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DYNAMIC STABILITY OF A STRAIGHT PIPE CONVEYING FLUID, ELASTICALLY RESTRAINED AGAINST ROTATION AT ITS BOTH ENDS

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Abstract: *The article presents a dynamic analysis of a straight pipe, elastically clamped at both ends. The stiffnesses of the two angular springs are equal. The flowing fluid is incompressible and heavy, moving through the pipe at a constant speed. For three different values of the stiffness of the elastic clamping of the pipe, the relationships between the fluid velocity and the natural circular frequencies of the pipe are determined. In the calculations, the influence of the Coriolis force has been neglected. Considering the fact that pipeline failures and associated fluid leaks lead to environmental, financial, and health issues, the main objective of the article is to determine the critical fluid velocity for each of the studied pipes, at which they lose stability. The study provides insights into how the rigidity of both elastic springs affects the system's stability.*

Key words: *pipe, fluid, natural frequency, critical velocity, elastic supports, dynamic stability, Coriolis force.*

1. INTRODUCTION

In recent years, extensive research has focused on the interaction between fluids and structures, with the dynamic behavior of fluid-carrying pipes being a key issue in this field. The dynamic stability of these structures has garnered significant attention from both scientific research and industry. Since fluid-conveying pipes are integral to many engineering systems and serve as a primary means of transporting oil and gas, ensuring their stability is crucial. Any loss of stability in these pipes can result in damage with far-reaching economic, environmental, and societal consequences.

In their study, Zhang, Gorman and Reese [1] investigated the dynamic stability of pipes with fluid flow. The finite element method was employed to analyze the vibrations of the fluid-conveying pipes. The pipes in question were supported at both ends by joints or clamps. Lagrange's principle and the Ritz method were also applied in their analysis.

Jum'a, Al-hilli and Sattar [2] examined pipes with fluid flow under various support conditions, including simply supported beams, cantilever beams, beams clamped at both ends, and beams composed of separate sections connected by joints. Both straight and curved pipes are considered. To increase energy dissipation, dampers were added to the pipes. Theoretical conclusions, numerical modeling, and experiments on the vibrations of fluid-conveying pipes are discussed.

Bao [3] investigated the dynamic stability of pipes with various types of supports under the influence of fluid flow. The differential equation for the transverse displacements of the pipe's axis was discretized using the Galerkin method. The critical fluid velocity was determined. The stability of the pipe was also studied using dynamic simulation methods.

Once more, Bao [4] conducted a study on the dynamic characteristics of the free vibrations of a fluid-conveying pipe with various types of supports. A numerical analysis was performed, and the natural circular frequency was determined based on the stiffness of the elastic supports.

Siba et al. [5] conducted a review of studies related to the analytical, numerical, and

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experimental investigation of vibrations in fluid-conveying pipes—specifically water, oil, gas, and steam. Mathematical models have been proposed.

Shankarachar and Radhakrishna [6] investigate the dynamic stability of a pipe supported at both ends by a linear and a rotational spring. The frequency equation is derived using Euler-Bernoulli theory. An energetic approach using Hamilton's principle is applied to the differential equation describing the vibrations of the fluid-conveying pipe. It is found that as the velocity of the flowing fluid increases, the circular frequency of the free vibrations decreases.

In their work, Mahato and Luintel [7] present a dynamic study of a pipe with fluid flow, supported by various types of supports. They derive a formula for the fluid acceleration and subsequently establish the differential equation for the transverse displacements of the pipe's axis. The Galerkin method is applied to this equation, and the numerical investigation is conducted using the Runge-Kutta method. The results indicate that as the fluid velocity increases, the transverse displacements of the pipe's axis and its bending moments also increase. The study examines two different materials for the pipe.

The influence of the Coriolis force on the dynamic stability of fluid-conveying pipes has been studied in several articles.

Kuye and Olayiwola [8] investigated the dynamic stability of a pipe with fluid flow, supported at both ends as a simple beam. They solved the differential equation for the transverse displacements of the beam using integral Fourier-Laplace transformation. Their findings revealed that ignoring the Coriolis force increased the instability of the pipe.

Santi et al. [9] conducted an investigation on two types of tubes with fluid flow. In one case, the pipe ends are clamped, while in the other, they are freely supported. They applied the finite element method using Hermitian shape functions to address the dynamic problem. By considering two finite elements of equal length, their results were found to align with the analytical solution using the Hermitian shape function, particularly for the first circular frequency of oscillation.

In his article, Udoetok [10] investigates the oscillations of pipes that are clamped at both ends and those that are hinged. The differential equation governing the transverse displacements of the pipe axis does not account for the influence of Coriolis acceleration. However, the applied method for studying oscillations incorporates the effect of centripetal force resulting from the transverse displacements of the pipe axis. The author presents expressions for the circular frequency of oscillations, the function of transverse displacements, the critical fluid velocity, and the maximum stress. The circular oscillation frequencies obtained are compared with published experimental results.

The objective of this study is to investigate the dynamic stability of straight pipes with elastic restraints against rotation that convey fluid, without considering the Coriolis force.

2. PROBLEM FORMULATION

In this study, the dynamic stability of a fluid-conveying pipe is analyzed using the Euler-Bernoulli beam theory, with the Coriolis force omitted from the calculations.

The investigated beam is elastically clamped at both ends. Its static scheme shown in Fig.1.

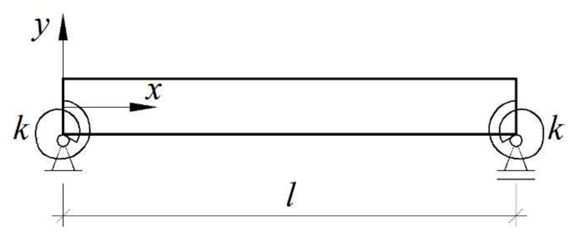

Fig. 1. Static scheme of the pipe under investigation

The rigidity of the rotational springs is denoted by k in Fig. 1

The following differential equation describes the transverse oscillations of a straight pipe conveying an inviscid fluid.

$$EI\frac{\partial^4 w}{\partial x^4}+m_f V^2\frac{\partial^2 w}{\partial x^2}+2m_f V\frac{\partial^2 w}{\partial x \partial t}+ +\left(m_f+m_p\right)\frac{\partial^2 w}{\partial t^2}=0. \quad (1)$$

In equation (1), the function $w(x,t)$ represents the lateral displacement of the pipe's axis. The other symbols in equation (1) include: the time t, the pipe's cross-sectional rigidity EI, the velocity of the flowing fluid V, the mass of the pipe per unit length m_p, and the mass of the conveyed fluid per unit length of the pipe m_f.

When the Coriolis acceleration is neglected, equation (1) takes the following simplified form [10].

$$EI\frac{\partial^4 w}{\partial x^4}+m_f V^2\frac{\partial^2 w}{\partial x^2}+ \\ +\left(m_f+m_p\right)\frac{\partial^2 w}{\partial t^2}=0. \qquad (2)$$

The boundary conditions for solving equation (2) for the beam with the static scheme shown in Fig. 1 are:

$$w(0,t)=w(l,t)=0; \\ EIw''(0,t)+k\,w'(0,t)=0; \\ EIw''(l,t)+k\,w'(l,t)=0. \qquad (3)$$

The solution to the differential equation (2) is sought in the form

$$w(x,t)=W(x)\sin(\omega t). \qquad (4)$$

In (4) ω is the circular frequency of the pipe.

A polynomial solution is assumed for the function $W(x)$.

$$W(x)=B_1+B_2x+B_3x^2+B_4x^3+B_5x^4. \qquad (5)$$

Applying the boundary conditions (3) yields the following results for the coefficients in (5).

$$B_1=0. \qquad (6)$$

$$B_2=\frac{2EI\,l^3(6EI-kl)}{12E^2I^2+k^2l^2-8EIkl}B_5. \qquad (7)$$

$$B_3=\frac{k\,l^3(kl-6EI)}{12E^2I^2+k^2l^2-8EIkl}B_5. \qquad (8)$$

$$B_4=\frac{l\left(16EIkl-2k^2l^2-24E^2I^2\right)}{12E^2I^2+k^2l^2-8EIkl}B_5. \qquad (9)$$

Substituting (4) into (2) yields the following equation.

$$24EIB_5+m_fV^2\frac{d^2W}{dx^2}- \\ -\left(m_f+m_p\right)\omega^2W(x)=0. \qquad (10)$$

By using the average values of centripetal force and deflection, x will be eliminated from the equation (10) [10].

The average value of the centripetal force is determined using the sketch presented in Fig. 2.

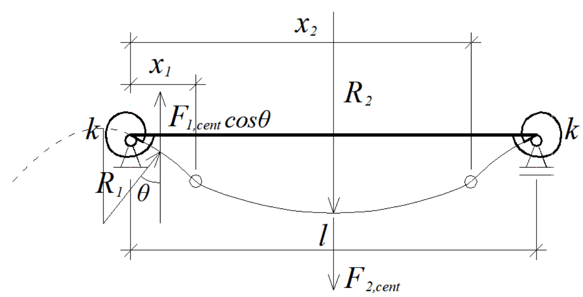

Fig. 2. Centripetal forces acting on the investigated pipe

The axis of the pipe exhibits varying curvature with x and experiences two changes in direction along the deflected pipe at x_1 and x_2. These points, where the direction changes and centripetal force is zero, are identified by solving the following equation for zero.

$$\frac{d^2W}{dx^2}=0. \qquad (11)$$

As a result, the following quadratic equation is obtained.

$$12\left(12E^2I^2+k^2l^2-8EI\,kl\right)x^2+ \\ +6l\left(16EI\,kl-24E^2I^2-2k^2l^2\right)x+ \\ +2kl^3(kl-6EI)=0. \qquad (12)$$

The roots of equation (12) correspond to the deflection points of the pipe's axis at x_1 and x_2.

The average curvatures in each of the three zones along the length of the pipe are as follows:

For $x \in [0, x_1]$ and $x \in [x_2, l]$

$$\frac{1}{\overline{R}_1} = \frac{1}{x_1} \int_0^{x_1} \frac{d^2W}{dx^2} dx . \qquad (13)$$

$$\frac{1}{\overline{R}_1} = \frac{A}{x_1} \Big[2kl^3 (kl - 6EI) x_1 + 3l \left(16EI\, kl - 24E^2 I^2 - 2k^2 l^2\right) x_1^2 + 4\left(12E^2 I^2 + k^2 l^2 - 8EI\, k\, l\right) x_1^3 \Big]. \qquad (14)$$

For $x \in [x_1, 0{,}5l]$

$$\frac{1}{\overline{R}_2} = \frac{1}{0{,}5l - x_1} \int_{x_1}^{0{,}5l} \frac{d^2W}{dx^2} dx . \qquad (15)$$

$$\frac{1}{\overline{R}_2} = \frac{A}{0{,}5l - x_1} \Big[2kl^3 (kl - 6EI)(0{,}5l - x_1) + 3l\left(16EI\, kl - 24E^2 I^2 - 2k^2 l^2\right)(0{,}25l - x_1)^2 + 4\left(12E^2 I^2 + k^2 l^2 - 8EI\, k\, l\right)(0{,}125l - x_1)^3 \Big] \qquad (16)$$

In (14) and (16)

$$A = B_5 \frac{1}{12E^2 I^2 + k^2 l^2 - 8EIkl} . \qquad (17)$$

From the geometry in the region $x \in [0, x_1]$, the curvature of the pipe is approximated as follows [10]

$$\frac{1}{\overline{R}_1} = \frac{2\, w(x_1)}{w^2(x_1) + 0.25l^2} . \qquad (18)$$

By equating equations (14) and (18), the value of the coefficient B_5 in the region $x \in [0, x_1]$ is obtained.

The value of the angle θ can be determined based on the geometric relations (Fig.2).

Knowing θ, one could calculate the centripetal force for the entire beam.

In formula (10) $W(x)$ is represented as the peak of the pipe's deflection

$$W(x) = W\left(\frac{l}{2}\right) = A\, \big[EIl^4 (6EI - kl) + 0{,}25kl^5 (kl - 6EI) + 0{,}125l^4 \left(16EI\, kl - 2k^2 l^2 - 24E^2 I^2\right) + 0{,}0625l^4 \left(12E^2 I^2 + k^2 l^2 - 8EI\, kl\right) \big]. \qquad (19)$$

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Knowing the centripetal force of the beam and the peak value of $W(x)$ through the use of equation (10), the circular frequency of the system can be calculated.

3. RESULTS AND DISCUSSION

Numerical analyses have been conducted for the pipe depicted in Fig. 1.

The pipe's dimensions and material properties are as follows: inner radius of the cross-section $R_{in} = 0.05\, m$, outer radius of the cross-section $R_{out} = 0.055\, m$, modulus of elasticity $E = 210\, GPa$, density of the flowing fluid - $\rho = 1000 kN/m^3$, density of the material of the pipe - $\rho = 7850\, kN/m^3$.

When the circular frequency reaches zero, the system is on the verge of losing stability. The fluid velocity corresponding to the circular frequency $\omega = 0$ is referred to as the critical velocity V_{cr} .

The natural frequency of the system is influenced by all its parameters. Therefore, if all parameters are kept constant (except for fluid velocity), the corresponding critical velocity can be determined.

Fig. 3 illustrates the relationship between the first circular frequency ω of the system and the velocity of the conveyed fluid V for three different rigidities of the elastic clamping of the pipe.

From the results presented in Fig. 3, it can be concluded how the stiffness of the elastic clamping affects the stability of the analyzed pipe.

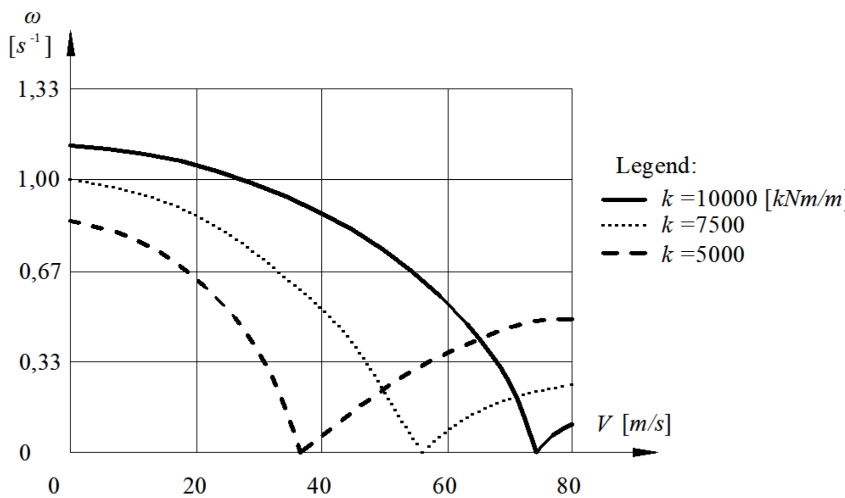

Fig. 3. *The relationship between the first natural frequency* ω *of the pipe and the velocity of the conveyed fluid* V

4. CONCLUSION

Neglecting the Coriolis force when studying the dynamic stability of fluid-conducting pipes allows for a quick and easy determination of the critical fluid velocity. The suggested method can be viewed as a strong competitor to other well-established techniques, like the Matrix method, for the dynamic analysis of fluid-conveying pipes.

This study seeks to assess the impact of elastic clamping stiffness on the dynamic stability of a fluid-conveying pipe, elastically restrained against rotation at its both ends.

The findings demonstrate that higher clamping stiffness enhances the stability of the pipe, with increased rigidity leading to a rise in the critical velocity.

The results also show that fluid velocity has a significant effect on the system's dynamic behavior and may affect its safety. To avoid potential damage, operators should ensure that transport velocities do not exceed the system's critical velocity. ¶

5. REFERENCES

[1] Zhang, Y., Gorman, D., Reese, J., *Analysis of the vibration of pipes conveying fluid*, Proceedings of the Institution of Mechanical Engineers, Part C, pp. 849-860, 1999.

[2] Jum'a, M., Al-hilli, A., Sattar, M., *Stability and vibration of pipe conveying flowing fluid for different support pipe: Review,* Library Progress International, 44 (2), pp. 1236-1247, 2024.

[3] Bao, R., *The Influence of Supported and Restrained Elastic Coefficient on the Stability of Fluid Conveying Pipe*, Advanced Materials Research, 479-481, pp. 292-298, 2012.

[4] Bao, R., *The Natural Characteristics of Fluid Conveying Pipe under Different Supporting Conditions*, Applied Mechanics and Materials, 372, pp. 612-619, 2013.

[5] Siba, M., et al., *Flow-Induced Vibration in Pipes: Challenges and Solutions*, Journal of Engineering Science and Technology, 1 (3), pp. 362-382, 2016.

[6] Shankarachar, S., Radhakrishna, M., *Fluid Induced Piping Vibration with Elastically Restrained Different End Supports*, International Journal of Engineering Research & Technology, 4 (10), pp. 424-429 2013.

[7] Mahato, J., Luintel, M., *The Natural Characteristics of Fluid Conveying Pipe under Different Supporting Conditions*, Proceedings of 14th IOE Graduate Conference, 14, pp. 1217-1226, 2023.

[8] Kuye, S. I., Olayiwola, P. S., *Transverse vibration and stability parameters analyses of an offshore fluid-conveying pipe*, FULafia

Journal of Science & Technology, 4, pp. 35-41, 2018.
[9] Siba, M., et al., *Effect of Coriolis Force on Vibration of Annulus Pipe*, Applied Sciences, 25, pp. 1-15, 2021.
[10] Udoetok, E., *Internal Fluid Flow Induced Vibration of Pipes*, Journal of Mechanical Design and Vibration, 6, pp. 1-8, 2018.

STABILITATEA DINAMICĂ A UNEI CONDUCTE DREPTE CARE TRANSPORTĂ FLUID, FIXATĂ ELASTIC ÎMPOTRIVA ROTAȚIEI LA AMBELE CAPETE

Rezumat: Articolul prezintă o analiză dinamică a unei conducte drepte, fixată elastic la ambele capete. Rigiditățile celor două arcuri unghiulare sunt egale. Fluidul care curge este incomprimabil și greu, deplasându-se prin conductă cu o viteză constantă. Pentru trei valori diferite ale rigidității fixării elastice a conductei, sunt determinate relațiile dintre viteza fluidului și frecvențele circulare proprii ale conductei. În calcule, influența forței Coriolis a fost neglijată. Având în vedere faptul că defecțiunile conductelor și scurgerile de fluid asociate duc la probleme de mediu, financiare și de sănătate, obiectivul principal al articolului este de a determina viteza critică a fluidului pentru fiecare dintre conductele studiate, la care acestea își pierd stabilitatea. Studiul oferă informații despre modul în care rigiditatea celor două arcuri elastice influențează stabilitatea sistemului.
Cuvinte cheie: *țeavă, fluid, frecvență naturală, viteză critică, suporturi elastice, stabilitate dinamică, forță Coriolis.*

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