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SOME PARTICULARITIES OF INTERNAL-EXTERNAL GEAR PAIRS WITH ZERO DIFFERENCE IN NUMBERS OF TEETH

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Abstract: *Geared mechanisms using internal-external gear pairs with a small difference in the number of teeth are well-known for the advantages they offer. Internal-external gear pairs with zero difference in the number of teeth represent a special case of this category of gear drives. Even if they have a transmission ratio equal to unity these gear pairs can be used both for transmitting rotational motion in the same direction between two shafts with a very small distance between their axes, and in the construction of cycloidal reducers. Because the difference in the number of teeth is zero, most of the mathematical equations used for internal-external gear drive with this difference bigger than zero are not valid any more. In this paper, a part of the mathematical equations which are only specific to these gear drives will be presented and discussed. More specific equations will be presented in future work.*

Key words: *internal-external gear drives, zero difference, number of teeth, cycloidal reducers.*

1. INTRODUCTION

Internal-external gear pairs with a small difference in the number of teeth are well-known for the advantages they offer when used in the construction of cycloidal reducers [1-5]. Among these advantages, the following can be mentioned: achieving high transmission ratios with reduced overall dimensions of the reducers; compact designs; both gears rotate in the same direction, etc. By appropriately selecting the (small) distance between the axes of the internal and the external gears, the difference in the number of teeth between these gears can be reduced to one or even to zero. Despite the aforementioned advantages, these gear pairs also have several disadvantages, such as a low contact ratio and the occurrence of interference phenomena. For this reason, studies are necessary to determine the optimal values of the addendum modification coefficients, which can counteract these disadvantages [6-11].

Internal-external gear pairs with zero difference in the number of teeth represent a special case. Even if they have a transmission ratio equal to unity, these gear pairs can be used both for transmitting rotational motion in the same direction between two shafts with a very small distance between their axes (Figure 1.a) and in the construction of cycloidal reducers, to prevent the satellite from rotating around its own axis or to collect the motion from the satellite and transmit it to the output shaft (Figure 1.b), [12-16]. For all these applications, these special gears can replace couplings with various construction solutions. In Figure 1, the following notations were made: *a* is the distance between the rotation axes of shafts 1 and 2; z_1 and $z_{1'}$ are the numbers of teeth of the external gears; z_2 and z_3 are the numbers of teeth of the internal gears. The differences in the number of teeth are: $z_2 - z_1 = 0$ and $z_3 - z_{1'} \geq 1$, when we want to prevent the satellite from rotating around its own axis; $z_2 - z_1 \geq 1$ and $z_3 - z_{1'} = 0$, when we want to collect the motion from the satellite and transmit it to the output shaft.

Some mathematical equations for these types of gear pairs are similar to those used for conventional internal gear drives, but most of them have specific characteristics, and their

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derivation requires special considerations. As we know, not any paper in literature is discussing the mathematical equations specific to internal-external gear pairs with zero difference in the number of teeth.

This paper will present and discuss a part of the mathematical equations which are only specific to these gear drives. More specific equations will be presented in future work.

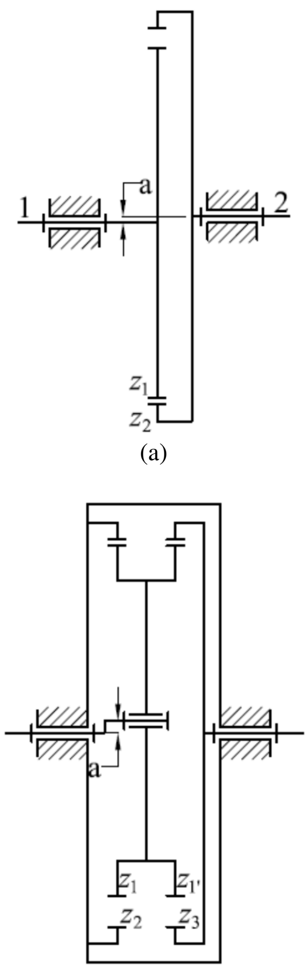

Fig. 1. Geared mechanisms using internal-external gear pair with zero difference in numbers of teeth: a) simple spur gear pair; b) cycloidal reducer

2. SOME PARTICULAR MATHEMATICAL EQUATIONS

2.1. The meshing angle

The base circles, C_{b1} and C_{b2}, have equal diameters. The common tangent T_b to the base circles is, therefore, parallel to the line O_1O_2 connecting the centers. The common tangent to the rolling circles at the instantaneous center of rotation C is perpendicular to the line O_1O_2 connecting the centers, and thus also perpendicular to T_b. Under these conditions, the meshing angle in the frontal plane α_{wt} has a value of $\pi/2$ (Figure 2).

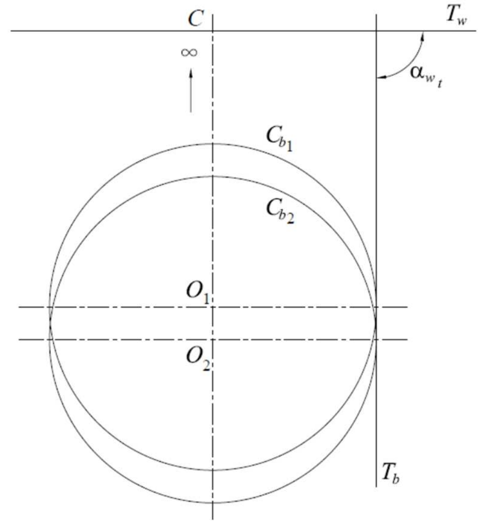

Fig. 2. The meshing angle

As an immediate consequence, the diameters of the pitch circles, d_{w1} and d_{w2}, will have infinite values, so the point C is at infinity on the line O_1O_2 connecting the centers:

$$d_{w1} = \left(d_1 \cdot \cos\alpha_t\right) / \cos\alpha_{wt} \to \infty, \tag{1}$$

$$d_{w2} = \left(d_2 \cdot \cos\alpha_t\right) / \cos\alpha_{wt} \to \infty. \tag{2}$$

where: d_1 and d_2 are the diameters of the reference circles for the external and the internal gears; α_t is the transverse pressure angle; α_{wt} is the transverse meshing angle.

2.2 Centre distance

The classical equation for working centre distance,

$$a_w = \left(a \cdot \cos\alpha_t\right) / \cos\alpha_{wt}, \tag{3}$$

leads to an indeterminate form $a_w = 0/0$, since $a = 0$ and $\alpha_{wt} = \pi/2$. L'Hôpital's theorem resolves the indeterminate form and provides the specific relationship for calculating the distance between axes for inclined teeth,

$$a_w = m_n \cdot \left(x_{n2} - x_{n1}\right) \cdot \sin \alpha_t , \qquad (4)$$

and for straight teeth,

$$a_w = m \cdot \left(x_2 - x_1\right) \cdot \sin \alpha , \qquad (5)$$

where: m is the module of the tooth; x_1 and x_2 are the addendum modification coefficients.

The equations (2) and (3) can also be obtained through a simple reasoning, using the Figure 3.

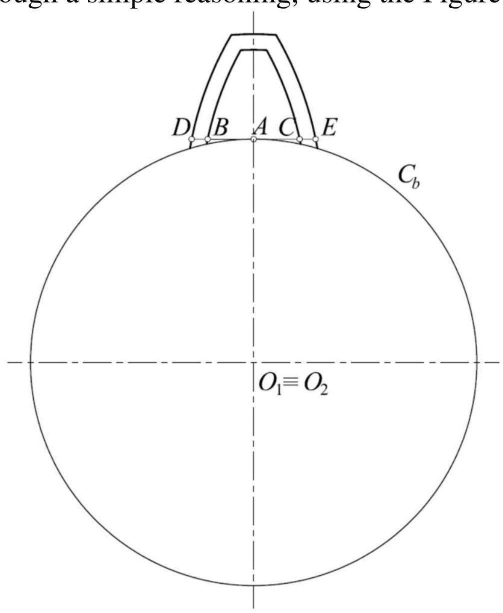

Fig. 3. Sketch for determining the centre distance

By overlapping the centers of the two gears and the symmetry axes of a tooth on gear 1 and a gap between the teeth of gear 2, equal clearance is obtained on both flanks. Achieving a gear mesh without clearance results in an axial distance equal to the clearance on a single flank (left or right):

$$a_w = \frac{g_{b2} - S_{b1}}{2} , \qquad (6)$$

where:

$$g_{b2} = d_b \cdot \left(\frac{\pi + 4 \cdot x_{n2} \cdot tg\alpha_n}{2 \cdot z} + inv\alpha_t \right) \qquad (7)$$

is the tooth thickness at the base circle for the external gear, z is the number of teeth, and

$$S_{b1} = d_b \cdot \left(\frac{\pi + 4 \cdot x_{n1} \cdot tg\alpha_n}{2 \cdot z} + inv\alpha_t \right) \qquad (8)$$

is the width of the gap between the teeth at the base circle for the internal gear.

2.3 The approach of the gears centers

The approach of the gear centers is calculated according to the diagrams in Figure 4, and the equation obtained is

$$\Delta = m_n \cdot \left(x_{n2} - x_{n1}\right) \cdot \left(1 - \sin \alpha_t\right), \qquad (9)$$

In the same figure, the normal coefficient of backlash increase in the reference section could be determined with

$$\Delta y_n = \left(x_{n2} - x_{n1}\right) \cdot \left(1 - \sin \alpha_t\right) / m_n \qquad (10)$$

respectively.

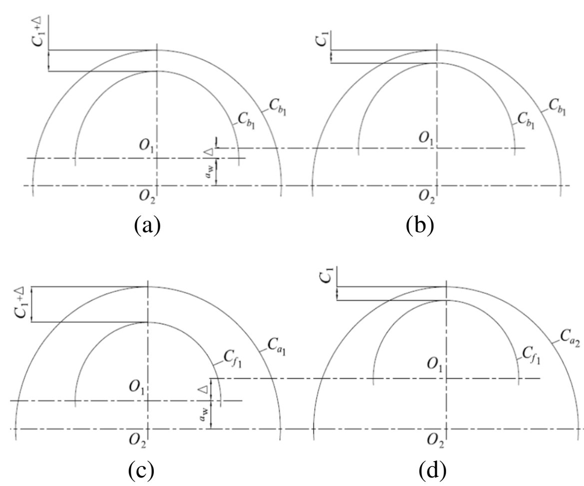

Fig. 4. The approach of the gears centers

2.4 The continuity of meshing

The diagram in Figure 5 allows for the calculation of the frontal contact ratio:

$$\varepsilon_\alpha = \frac{\overline{AE}}{p_{bt}} = \frac{\overline{KE_1} + \overline{K_1K_2} - \overline{K_2A}}{p_{bt}} \qquad (11)$$

where p_{bt} is the base circle pitch.

After substitutions, we get:

$$\varepsilon_\alpha = \left[tg\alpha_{a_{1t}} - tg\alpha_{a_{2t}} + \frac{2(x_{n2}-x_{n1})tg\alpha_n}{z} \right] 2\pi z \quad (12)$$

or

$$\varepsilon_\alpha = \frac{\sqrt{d_{a_1}^2 - d_{b_1}^2} - \sqrt{d_{a_2}^2 - d_{b2}^2} + 2a_w}{2p_t \cos\alpha_t} \quad (13)$$

where: d_{a1} and d_{a2} are the diameters of the tip circles.

The additional contact ratio:

$$\varepsilon_\beta = \frac{b \cdot \sin\beta}{\pi \cdot m_n} \quad (14)$$

where: b is the tooth width; β is the tooth angle.

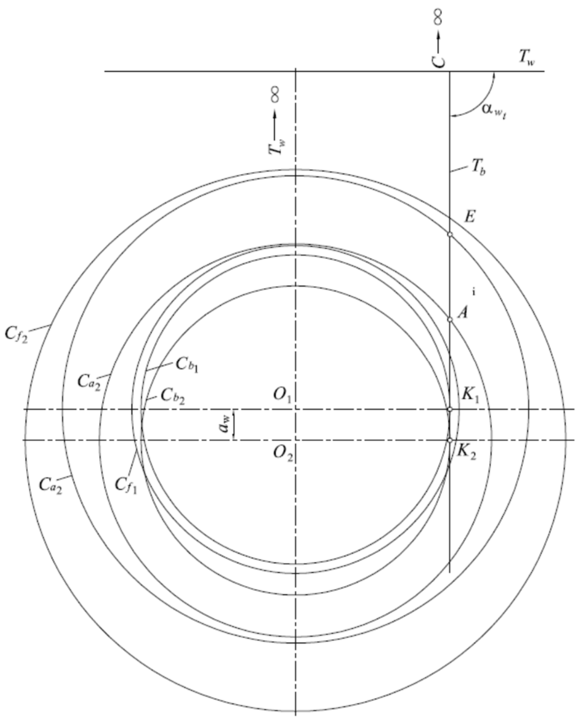

Fig. 5. Sketch for determining the contact ratio

2.4 Primary interference

Avoiding interference between the tip of the tooth of gear 2 and the root of the tooth of gear 1 requires that point A be located above points K_1 and K_2 (Figure 6).

A conventional gear (with $\Delta z \neq 0$) requires the condition:

$$tg\alpha_{a_{2t}} \geq \left(1 - \frac{z_1}{z_2}\right) \cdot tg\alpha_{wt} \quad (15)$$

The replacement of the difference $z_2 - z_1$ and the elimination of the indeterminacy leads to the relation:

$$tg\alpha_{a_{2t}} \geq \frac{2 \cdot (x_{n2} - x_{n1}) \cdot tg\alpha_n}{z_2} \quad (16)$$

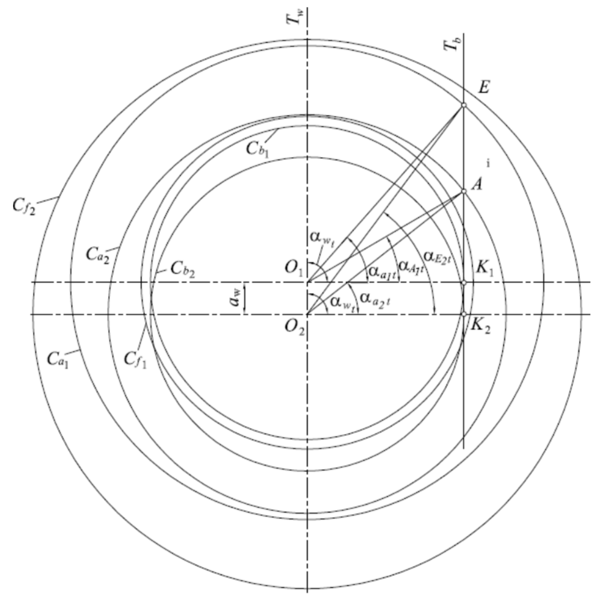

Fig. 6. Primary interference

The same equation (15) is also obtained based on the diagram in Figure 6.

The interference between the tip of the tooth of gear 1 and the root of the tooth of gear 2 is excluded because point E cannot reach between K_1 and K_2.

2.5 The interference in the fillet area

Interference in the fillet area occurs if the active profile of the tooth extends beyond the involute profile. The gear mechanism formed by gear 1, externally toothed with a rack-type tool, and gear 2, internally toothed with a wheel-cutter, requires the following condition (Figure 6):

$$d_{A1} \geq d_{e1} \quad (18)$$

$$d_{E2} \leq d_{e2} \quad (19)$$

where d_e, d_{e1} and d_{e2} are determined using the same equations as for conventional gears:

$$d_e = d_{b1}\sqrt{1+\left\{\frac{tg\alpha_t-\left[4\left(h^*_{an}-x_{n1}\right)\cos\beta\right]}{\left[z_1\sin\left(2\alpha_t\right)\right]}\right\}^2}, \quad (20)$$

$$d_{e1} = d_{b1}\sqrt{1+\left[tg\alpha_{w_{01t}}+\left(tg\alpha_{w_{o1t}}-tg\alpha_{a_{o1t}}\right)\frac{z_0}{z_1}\right]^2}, \quad (21)$$

$$d_{e2} = d_{b2}\sqrt{1+\left[tg\alpha_{w_{02t}}+\left(tg\alpha_{w_{o2t}}-tg\alpha_{a_{o2t}}\right)\frac{z_0}{z_2}\right]^2}, \quad (22)$$

and the equations for d_{A1} and d_{E2} are determined using the diagram in Figure 6:

$$d_{A1} = d_{b1}\sqrt{1+\left(tg\alpha_{a_{2t}}-2\frac{a_w}{d_{b1}}\right)^2}, \quad (23)$$

$$d_{E2} = d_{b2}\sqrt{1+\left(tg\alpha_{a_{1t}}+2\frac{a_w}{d_{b2}}\right)^2}. \quad (23)$$

2.6 Interference of the teeth tips

The interference of the teeth tips is, in principle, not possible because each tooth of the external gear 1 remains permanently in the same gap between the teeth of the internal gear 2 (see Figure 7).

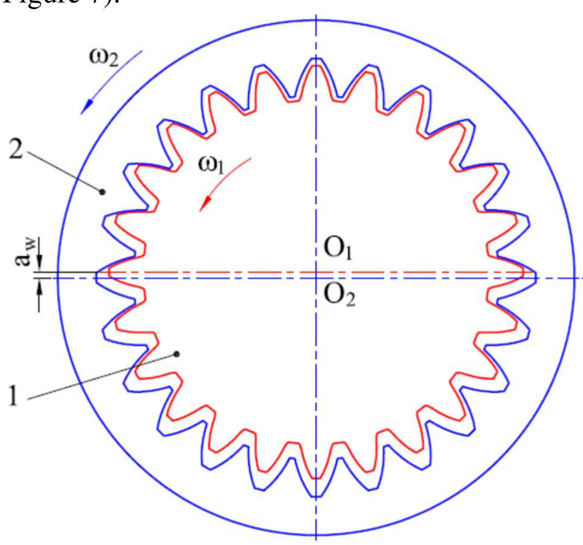

Fig. 7. Internal-external gear pair with zero difference in numbers of teeth

3. CONCLUSION

Internal-external gear pairs with zero difference in the number of teeth represent a special case of gear drives. Even if they have a transmission ratio equal to unity and one cannot consider them as gear transmissions because of this fact, we want to bring into consideration the fact that the same thing is not said when discussing an external-external gear pair with a transmission ratio equal to unity. These gear pairs can be used both for transmitting rotational motion in the same direction between two shafts with a very small distance between their axes, and to prevent the satellite from rotating around its own axis or to collect the motion from the satellite and transmit it to the output shaft, when they are used in the construction of cycloidal reducers. Because the difference in the number of teeth is zero, most of the mathematical equations used for internal-external gear drive with this difference bigger than zero are not valid any more. In this paper, a part of the mathematical equations which are only specific to these gear drives have been presented and discussed. More specific equations will be presented in future work.

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Unele particularități ale angrenajelor interioare cu diferență zero între numerele de dinți

Mecanismele cu roți dințate care utilizează angrenaje interioare cu o diferență mică dintre numărele de dinți sunt bine cunoscute pentru avantajele pe care acestea le oferă. Angrenajele interioare cu această diferență egală cu zero reprezintă un caz special din această categorie. Chiar dacă aceste angrenaje au un raport de transmitere egal cu unitatea, pot fi utilizate atât pentru a transmite mișcarea de rotație, în același sens, între doi arbori cu o distanță foarte mică între axele lor, cât și în construcția reductoarelor cicloidale. Deoarece diferența dintre numerele de dinți este zero, majoritatea ecuațiilor matematice utilizate pentru angrenajele interioare cu această diferență mai mare decât zero nu mai sunt valabile. În această lucrare, vor fi prezentate și discutate o parte din ecuațiile matematice care sunt specifice acestor angrenaje. Mai multe ecuații specifice acestora vor fi prezentate în lucrări viitoare.

Cuvinte cheie: *angrenaje interne-externe, diferență zero, număr de dinți, reductoare cicloidale*

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