



TECHNICAL UNIVERSITY OF CLUJ-NAPOCA

ACTA TECHNICA NAPOCENSIS

Series: Applied Mathematics, Mechanics, and Engineering
Vol. 68, Issue I, March, 2025

MECHANICAL VIBRATIONS STUDY PRODUCED BY MACHINE-TOOLS OVER A HUMAN OPERATOR, WHO SERVES IT. PART I: OPERATOR MECHANICAL CHARACTERISTICS AND THE SYSTEM OF DIFFERENTIAL EQUATIONS

Adela Bianca CRĂCIUN (HOSU), Ovidiu Aurelian DETEȘAN, Mariana ARGHIR

Abstract: The paper is part of the doctoral thesis of the first author, in which the action of vibrations on an operator serving a machine tool during the production process is studied. In this paper, the theoretical study of the transmission of vibrations from the machine tool through the operator's feet is carried out, which is placed on a pedestal on which there is a rubber mattress. It is considered the vertical vibrations, through both legs, up to the abdomen, which summarize them and form together with the upper part of the body an undeformable system. In the first part of the paper, the mechanical model of the material system, assimilated to the two legs, as well as the mechanical characteristics of the segmental component parts, are presented. The mechanical diagram of the system is drawn up in its entirety and on the elements, then the system of differential equations is written, which governs the vibratory movement of the material system, during the machining process on the machine tool

Key words: machine tool vibrations, human operator, mechanical model.

1. INTRODUCTION

The study corresponding to this doctoral thesis refers to the vertical transmission of vibrations through the operator, from the soles of the feet, through both feet, to the abdomen, which is the part that sums up / cumulates the vibrations transmitted through both feet. The body segments referred to in this study are: the foot, ankle, calf, knee joint, femur, hip joint, abdomen – which together with the upper body form an assembly assimilated to a rigid solid.

The study will be carried out on two human operators, of medium age, who have experience in servicing machine tools with numerical control. They were warned that their characteristic data will be used in the theoretical study of the vibrations action on the body. The study will not cause health damage, but will use their mechanical characteristics of: shock absorber, mass, and spring on the different segments of the body. as it will result later. The two operators gave their consent for their mechanical characteristics to be used. The operators are:

- a. A 46-year-old male operator, weighing 83 kg and 187cm tall;
- b. A 45-year-old female operator, weighing 78 kg and 170 cm tall.

2.THE BIOMECHANICAL MODEL OF OPERATOR'S FOOT

A human operator, who serves a machine tool with numerical control, is required to vibrate, to supervise the processing process, being placed on an elastic rubber support, positioned in front of the machine at a distance appropriate to the supervision and in which he can perform his tasks [1]. It is positioned on both legs, and the vibration is transmitted evenly through them. The important load is the vertical one, according to the "z" direction of each segment. For the theoretical study, each foot is considered discretized / segmented into: foot, ankle, calf, knee joint, femur, thigh. Vibration is considered, from both legs with action on the abdomen, the vulnerable mass of the human body, due to the internal organs that are found here, but it is studied with further transmission, because the upper parts of the body are more compact, have

much greater possibilities of damping vibrations, so for a human operator, the body is assimilated to a rigid solid [2], [3].

The mechanical system will be characterized by six independent coordinates for each leg, so there will also be six degrees of freedom for the right leg and six degrees of freedom for the left leg, to which is added a degree of freedom for

the body. The two operator's legs are considered to have slightly different mechanical characteristics, due to the fact that the two operators are right-handed. In this way, the material system on which the study is carried out has 13 degrees of freedom and will be called 13DEGOFR. (so, 13 DEGRess Of FREedom). The mechanical model is represented in figure 1.

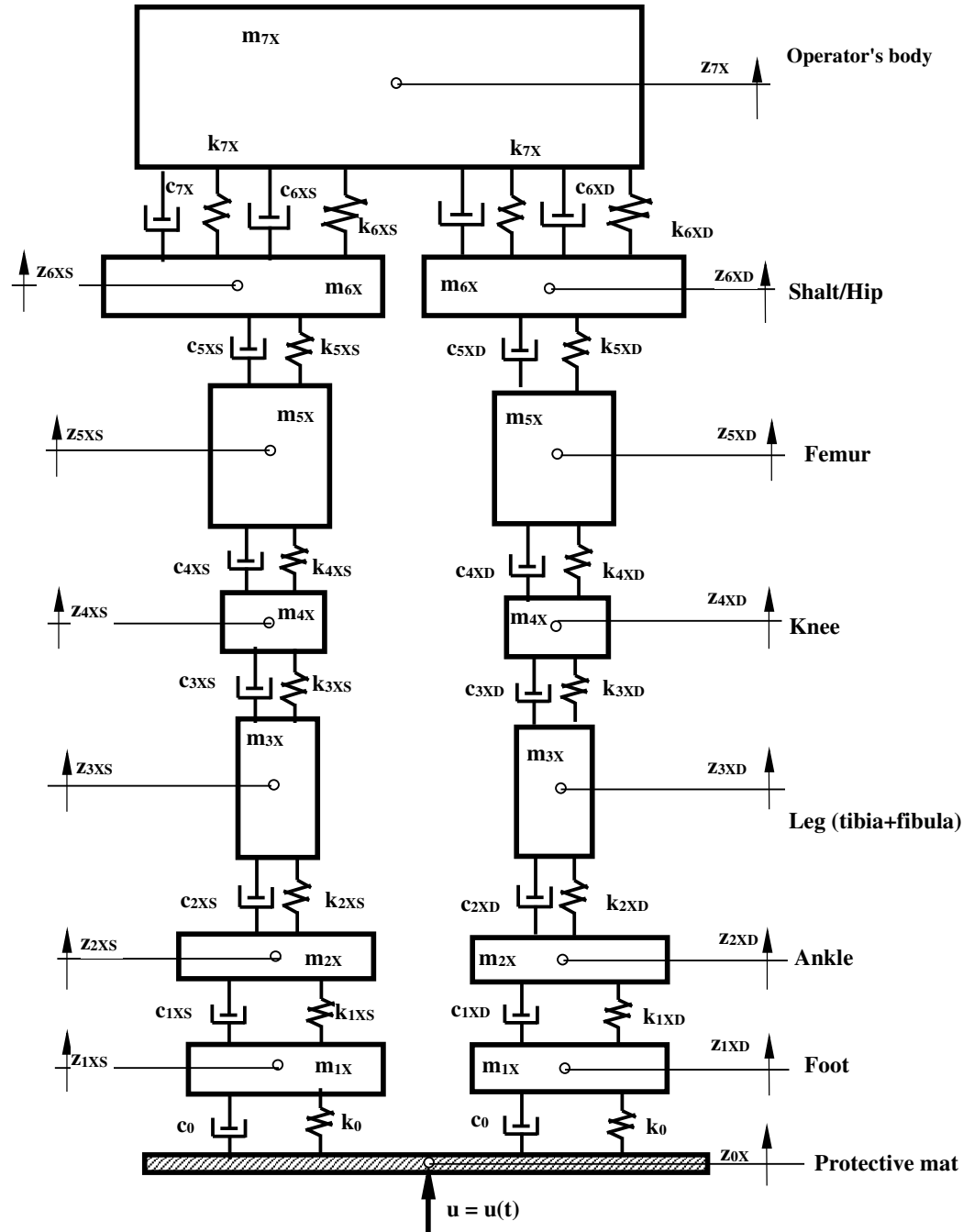


Fig. 1. The mechanical model of the 13DEGOFR system, corresponding to the operator

The meaning of the notations in Figure 1, are as follow.

u – gives the law on the request of the mechanical system, assimilated to the human body, which serves the machine-tool. The law of variation depends on the mass of the operator (male or female), on the speed of the main shaft of the machine tool during the period of operation, but also on the failure of the rubber mat at the initial moment, under the weight of the operator. The spindle speed $n = 10,000$ rpm is considered.

z – represents the generalized coordinate of the vertical displacement of the element, to which it refers;

0 – is the index of the rubber protection mat, on which the operator is positioned during the supervision of the processing process. The mechanical characteristics of the carpet are:

$k_0 = 310,000$ N/m, elasticity constant of rubber;

$c_0 = 3,900$ Ns/m, the damping constant of the rubber.

1 – is the index of the foot of the foot;

2 – corresponds to the ankle;

3 – refers to the calf (both the bone and the muscle part);

4 – represents the index of the knee in the translational movement;

5 – is the index of the femur;

6 – is the hip (haunch);

7 – the operator's body, which summarizes the vibrations transmitted through both legs;

m – the mass of the element obtained by discretizing the real system;

k – the elasticity constant of the element;

c – the damping constant of the element;

B – is the index given to the "male" operator;

F – is the index given to the "female" operator;

X – is replaced by F or B, as the case may be;

S – refers to the left leg;

D – refers to the right leg of the human operator.

3. DISTRIBUTION OF THE HUMAN ORGANISM MASSES / OPERATOR

From the literature [6] the distribution of masses is a function of the total mass of the organism and its height. Table 1 shows the mass distribution of the segmental elements taken in

the study for the "male" operator and in Table 2 for the "female" operator.

Table 1
Mass distribution for the "male" operator

Crt. No.	Segment Name	Mass [kg]	Symbol
1.	Foot	0,71	m_{1B}
2.	Ankle	0,49	m_{2B}
3.	Leg (Tibia+fibula)	3,86	m_{3B}
4.	Knee	1,66	m_{4B}
5.	Femur	6,64	m_{5B}
6.	Hip	5,89	m_{6B}
7.	Operator's body	44,50	m_{7B}

Table 2
Mass distribution for the "female" operator

Crt. No.	Segment Name	Mass [kg]	Symbol
1.	Foot	0,67	m_{1F}
2.	Ankle	0,46	m_{2F}
3.	Leg (Tibia+fibula)	3,63	m_{3F}
4.	Knee	1,56	m_{4F}
5.	Femur	6,24	m_{5F}
6.	Hip	5,54	m_{6F}
7.	Operator's body	41,80	m_{7F}

4. MECHANICAL CHARACTERISTICS OF OPERATORS, WHO SERVICE THE MACHINE TOOL

The study corresponding to this work refers to the vertical transmission of vibrations through the operator, from the soles of the feet, through both feet, to the abdomen, which is the part that sums up / cumulates the vibrations transmitted through both feet.

The body segments referred in this study are: the foot, ankle, calf, knee joint, femur, hip joint, abdomen – which together with the upper body form an assembly assimilated to a rigid solid (Fig. 1).

Each segment of the human body, like any system in nature located on the face of the earth, has the three basic mechanical characteristics: mass, shock absorber and spring. Selected from the literature [5] the seven segmental elements of the human operator's body are presented in the following tables.

Table 3

Mechanical characteristics for the "male" operator

Crt. No	Characteristic name	Symbol	Value & U.M.	
			Left leg	Right leg
1.	Element Mass	m _{1B}	0,71 kg	
2.		m _{2B}	0,49 kg	
3.		m _{3B}	3,86 kg	
4.		m _{4B}	1,66 kg	
5.		m _{5B}	6,64 kg	
6.		m _{6B}	5,89 kg	
7.		m _{7B}	44,50 kg	
8.	Coefficient of elasticity	k _{1BS/D}	100000 N/m	110000 N/m
9.		k _{2BS/D}	108000 N/m	120000 N/m
10.		k _{3BS/D}	105000 N/m	120000 N/m
11.		k _{4BS/D}	110000 N/m	120000 N/m
12.		k _{5BS/D}	60000 N/m	50000 N/m
13.		k _{6BS/D}	103000 N/m	105000 N/m
14.		k _{7B}	105000 N/m	
15.	Depreciation coefficient	c _{1BS/D}	900 Ns/m	850 Ns/m
16.		c _{2BS/D}	1100 Ns/m	1000 Ns/m
17.		c _{3BS/D}	11000 Ns/m	10000 Ns/m
18.		c _{4BS/D}	2300 Ns/m	2200 Ns/m
19.		c _{5BS/D}	1200 Ns/m	1100 Ns/m
20.		c _{6BS/D}	2000 Ns/m	1800 Ns/m
21.		c _{7B}	1600 Ns/m	

Table 4

Mechanical characteristics for the "female" operator

Crt. No	Characteristic name	Symbol	Value & U.M.	
			Left leg	Right leg
1.	Element Mass	m _{1F}	0,67kg	
2.		m _{2F}	0,46kg	
3.		m _{3F}	3,63kg	
4.		m _{4F}	1,56kg	
5.		m _{5F}	6,24kg	
6.		m _{6F}	5,54 kg	
7.		m _{7F}	41,80 kg	
8.	Coefficient of elasticity	k _{1FS/D}	90000 N/m	110000 N/m
9.		k _{2FS/D}	105000 N/m	120000 N/m
10.		k _{3FS/D}	100000 N/m	120000 N/m
11.		k _{4FS/D}	110000 N/m	120000 N/m
12.		k _{5FS/D}	60000 N/m	50000 N/m
13.		k _{6FS/D}	100000 N/m	105000 N/m
14.		k _{7F}	105000 N/m	
15.	Depreciation coefficient	c _{1FS/D}	910 Ns/m	850 Ns/m
16.		c _{2FS/D}	1100 Ns/m	1000 Ns/m
17.		c _{3FS/D}	11500 Ns/m	10000 Ns/m
18.		c _{4FS/D}	2320 Ns/m	2200 Ns/m
19.		c _{5FS/D}	1250 Ns/m	1100 Ns/m
20.		c _{6FS/D}	2000 Ns/m	1800 Ns/m
21.		c _{7F}	1600 Ns/m	

4. THE SYSTEM OF DIFFERENTIAL EQUATIONS, WHICH GOVERNS THE VIBRATORY MOVEMENT OF THE MECHANICAL SYSTEM

The establishment of the system of differential equations can be done starting from the schema of the mechanical model assimilated to the body of the human operator, which

supervises the development of the processing process [7], [8]. The diagram is shown in Figure 1, with the operator standing on a protective tyre mat that transmits vibrations from the machine foundation to the operator. The operator's feet are discretized into six distinct segments, each of which has mechanical characteristics of mass, shock absorber and spring well defined in the literature. In the paper they are given in tables 1 – 4.

4.1. Preparatory phase

Each element of the mechanical structure shown in Figure 1 shall be isolated and applied to it all the active, resistance, inertia, and damping forces, applied directly to the element under consideration, according to the second principle of mechanics.

The transition from one element of the material system to another is made by applying the principle of action and reaction. The forces are considered positive if they act in the direction of increasing the generalized coordinate, and they are considered negative if they act in the direction of decreasing the generalized coordinate, which gives the position of the center of the masses of the element.

For each generalized coordinate, the element is presented charged with all the active, linking, inertial forces. The part of component elements of the material system are presented in the sequence of the following figures.

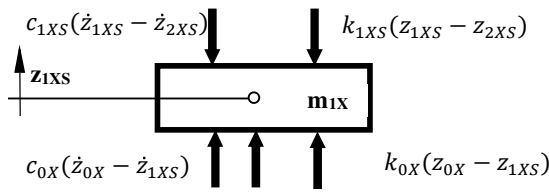


Fig. 2. Loading the left foot from the system 13DEGOFR

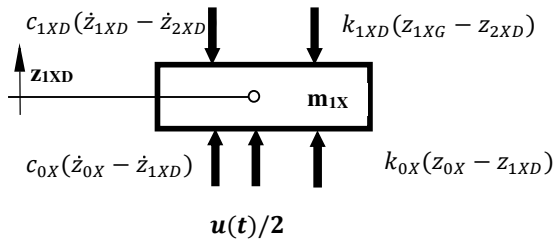


Fig. 3. Loading the right foot from the system 13DEGOFR

The same will be done for the ankle, calf, knees, femur, both for the left and right leg. For the left thigh, for the right thigh, as well as for the body of the operator, the figures are presented by analogy with figure 2 for the left leg and by analogy with figure 3 for the right leg.

Due to the complexity of the vibrational activity accumulated in the hip / thigh, the mechanical load schemes will be presented, as in figure 4 for the left hip and in figure 5 for the right hip.

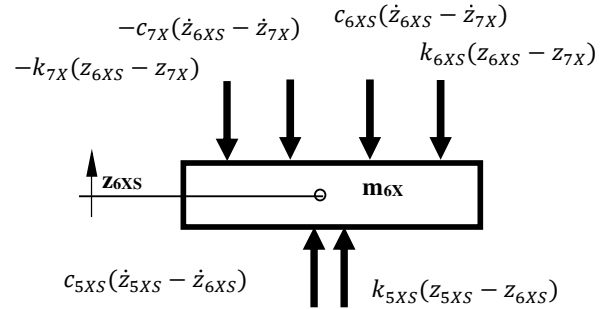


Fig. 4. Loading the hip of the left leg in the system 13DEGOFR

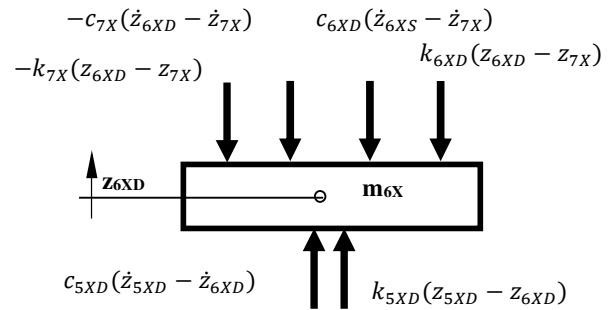


Fig. 5. Loading the hip of the right leg in the system 13DEGOFR

The mechanical load of the operator's body, which serves the machine tool during the machining process, is shown in Figure 6 and summarizes the vibrations transmitted through both legs to the abdomen.

4.2. Differential equations governing the dynamics of the mechanical system 13DEGOFR

The representations in Figures 2, 3, 4 and 5 contain the loads and mechanical diagrams related to the concentrated masses (foot and hip) of an operator, who serves the machine tool during the machining process, with the main shaft in the rpm motion, in idle. In this situation,

the operator, standing with both feet, is positioned on a protective rubber mat and is stressed by the sole of the foot by a vibrating force, with harmonic variation depending on the operator's mass (M_x), the oscillation pulsation (ω) due to the rotational movement of the main shaft of the machine and the initial failure of the rubber mat under the weight of the operator.

The law of motion of the point of stress to vibrations of the organism is symmetrical with respect to the two legs and presents the law:

$$u = u(t) = M_x \omega^2 u_0 \sin(\omega t) \quad (1)$$

In which M_x is the mass of an operator.

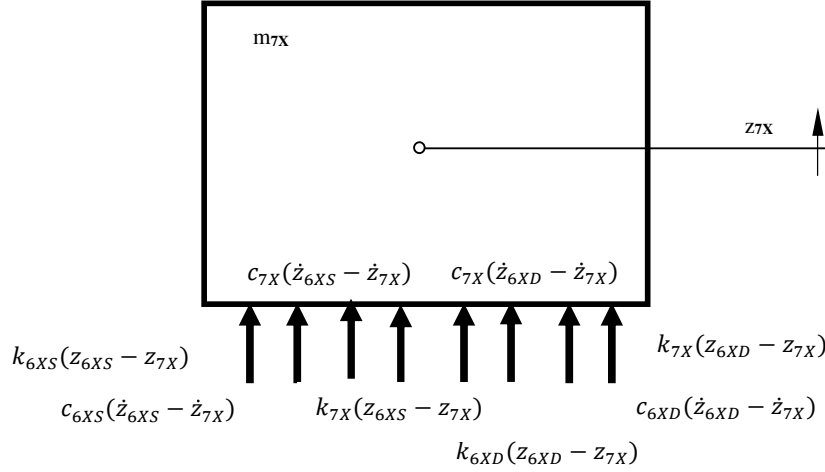


Fig. 6. Loading of the operator's body in the 13DEGOF system due to the stress of vibrations through the feet, when supervising the cutting process of a machine tool

The elaboration of the system of differential equations, which governs the transmission of vibrations through the operator's legs to the abdomen (body), is done by applying principle II of the mechanics of each body segment, according to the representations in figures 2 - 6. Thus, from figure 2, it follows:

$$m_{1X} \ddot{z}_{1XS} = u(t)/2 + k_{0X}(z_{0X} - z_{1XS}) + c_{0X}(\dot{z}_{0X} - \dot{z}_{1XS}) - k_{1XS}(z_{1XS} - z_{2XS}) - c_{1XS}(\dot{z}_{1XS} - \dot{z}_{2XS}) \quad (2)$$

and represents the differential equation in the dynamics of the operator's left foot. From figure 3, it follows:

$$m_{1X} \ddot{z}_{1XD} = u(t)/2 + k_{0X}(z_{0X} - z_{1XD}) + c_{0X}(\dot{z}_{0X} - \dot{z}_{1XD}) - k_{1XD}(z_{1XD} - z_{2XD}) - c_{1XD}(\dot{z}_{1XD} - \dot{z}_{2XD}) \quad (3)$$

It is the differential equation of the operator's right foot. From figures analogous to figures 2 and 3 can be made of the ankle, the calf, knee, femur for both the left and right legs. The relationship corresponding to each element will be explained.

$$m_{2X} \ddot{z}_{2XS} = k_{1XS}(z_{1XS} - z_{2XS}) + c_{1XS}(\dot{z}_{1XS} - \dot{z}_{2XS}) - k_{2XS}(z_{2XS} - z_{3XS}) - c_{2XS}(\dot{z}_{2XS} - \dot{z}_{3XS}) \quad (4)$$

containing the differential equation of the ankle of the left foot.

$$m_{2X} \ddot{z}_{2XD} = k_{1XD}(z_{1XD} - z_{2XD}) + c_{1XD}(\dot{z}_{1XD} - \dot{z}_{2XD}) - k_{2XD}(z_{2XD} - z_{3XD}) - c_{2XD}(\dot{z}_{2XD} - \dot{z}_{3XD}) \quad (5)$$

establishes the differential equation of the ankle of the right foot.

$$m_{3X} \ddot{z}_{3XS} = k_{2XS}(z_{2XS} - z_{3XS}) + c_{2XS}(\dot{z}_{2XS} - \dot{z}_{3XS}) - k_{3XS}(z_{3XS} - z_{4XS}) - c_{3XS}(\dot{z}_{3XS} - \dot{z}_{4XS}) \quad (6)$$

has the differential equation of the calf of the left foot.

$$m_{3X} \ddot{z}_{3XD} = k_{2XD}(z_{2XD} - z_{3XD}) + c_{2XD}(\dot{z}_{2XD} - \dot{z}_{3XD}) - k_{3XD}(z_{3XD} - z_{4XD}) - c_{3XD}(\dot{z}_{3XD} - \dot{z}_{4XD}) \quad (7)$$

This is what the differential equation of the calf of the right foot looks like.

$$m_{4X}\ddot{z}_{4XS} = k_{3XS}(z_{3XS} - z_{4XS}) + c_{3XS}(\dot{z}_{3XS} - \dot{z}_{4XS}) - k_{4XS}(z_{4XS} - z_{5XS}) - c_{4XS}(\dot{z}_{4XS} - \dot{z}_{5XS}) \quad (8)$$

with the differential equation of the knee of the left leg.

$$m_{4X}\ddot{z}_{4XD} = k_{3XD}(z_{3XD} - z_{4XD}) + c_{3XD}(\dot{z}_{3XD} - \dot{z}_{4XD}) - k_{4XD}(z_{4XD} - z_{5XD}) - c_{4XD}(\dot{z}_{4XD} - \dot{z}_{5XD}) \quad (9)$$

It is the differential equation of the knee of the right foot.

$$m_{5X}\ddot{z}_{5XS} = k_{4XS}(z_{4XS} - z_{5XS}) + c_{4XS}(\dot{z}_{4XS} - \dot{z}_{5XS}) - k_{5XS}(z_{5XS} - z_{6XS}) - c_{5XS}(\dot{z}_{5XS} - \dot{z}_{6XS}) \quad (10)$$

This is how the femur of the left leg will move under the action of vibrations.

$$m_{5X}\ddot{z}_{5XD} = k_{4XD}(z_{4XD} - z_{5XD}) + c_{4XD}(\dot{z}_{4XD} - \dot{z}_{5XD}) - k_{5XD}(z_{5XD} - z_{6XD}) - c_{5XD}(\dot{z}_{5XD} - \dot{z}_{6XD}) \quad (11)$$

The right femur responds to this law.

$$m_{6X}\ddot{z}_{6XS} = k_{5XS}(z_{5XS} - z_{6XS}) + c_{5XS}(\dot{z}_{5XS} - \dot{z}_{6XS}) - k_{6XS}(z_{6XS} - z_{7X}) - c_{6XS}(\dot{z}_{6XS} - \dot{z}_{7X}) + k_{7X}(z_{6XS} - z_{7X}) + c_{7X}(\dot{z}_{6XS} - \dot{z}_{7X}) \quad (12)$$

The left hip responds to this request.

$$m_{6X}\ddot{z}_{6XD} = k_{5XD}(z_{5XD} - z_{6XD}) + c_{5XD}(\dot{z}_{5XD} - \dot{z}_{6XD}) - k_{6XD}(z_{6XD} - z_{7X}) - c_{6XD}(\dot{z}_{6XD} - \dot{z}_{7X}) + k_{7X}(z_{6XD} - z_{7X}) + c_{7X}(\dot{z}_{6XD} - \dot{z}_{7X}) \quad (13)$$

And the right hip responds to the requests, according to the differential equation in relation (13).

$$m_{7X}\ddot{z}_{7X} = +k_{6XS}(z_{6XS} - z_{7X}) + c_{6XS}(\dot{z}_{6XS} - \dot{z}_{7X}) + k_{6XD}(z_{6XD} - z_{7X}) + c_{6XD}(\dot{z}_{6XD} - \dot{z}_{7X}) + k_{7X}(z_{6XS} - z_{7X}) + c_{7X}(\dot{z}_{6XS} - \dot{z}_{7X}) + k_{7X}(z_{6XD} - z_{7X}) + c_{7X}(\dot{z}_{6XD} - \dot{z}_{7X}) \quad (14)$$

The relation (14) contains the law of motion of the center of mass of the body of the operator

subjected to the action of vibrations, produced by the machine-tool in the motion of the regime.

The relations (2) – (14) together form a system of 13 differential equations of the second order, in the generalized coordinates of the operator's feet discretized into six distinct segments, for which the mechanical characteristics of each segment are known, as well as the position of the center of mass.

5. CONCLUSIONS

This paper studies the action of vibrations, which act on the operator, who serves a machine tool, with numerical control during the production process.

The study is carried out in several stages:

1. The mechanical diagram of the material system assimilated to the operator's feet, which serves the machine tool, is drawn up in sequence: foot, ankle, calf, knee, femur.
2. The movement is transmitted through both legs, from the sole of the foot to the hip / thigh.
3. The upper part of the body sums up the vibrations transmitted through the legs; the entrance being made in the abdomen.
4. The abdomen together with the upper part of the body is considered to form a rigid solid, which reacts to the vibrations transmitted.
5. It is considered, from the specialized literature, the mechanical characteristics of each part of the system, with their differentiation on the left leg and the right leg, because the operators are right-handed.
6. The mechanical diagrams of the elements of the system are made, applying the principles of mechanics.
7. The differential equations of the component elements of the material system are written, which will form a differential system of the second order in the 13 generalized coordinates, corresponding to the vertical translations of the elements of the material system, under the action of the vibrations of the machine tool.

8. The paper contributes to the study of vibrations acting on the operator, who serves a numerically controlled machine tool, without causing health damage to the human body during processing.

6. REFERENCES

- [1] Potîrniche, AM; Vasile, O; Capatana, GF, Modal Analysis of a Mechanical System Modeled as a 6 Degrees-of-Freedom Solid Body with Elastic Bearings and Structural Symmetries, *Romanian Journal of Acoustics and Vibration*, 2022, Vol. 19, Issue 1, pp.36-40
- [2] Zhivomirov, H; Nedelchev, I, A method for signal stationarity estimation, *Romanian Journal of Acoustics and Vibration*, 2020, Vol. 17, Issue 2, pp. 149-155;
- [3] Stanescu, ND, Multibody Systems - approaches and challenges, , *Romanian Journal of Acoustics and Vibration*, 2020, Vol. 17, Issue 1, pp. 2-2;
- [4] Stan, AF; Pandrea, N; Stanescu, ND, On the dynamics of the constrained rigid solid acted by conservative forces. part I: Theory, *Romanian Journal of Acoustics and Vibration*, 2020, Vol. 17, Issue 1, pp. 3-9;
- [5] Bratu, P; Buruga, A; Chilari, O; Ciocodeiu, AI; Oprea, I, Evaluation of the linear viscoelastic force for a dynamic system (m, c, k) excited with a rotating force, *Romanian Journal of Acoustics and Vibration*, 2019, Vol. 16, Issue 1, pp. 39-46;
- [6] Alexandra-Maria Macovei, Studii și cercetări privind mediul vibrational produs de utilajele din industria alimentară cu acțiune asupra operatorului uman, Teză de doctorat, Universitatea Tehnică din Cluj-Napoca, 2019
- [7] Mariana Runcan, Mărimi cinematice și măsurări locale și globale în interacțiunea om-structură, Teză de doctorat, Universitatea Tehnică din Cluj-Napoca, 2010.
- [8] Aurica Truta, Modelări biomecanice ale organismului uman sub acțiunea vibrațiilor mecanice produse de mașini și utilaje în mediul de lucru, Teza de doctorat, 2009, Universitatea Tehnică din Cluj-Napoca

Studiul vibrațiilor mecanice produse de o mașină-unealtă asupra unui operator uman, care o servește. Partea I: caracteristici mecanice ale operatorului și sistemul ecuațiilor diferențiale

Rezumat: *Lucrarea este parte a tezei de doctorat a primului autor, în care se studiază acțiunea vibrațiilor asupra unui operator, ce deservește o mașină-unealtă în timpul procesului de producție. În această lucrare se realizează studiul teoretic al transmiterii vibrațiilor de la mașină-unealtă prin picioarele operatorului, care este așezat la locul de muncă pe un postament, pe care se găsește o saltea de cauciuc. Se consideră vibrațiile verticale, prin ambele picioare, până la abdomen, care le însumează și formează împreună cu partea superioară a corpului un sistem indeformabil. În prima parte a lucrării se prezintă modelul mecanic al sistemului material, asimilat celor două picioare, cu caracteristicile mecanice ale părților componente segmentare. Se întocmește schema mecanică a sistemului în totalitate și pe elemente, apoi se întocmește sistemul ecuațiilor diferențiale, care guvernează mișcarea vibratorie a sistemului material, în timpul procesului de prelucrare pe mașină-unealtă*

Cuvinte cheie: *vibrațiile mașinii-unelte, operator uman, model mecanic, sistemul ecuațiilor diferențiale.*

Adela Bianca CRĂCIUN (căs: HOSU), PhD Stud., Eng., Technical University of Cluj-Napoca, Department of Mechanical System Engineering, B-dul Muncii, Nr. 103-105, Cluj-Napoca, ROMANIA.

Ovidiu Aurelia DETEȘAN, Assoc. Prof. Dr. Eng. Technical University of Cluj-Napoca, Department of Mechanical System Engineering, B-dul Muncii, Nr. 103-105, Cluj-Napoca, ROMANIA.

Mariana ARGHIR, Professor, dr. eng., Technical University of Cluj-Napoca, Department of Mechanical System Engineering, e-mail: marianaarghir@yahoo.com, Tel: 0264.401.657, B-dul Muncii, Nr. 103-105, Cluj-Napoca, ROMANIA.