bmatrix} \prod_{m=3}^{3} \exp\left\{\left\{\bar{k}_m^{(0)} \times\right\} \cdot q_m \cdot \delta_m \cdot \Delta_m\right\} \end{bmatrix}\right\} & \sigma_5 \\ & \delta_m = \begin{Bmatrix} \{0; m=3\}; \\ \{1; m \geq 4\} \end{Bmatrix} & \end{bmatrix} \tag{26}$$

Where: $\sigma_5 = \prod_{k=4}^{3} \exp\left\{\left\{\bar{k}_k^{(0)} \times\right\} q_k \cdot \Delta_k\right\}$

$$\underset{(6x9)}{ME}(V_{43}) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & cq_4 & -sq_4 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & sq_4 & cq_4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \tag{27}$$

## 2.2. THE ANGULAR VELOCITY TRANSFER MATRIX

To obtain the Jacobian matrix, it is necessary to determine both the linear and angular velocity transfer matrices. The general formulations of angular velocities, according to the matrix exponentials algorithm in direct kinematics from [1]-[10], are presented:

$$\underset{(6\times6)}{ME}\{J_{i1}\} = \begin{bmatrix} ME\{V_{i1}\} & [0] \\ [0] & ME\{V_{i1}\} \end{bmatrix} \tag{28}$$

$$\underset{(6\times9)}{ME}\{J_{i2}\} = \begin{bmatrix} ME\{V_{i2}\} & [0] \\ [0] & I_3 \end{bmatrix} \tag{29}$$

$$\underset{\{9\times[12+3\cdot(3-i)]\}}{ME\{J_{i3}\}} = \begin{bmatrix} ME\{V_{i3}\} & [0] \\ [0] & I_3 \end{bmatrix} \tag{30}$$

Applying the previous expressions to the four kinematic pairs of the studied robot structure, we obtain:
For the first kinematic pair (i=1):

$$\underset{(6\times6)}{ME}\{J_{11}\} = \left\{\begin{bmatrix} ME\{V_{11}\} & [0] \\ [0] & ME\{V_{11}\} \end{bmatrix}\right\} \tag{31}$$

$$\underset{(6\times6)}{ME}\{J_{11}\} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{32}$$

$$\underset{(6\times9)}{ME}\{J_{12}\} = \left\{\begin{bmatrix} ME\{V_{12}\} \equiv \begin{bmatrix} I_3 & \left\{\bar{k}_1^{(0)} \times\right\} \end{bmatrix} & [0] \\ [0] & I_3 \end{bmatrix}\right\} \tag{33}$$

$$\underset{(6\times9)}{ME}\{J_{12}\} = \begin{bmatrix} 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{34}$$

$$\underset{(9\times18)}{ME}\{J_{13}\} = \begin{bmatrix} ME\{V_{13}\} & [0] \\ [0] & I_3 \end{bmatrix} \tag{35}$$

$$\underset{(9\times18)}{ME}\{J_{13}\}=\left[\begin{array}{cccccccccccccccccc} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & cq_1 & -sq_1 & 0 & cq_1 & -sq_1 & 0 & c(q_1+q_2) & -s(q_1+q_2) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & sq_1 & cq_1 & 0 & sq_1 & cq_1 & 0 & s(q_1+q_2) & c(q_1+q_2) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right] \tag{36}$$

Continuing, the matrices for the second kinematic pair will be determined as follows:

$$\underset{(6\times6)}{ME}\{J_{21}\}=\begin{bmatrix} ME\{V_{21}\} & [0] \\ [0] & ME\{V_{21}\} \end{bmatrix} \tag{37}$$

$$\underset{(6\times6)}{ME}\{J_{21}\}=\begin{bmatrix} cq_2 & -sq_2 & 0 & 0 & 0 & 0 \\ sq_2 & cq_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & cq_2 & -sq_2 & 0 \\ 0 & 0 & 0 & sq_2 & cq_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{38}$$

$$\underset{(6\times9)}{ME}\{J_{22}\}=\begin{bmatrix} ME\{V_{22}\}\equiv\begin{bmatrix} I_3 & [0] \end{bmatrix} & [0] \\ [0] & I_3 \end{bmatrix} \tag{39}$$

$$\underset{(6\times9)}{ME}\{J_{22}\}=\begin{bmatrix} 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{40}$$

$$\underset{(9\times15)}{ME}\{J_{23}\}=\begin{bmatrix} ME\{V_{23}\} & [0] \\ [0] & I_3 \end{bmatrix} \tag{41}$$

$$\underset{(9\times15)}{ME}\{J_{23}\}=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & cq_2 & -sq_2 & 0 & cq_2 & -sq_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & sq_2 & cq_2 & 0 & sq_2 & cq_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{42}$$

The matrices for i=3 are determined as follows:

$$\underset{(6\times6)}{ME}\{J_{31}\}=\begin{bmatrix} ME\{V_{31}\} & [0] \\ [0] & ME\{V_{31}\} \end{bmatrix} \tag{43}$$

$$\underset{(6\times6)}{ME}\{J_{31}\}=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{44}$$

$$\underset{(6\times9)}{ME}\{J_{32}\}=\left\{\begin{bmatrix} ME\{V_{32}\} & [0] \\ [0] & I_3 \end{bmatrix}\right\} \tag{45}$$

$$\underset{(6\times9)}{ME}\{J_{32}\}=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{46}$$

$$\underset{(9\times12)}{ME}\{J_{33}\}=\begin{bmatrix} ME\{V_{33}\} & [0] \\ [0] & I_3 \end{bmatrix} \tag{47}$$

$$\underset{(9\times12)}{ME}\{J_{33}\}=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{48}$$

The matrices for the last kinematic pair, where i=4, are determined as follows:

$$\underset{(6\times6)}{ME}\{J_{41}\}=\begin{bmatrix} ME\{V_{41}\} & [0] \\ [0] & ME\{V_{41}\} \end{bmatrix} \tag{49}$$

$$\underset{(6\times6)}{ME}\{J_{41}\}=\begin{bmatrix} cq_4 & -sq_4 & 0 & 0 & 0 & 0 \\ sq_4 & cq_4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & cq_4 & -sq_4 & 0 \\ 0 & 0 & 0 & sq_4 & cq_4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{50}$$

$$\underset{(6\times9)}{ME}\{J_{42}\}=\left\{\begin{bmatrix} ME\{V_{42}\} & [0] \\ [0] & I_3 \end{bmatrix}\right\} \tag{51}$$

$$\underset{(6\times9)}{ME}\{J_{42}\}=\begin{bmatrix} 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{52}$$

$$\underset{(9\times12)}{ME}\{J_{43}\}=\begin{bmatrix} ME\{V_{43}\} & [0] \\ [0] & I_3 \end{bmatrix} \tag{53}$$

$$\underset{(9\times12)}{ME}\{J_{43}\}=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & cq_4 & -sq_4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & sq_4 & cq_4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{54}$$

## 2.3. THE JACOBIAN MATRIX

In the matrix exponentials algorithm presented in [1]-[8], the Jacobian matrix contains both the component ${}^0J_V(\bar{\theta})$ and the ${}^0J_\Omega(\bar{\theta})$. Further, the transfer matrix corresponding to the first column of the Jacobian matrix is determined:

$$\underset{(6\times18)}{ME}\{{}^0J_1\}=ME\{J_{11}\}\cdot ME\{J_{12}\}\cdot ME\{J_{13}\} \tag{55}$$

$$\underset{(6\times18)}{ME}\{{}^0J1\}=\left[\begin{array}{cccccccccccccccccc} 1 & 0 & 0 & 0 & -1 & 0 & -cq_1 & -cq_1 & 0 & -cq_1 & -cq_1 & 0 & -s(q_1+q_2) & -c(q_1+q_2) & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & cq_1 & -sq_1 & 0 & cq_1 & -sq_1 & 0 & c(q_1+q_2) & -s(q_1+q_2) & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right] \tag{56}$$

The column vector corresponding to the linear component of the Jacobian matrix is determined using the expression:

$$\underset{\{18\times1\}}{M_{1v\omega}}=\begin{bmatrix} \bar{v}_1^{(0)T} & \begin{bmatrix} \bar{b}_k\ ;k= \\ =1\to3 \end{bmatrix}^T & \bar{p}_3^{(0)T} & \Delta_1\cdot\bar{k}_1^{(0)T} \end{bmatrix}^T \tag{57}$$

By substituting and performing the calculation, the column vector is obtained as follows:

$$\underset{\{18\times1\}}{M_{1v\omega}}=[0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ -l_1\cdot(cq_2-1)\ \ -l_1\cdot sq_2\ \ 0\ \ 0\ \ 0\ \ 0\ \ l_1+l_3\ \ 0\ \ l_0+l_2\ \ 0\ \ 0\ \ 1]^T \tag{58}$$

By matrix multiplication of the last two matrix functions mentioned earlier (56 and 58), the first column of the Jacobian matrix is generated, and its expression is as follows:

$$\underset{(6\times1)}{{}^0J_1}=ME\{{}^0J_1\}\cdot M_{1v\omega}=\begin{bmatrix} -l_1\cdot sq_1-l_3\cdot s(q_1+q_2) \\ l_1\cdot cq_1+l_3\cdot c(q_1+q_2) \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \tag{59}$$

Performing the necessary calculations according to the matrix exponentials algorithm in [1]-[10],, the transfer matrix corresponding to the second column of the Jacobian matrix is further determined:

$$\underset{(6\times15)}{ME}\left\{{}^{0}J_2\right\} = ME\{J_{21}\}\cdot ME\{J_{22}\}\cdot ME\{J_{23}\} \tag{60}$$

$$\underset{(6\times15)}{ME}\left\{{}^{0}J_2\right\} = \begin{bmatrix} cq_1 & -sq_1 & 0 & -sq_1 & -cq_1 & 0 & -s(q_1+q_2) & -c(q_1+q_2) & 0 & -s(q_1+q_2) & -c(q_1+q_2) & 0 & 0 & 0 & 0 \\ sq_1 & cq1 & 0 & cq_1 & -sq_1 & 0 & c(q_1+q_2) & -s(q_1+q_2) & 0 & c(q_1+q_2) & -s(q_1+q_2) & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & cq_1 & -sq_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & sq_1 & cq_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{61}$$

The column vector corresponding to the linear component of the Jacobian matrix is determined using the expression described in [N03]:

$$\underset{\{15\times1\}}{M_{2v\omega}} = \begin{bmatrix} \overline{v}_2^{(0)T} & \begin{bmatrix} \overline{b}_k\ ;k= \\ =2\to3 \end{bmatrix}^T & \overline{p}_3^{(0)T} & \Delta_2\cdot\overline{k}_2^{(0)T} \end{bmatrix}^T \tag{62}$$

Performing the calculation yields the column vector:

$$\underset{\{15\times1\}}{M_{2v\omega}} = \begin{bmatrix} 0 & -l_1 & 0 & -l_1\cdot(cq_2-1) & -l_1\cdot sq_2 & 0 & 0 & 0 & 0 & l_1+l_3 & 0 & l_0+l_2 & 0 & 0 & 1 \end{bmatrix}^T \tag{63}$$

By matrix multiplication of the last two matrix functions presented above (61) and (63), the second column of the Jacobian matrix is obtained, and its expression is as follows:

$$\underset{(6\times1)}{{}^{0}J_2} = ME\left\{{}^{0}J_2\right\}\cdot M_{2v\omega} = \begin{bmatrix} -l_3\cdot s(q_1+q_2) \\ l_3\cdot c(q_1+q_2) \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \tag{64}$$

The transfer matrix included in the third column of the Jacobian matrix is obtained as follows:

$$\underset{(6\times12)}{ME}\left\{{}^{0}J_3\right\} = ME\{J_{31}\}\cdot ME\{J_{32}\}\cdot ME\{J_{33}\} \tag{65}$$

$$\underset{(6\times12)}{ME}\left\{{}^{0}J_3\right\} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{66}$$

The column vector corresponding to the linear component of the Jacobian matrix is determined using the expression described in [1]:

$$\underset{\{12\times1\}}{M_{3v\omega}} = \begin{bmatrix} \overline{v}_3^{(0)T} & \begin{bmatrix} \overline{b}_k\ ;k= \\ =3\to3 \end{bmatrix}^T & \overline{p}_3^{(0)T} & \Delta_3\cdot\overline{k}_3^{(0)T} \end{bmatrix}^T \tag{67}$$

Performing the specific calculation yields the following column vector:

$$\underset{\{12\times1\}}{M_{3v\omega}} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & l_1+l_3 & 0 & l_0+l_2 & 0 & 0 & 0 \end{bmatrix}^T \tag{68}$$

By matrix multiplication of the last two matrix functions presented above (65) and (68), the third column of the Jacobian matrix is obtained, and its expression is:

$$\underset{(6\times1)}{{}^{0}J_3} = ME\{J_{31}\}\cdot ME\{J_{32}\}\cdot ME\{J_{33}\}\cdot M_{3v\omega} \tag{69}$$

$$\underset{(6\times1)}{{}^{0}J_3} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}^T \tag{70}$$

Furthermore, the transfer matrix included in the fourth column of the Jacobian matrix is obtained:

$$\underset{(6\times12)}{ME}\left\{{}^{0}J_4\right\} = ME\{J_{41}\}\cdot ME\{J_{42}\}\cdot ME\{J_{43}\} \tag{71}$$

$$\underset{(6\times12)}{ME}\left\{{}^{0}J_4\right\} = \begin{bmatrix} cq_4 & -sq_4 & 0 & -sq_4 & -cq_4 & 0 & -s(2\cdot q_4) & -c(2\cdot q_4) & 0 & 0 & 0 & 0 \\ sq_4 & cq_4 & 0 & cq_4 & -sq_4 & 0 & c(2\cdot q_4) & -s(2\cdot q_4) & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & cq_4 & -sq_4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & sq_4 & cq_4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{72}$$

The column vector corresponding to the linear component of the Jacobian matrix is determined using the expression described in [1]-[8]:

$$\underset{\{12\times1\}}{M_{4v\omega}}=\begin{bmatrix}\bar{v}_4^{(0)T} & \begin{bmatrix}\bar{b}_k\ ;k= \\ =4\to4\end{bmatrix}^T & \bar{p}_4^{(0)T} & \Delta_4\cdot\bar{k}_4^{(0)T}\end{bmatrix}^T \quad (73)$$

Performing the calculation, the column vector is obtained as follows:

$$\underset{\{12\times1\}}{M_{4v\omega}}=\begin{bmatrix}0 & -l_1-l_3 & 0 & -(l_1+l_3)\cdot(cq_4-1) & -sq_4\cdot(l_1+l_3) \\ 0 & l_1+l_3 & 0 & l_0+l_2 & 0 \quad 0 \quad 1\end{bmatrix}^T \quad (74)$$

By matrix multiplication of the last two matrix functions presented above (72) and (74), the fourth column of the Jacobian matrix is obtained, and its expression is as follows:

$$\underset{(6\times1)}{{}^{0}J_4}=ME\left\{{}^{0}J_4\right\}\cdot M_{4v\omega} \quad (75)$$

$$\underset{(6\times1)}{{}^{0}J_4}=\begin{bmatrix}0 & 0 & 0 & 0 & 0 & 1\end{bmatrix}^T \quad (76)$$

Therefore, by substituting the four column matrices (59), (64), (70), and (76) into the definition expression, the final form of the Jacobian matrix is obtained, written in expanded form, according to [1]-[10], as follows:

$$\underset{(6\times4)}{{}^{0}J\left(\bar{\theta}\right)}=\begin{bmatrix}{}^{0}J_1 & {}^{0}J_2 & {}^{0}J_3 & {}^{0}J_4\end{bmatrix} \quad (77)$$

$$\underset{(6\times4)}{{}^{0}J\left(\bar{\theta}\right)}=\begin{bmatrix}-l_1\cdot sq_1-l_3\cdot s(q_1+q_2) & -l_3\cdot s(q_1+q_2) & 0 & 0 \\ l_1\cdot cq_1+l_3\cdot c(q_1+q_2) & l_3\cdot c(q_1+q_2) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1\end{bmatrix} \quad (78)$$

Taking into account expression (78), the corresponding matrix equation for the generalized (operational) absolute velocities of the end effector, projected onto the fixed system, is determined further in accordance with [1]-[6]:

$${}^{0}\dot{X}={}^{0}J(\theta)\cdot\dot{\theta} \quad (79)$$

Where: $\dot{\theta}=\begin{pmatrix}\dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4\end{pmatrix}$.

Replacing in expression (80), results:

$${}^{0}\dot{X}=\begin{pmatrix}-\dot{q}_1\cdot[l_1\cdot sq_1+l_3\cdot s(q_1+q_2)]-l_3\cdot s(q_1+q_2)\cdot\dot{q}_2 \\ \dot{q}_1\cdot[l_1\cdot cq_1+l_3\cdot c(q_1+q_2)]+l_3\cdot c(q_1+q_2)\cdot\dot{q}_2 \\ \dot{q}_3 \\ 0 \\ 0 \\ \dot{q}_1+\dot{q}_2+\dot{q}_4\end{pmatrix} \quad (80)$$

Continuing, the matrix equation corresponding to the generalized (operational) absolute accelerations of the end effector, projected onto the fixed system, is determined through the following expression in accordance with [1]-[6]::

$${}^{0}\ddot{X}=\begin{bmatrix}{}^{0}J(\bar{\theta}) & {}^{0}\dot{J}(\bar{\theta})\end{bmatrix}\cdot\begin{bmatrix}\ddot{\theta} \\ \dot{\theta}\end{bmatrix} \quad (81)$$

Where: $\ddot{\theta}=\begin{pmatrix}\ddot{q}_1 \\ \ddot{q}_2 \\ \ddot{q}_3 \\ \ddot{q}_4\end{pmatrix}$.

To obtain the derivative form of the Jacobian matrix, the determinant of ${}^{0}J\left(\bar{\theta}\right)$ determined by expression (79) is derived, thus yielding:

$${}^{0}\dot{J}(\theta)=\begin{pmatrix}-l_1\cdot cq_1\cdot\dot{q}_1-l_3\cdot c(q_1+q_2)\cdot(\dot{q}_1+\dot{q}_2) & -l_3\cdot c(q_1+q_2)\cdot(\dot{q}_1+\dot{q}_2) & 0 & 0 \\ -l_1\cdot sq_1\cdot\dot{q}_1-l_3\cdot s(q_1+q_2)\cdot(\dot{q}_1+\dot{q}_2) & -l_3\cdot s(q_1+q_2)\cdot(\dot{q}_1+\dot{q}_2) & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0\end{pmatrix} \quad (82)$$

By substituting (78), (82), $\ddot{\theta}$, and $\dot{\theta}$ into expression (81), the corresponding matrix equation for the generalized (operational) absolute accelerations of the end effector is obtained:

$$
{}^{0}\ddot{X} = \begin{pmatrix} \ddot{q}_3 - l_3 \cdot s(q_1 + q_2) \cdot (\ddot{q}_1 + \ddot{q}_2) - l_3 \cdot c(q_1 + q_2) \cdot (\dot{q}_1 + \dot{q}_2)^2 - \\ -l_1 \cdot cq_1 \cdot \dot{q}_1{}^2 - l_1 \cdot sq_1 \cdot \ddot{q}_1 \\ l_3 \cdot c(q_1 + q_2) \cdot (\ddot{q}_1 + \ddot{q}_2) - l_3 \cdot s(q_1 + q_2) \cdot (\dot{q}_1 + \dot{q}_2)^2 - \\ -l_1 \cdot sq_1 \cdot \dot{q}_1{}^2 + l_1 \cdot cq_1 \cdot \ddot{q}_1 \\ 0 \\ 0 \\ 0 \\ \ddot{q}_1 + \ddot{q}_2 + \ddot{q}_4 \end{pmatrix} \quad (83)
$$

## 3. CONCLUSIONS

In conclusion, the implementation of the matrix exponentials algorithm in the direct kinematic analysis of a 2RTR-type structure represents a significant contribution to the fields of mechanics and robotics. This algorithm stands as an essential tool for describing the complex motion of multi-body systems, providing remarkable precision and efficiency in determining the final position and orientation of the end effector in Cartesian space.

Compared to traditional methods, matrix exponentials highlight significant advantages. Their compact form, ease of geometric visualization, and notably, the avoidance of specific frames for each cinematic element constitute essential features. Thus, matrix exponentials become fundamental in defining linear and angular transfer matrices for the 2RTR structure.

Through the application of the algorithm and matrix exponentials, all relevant kinematic parameters for the 2RTR structure have been successfully determined. These parameters play a crucial role in characterizing the equations of direct kinematics and control for this specific robot structure. Regardless of the construction's complexity, matrix exponentials provide a robust foundation for defining and understanding the detailed cinematic behavior of the 2RTR structure, highlighting their utility and effectiveness in the field of mechanical research and robotics.

The matrix exponentials algorithm underscores significant advantages, including a concise and clear formulation, as well as the ability to handle specific constraints of the 2RTR structure. This research not only demonstrates the viability and effectiveness of the proposed algorithm but also opens new directions for the further development of complex multi-body systems.

Overall, the integration of this algorithm into kinematic analysis brings significant contributions to advancing knowledge in the field of mechanics, with the potential to influence the future development of robots or any multi-body system with similar structures.

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**Modelarea cinematică avansată a structurii de robot de tipul 2RTR**

Această lucrare explorează și detaliază aplicarea algoritmului exponențialelor de matrice în analiza cinematică directă a unui sistem multi-corp de tipul 2RTR. Cu o abordare meticuloasă și în profunzime, studiul dezvoltă un model matematic avansat pentru descrierea mişcării sistemului multi-corp în spațiu. Algoritmul propus demonstrează eficiență și precizie în determinarea poziției finale și a orientării end-effectuatorului în spațiul cartezian, având în vedere constrângerile particulare ale acestei structuri. Prin utilizarea metodelor avansate de analiză matriceală și tehnici specifice mecanicii, lucrarea contribuie la înțelegerea mai profundă a comportamentului cinematic al sistemului 2RTR. Datele obținute în studiul cinematic, sunt cruciale pentru investigarea caracteristicilor dinamice ale oricărui sistem multi-corp.

**Iulia Cristina MARE,** Ph.D. Student, Eng., Department of Mechanical System Engineering, Technical University of Cluj-Napoca, mareiulia123@gmail.com , Phone 0753676343

**Iuliu NEGREAN** Professor Ph.D., Eng., Department of Mechanical System Engineering, Technical University of Cluj-Napoca, iuliu.negrean@mep.utcluj.ro, Office Phone 0264/401616.