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SYNTHESIS OF THE COMPLEX MECHANISM OF THE QUADRUPED MOBILE ROBOT

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Abstract: *The paper presents two new kinematic schemes of articulated planar mechanisms used as legs in a quadruped mobile robot. These mechanisms consist of 5 mobile kinematic elements when it is compared to the Jansen mechanism which is made of 7 elements. The four leg mechanisms of the quadruped robot are driven by a single electric motor by means of spur gears. The structure of the mechanism schemes is consisting of kinematic chains with four articulated triadic or tetradic elements. For each of the proposed kinematic schemes, several geometrical options regarding the position of the articulations and the fulcrum have been considered. The mobility of each kinematic scheme is assessed and a geometrical synthesis method for generating the fulcrum of the reciprocating coupler curve is given.*

Key words: *Mechanism, quadruped leg, synthesis, mobile robot.*

1. INTRODUCTION

All currently used kinematic schemes [1, 3, 4, 5] for the configuration of the quadruped mobile robot legs include simple dyad chains in their topologic structure. The kinematic schemes proposed by the authors for the legs of a quadruped mobile robot (fig. 1a) are using complex kinematic chains as the triadic [1] or tetradic type [2].

For exemplification, the kinematic scheme of a quadruped mobile robot (figure 1b) is presented having the two legs with different structures, i.e. a triadic kinematic chain (left) and a tetradic kinematic chain (right).

In reality, the two mechanisms with articulated bars are the same and they are simultaneously driven by means of a central pinion *p* through spur gears (figure 1b).

Each crank 1 is fixed together with the corresponding gear (as the driven element in the gear), so that the cranks rotate in the same direction.

The articulated kinematic elements (2, 3, 4 and 5) form a triadic chain (left) LTr(2,3,4,5), and a tetradic chain (right) LTt(2,3,4,5) respectively.

We can remark that the Klann mechanism [6, 7], which has the same total number of kinematic elements, the chain consisting of the four elements (2, 3, 4 and 5) is formed of two simple dyad type kinematic chains: LD (2,3) + LD (4,5).

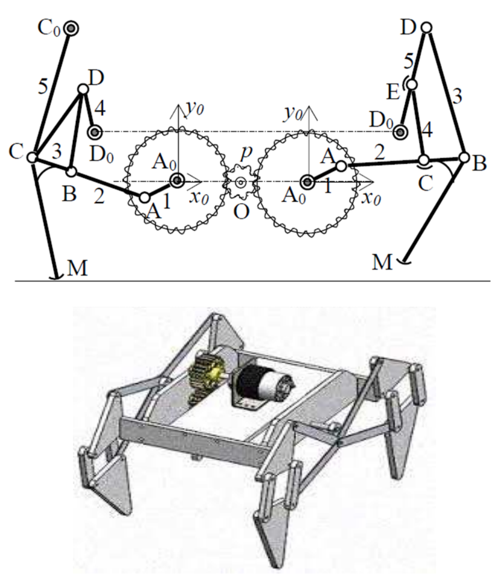

Fig. 1. Kinematic and 3D scheme of a quadruped mobile robot equipped with the Jansen mechanism.

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The structure of the Jansen mechanism [7] (figure 1a) contains a kinematic chain of 6 elements (2, 3, 4, 5, 6 and 7) assembled as three dyadic chains connected in parallel: LD(2,3) + LD(4,5) + LD(6,7).

For the proposed mechanisms (figure 1b), fulcrum M belongs to a planar movement kinematic element (reciprocating coupler) which can be the bars 2 or 3 (left), and 2, 3 or 4 (right).

2. STRUCTURE AND KINEMATICS OF THE COMPLEX MECHANISM

2.1 The triadic chain

Let us consider the kinematic scheme of the Triadic mechanism (figure 2a) for which the structural equation of the "motor" mechanism is written (with crank 1 as actuator):

$$MM = MA(0,1) + LTr(2,3,4,5) . \qquad (1)$$

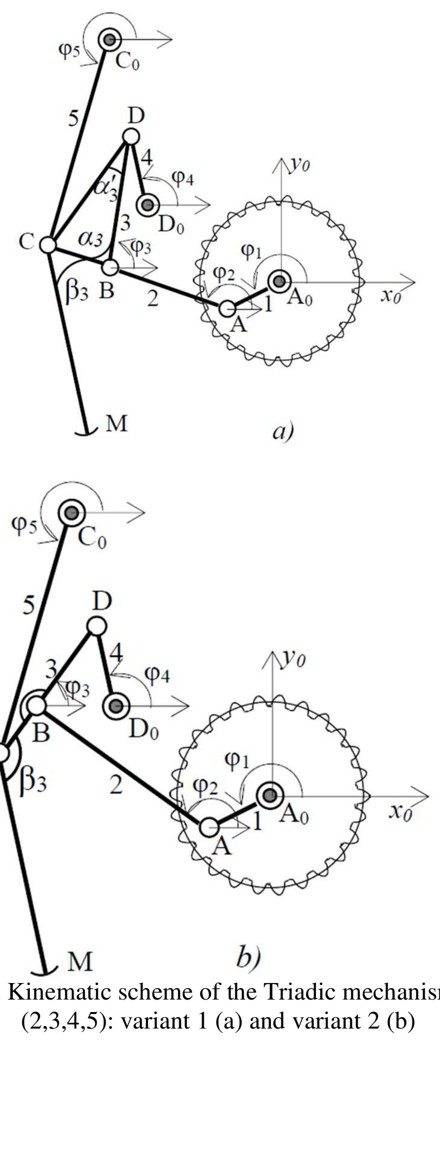


Fig. 2. Kinematic scheme of the Triadic mechanism LTr (2,3,4,5): variant 1 (a) and variant 2 (b)

The triadic chain L**Tr** (2,3,4,5) is defined by the characteristic lengths of the four component elements and the 4 variable angular parameters $(\varphi_2, \varphi_3, \varphi_4, \varphi_5)$.

The triad (2,3,4,5) is defined by two closed independent kinematic contours $(ABDD_0A)$ and $(C_0CDD_0C_0)$, for which the closing scalar equations are written as follows (fig. 2a):

$$l_2 \cos\varphi_2 + l_3 \cos\varphi_3 - l_4 \cos\varphi_4 = x_{D_0} - l_1 \cos\varphi_1 \; (2)$$

$$l_2 \sin\varphi_2 + l_3 \sin\varphi_3 - l_4 \sin\varphi_4 = y_{D_0} - l_1 \sin\varphi_1 \; (3)$$

$$l_5 \cos\varphi_5 + l_3' \cos(\varphi_3 - \alpha_3') - l_4 \cos\varphi_4 = x_{D_0} - x_{C_0} \; (4)$$

$$l_5 \sin\varphi_5 + l_3' \sin(\varphi_3 - \alpha_3') - l_4 \sin\varphi_4 = y_{D_0} - y_{C_0} \; (5)$$

For a certain value of the angle $\varphi_{1,}$ in the system of the four non-linear equations, angles $\varphi_2, \varphi_3, \varphi_4, \varphi_5.$ are calculated.

The position of the fulcrum M is defined by the Cartesian coordinates:

$$x_M = l_1 \cos\varphi_1 + l_2 \cos\varphi_1 + l_3'' \cos(\varphi_3 + \alpha_3) + l_3''' \cos(\varphi_3 + \alpha_3 + \pi - \beta_3) \qquad (6)$$

$$y_M = l_1 \sin\varphi_1 + l_2 \sin\varphi_1 + l_3'' \sin(\varphi_3 + \alpha_3) + l_3''' \sin(\varphi_3 + \alpha_3 + \pi - \beta_3) \qquad (7)$$

In the equations (2), (3)..., (7), we have denoted the following lengths:

$A_0A = l_1; AB = l_2; BD = l_3; CD = l_3'; BC = l_3''; DD_0 = l_4; CC_0 = l_5; CM = l_3''';$

If we consider variant 2 (fig. 2b), the central element 3 has the articulations B, C and D collinear, which results in changing certain geometrical parameters ($\alpha_3 = \pi;\ \alpha_3' = 0$). The fact that the three articulations C, B and D are collinear leads to $l_3' = l_3 + l_3''$.

$$x_M = l_1 \cos\varphi_1 + l_2 \cos\varphi_1 - l_3'' \cos\varphi_3 + l_3''' \cos(\varphi_3 + 2\pi - \beta_3) \qquad (6')$$

$$y_M = l_1 \sin\varphi_1 + l_2 \sin\varphi_1 - l_3'' \sin\varphi_3 + l_3''' \sin(\varphi_3 + 2\pi - \beta_3) \qquad (7')$$

2.2 The tetradic chain

Let us now consider the kinematic scheme of the Tetradic chain (fig. 3) for which the structural equation of the “motor” mechanism is written (with crank 1 as actuator):

$$MM = MA(0.1) + LTt(2,3,4,5) \qquad (8)$$

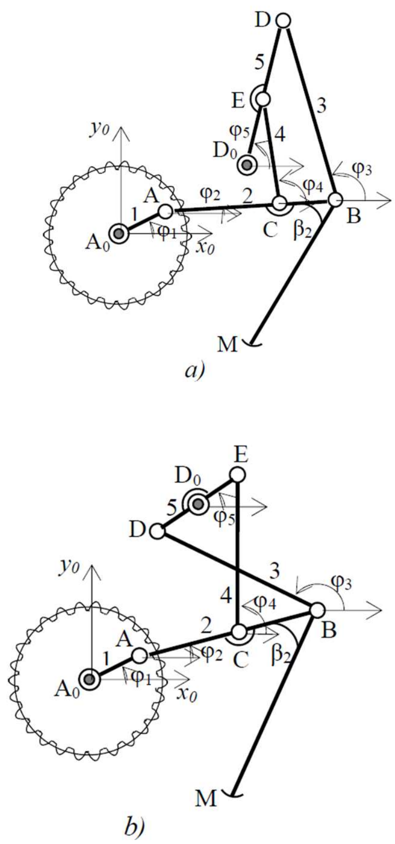


Fig. 3. Mechanism with **Te**t**r**adic chain **LTt** (2,3,4,5): variant 1 (*a*) and variant 2 (*b*)

Formula (8) shows that the degree of mobility M_b is given by the independent geometrical parameters. In this case, it is just one, represented by the angle φ_1 that positions the crank 1 (fig. 3).

The mobility of the mechanism is calculated by means of the general formula [2]:

$$M_b = \sum_{m=1}^{5} mC_m - \sum_{r=2}^{6} rN_r \qquad (9)$$

Let us identify in formula (9) the following symbols visible on the kinematic scheme:

m - mobility of the kinematic joint, $m \in (1,...,5)$;

C_m - number of the kinematic joints of class m;

r - rank of the space associated to the kinematic scheme $r \in (2,...,6)$;

N_r - number of the closed independent kinematic contours.

Observing the kinematic scheme (fig. 3), we deduce the numerical values:

$$m = 1, C_1 = 7;\ r = 3, N_3 = 2.$$

Introducing these data in the formula (9), we obtain:

$$M_b = 1 \times 7 - 3 \times 2 = 1 \qquad (10)$$

The calculation of the instantaneous positions of the driven kinematic elements (2, 3, 4, 5) implies the writing of the closing scalar equations of the two independent contours $(A_0ABDD_0A_0)$ and $(A_0ACED_0A_0)$:

$$l_2\cos\varphi_2 + l_3\cos\varphi_3 - l_5\cos\varphi_5 = x_{D_0} - l_1\cos\varphi_1 \quad (11)$$

$$l_2\sin\varphi_2 + l_3\sin\varphi_3 - l_5\sin\varphi_5 = y_{D_0} - l_1\sin\varphi_1 \quad (12)$$

$$l_2'\cos\varphi_2 + l_4\cos\varphi_4 - l_5'\cos\varphi_5 = x_{D_0} - l_1\cos\varphi_1 \quad (13)$$

$$l_2'\sin\varphi_2 + l_4\sin\varphi_4 - l_5'\sin\varphi_5 = y_{D_0} - l_1\sin\varphi_1 \quad (14)$$

In the equations (11) - (14), the following denotations have been used (fig. 3*a*):

$$A_0A = l_1; AB = l_2; AC = l_2'; BD = l_3; CE = l_4; D_0D = l_5; D_0E = l_5'.$$

Knowing the value of angle φ_1 (fig. 3a), using the system of the 4 non-linear equations (11), (12), (13), (14), we determine the angles $\varphi_2, \varphi_3, \varphi_4, \varphi_5$.

For the position of M, we obtain the Cartesian coordinates:

$$x_M = l_1\cos\varphi_1 + l_2\cos\varphi_2 + l_2''\cos(\pi + \varphi_2 + \beta_2) \quad (15)$$

$$y_M = l_1\sin\varphi_1 + l_2\sin\varphi_2 + l_2''\sin(\pi + \varphi_2 + \beta_2) \quad (16)$$

It has to be mentioned that the scalar equations (11) - (16) are the same in the variant 2 of the tetradic mechanism (fig. 3*b*).

For the variant 1 of the tetradic mechanism (fig.3a), a simulation along with a computer program have been made, tracing the reciprocating coupler curve by means of the fulcrum M (fig. 4).

3. SYNTHESIS OF THE TETRADIC MECHANISM

Let us consider the kinematic scheme of the tetradic mechanism in the variant 3 (fig. 5) where M belongs to the plane of the bar 3, so that the angle β_3 is constant.

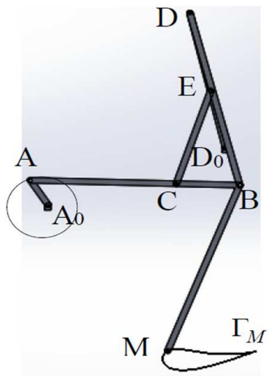


Fig. 4. Modelling the reciprocating coupler curve Γ_M

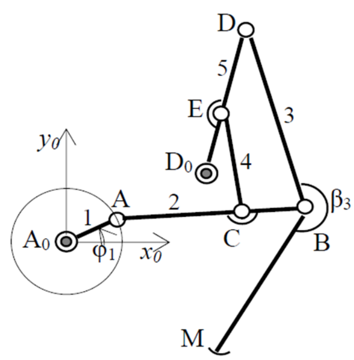


Fig. 5. Kinematic scheme of the mechanism in the variant 3 with the plane of the bar 3

For several numerical values imposed to the angle β_3 between 185° and 225° the reciprocating coupler curve Γ_M, has been traced, and whose configuration has clearly changed (fig. 6).

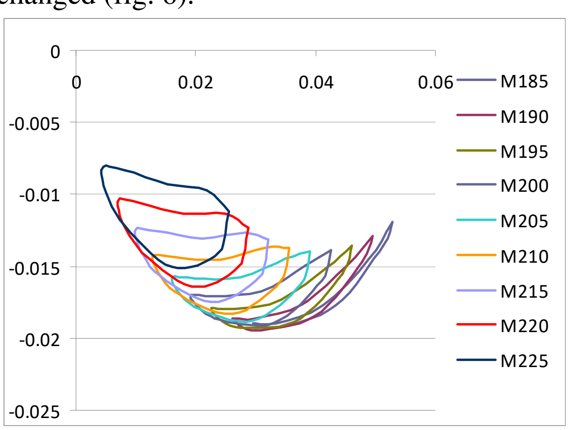


Fig. 6. The reciprocating coupler curve traced by M of the mobile robot leg

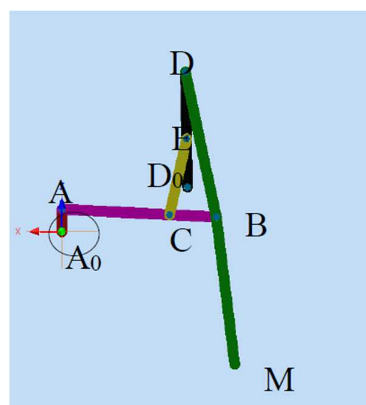
To trace the trajectories of the fulcrum M, we have gone through the following stages:

- writing the equations and solving them by means of MathCAD software;
- exporting the results (M point coordinates for each value of the angle β_3) into an Excel file;
- plotting the graphs that correspond to the trajectories of the point M by means of Excel software.

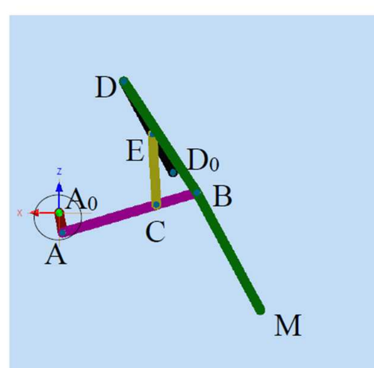
The Excel software automatically determines the best way to represent the data on the graph, unlike the MathCAD software that plots the graphs by uniting the points whose coordinates are specified for each of the axes (the values of the angle φ_{1k} increase by 45°).

We should notice (fig. 6) that only the first four closed curves correspond to an optimal trajectory of the fulcrum M ($M_{185}, M_{190}, M_{195}, M_{200}$).

For the first value of the angle β_3 (185^0), we have modeled and simulated the designed mechanism (fig. 7) in two positions of the crank ($\varphi_1 = 90^0, 270^0$).



a)



b)

Fig. 7. Screen captures in Inventor software for $\varphi_1 = 90°$ (*a*) and $\varphi_1 = 270°$ (*b*)

4. SYNTHESIS OF THE TETRADIC CROSSED-BAR MECHANISM

Let us now consider the solution of the tetradic chain LTt (2,3,4,5) where the bars 3 and 4 are crossed (fig. 8), having the motion in parallel planes. Two kinematic options are taken into consideration: the fulcrum M belongs to the bars 2 or 3. These are highlighted by the angle β_2 (fig. 8*a*) and angle β_3 (fig. 8*b*).

The synthesis procedure is the same, it consists of the simulation of the mechanism by means of the MathCAD software for several values of the angle β_2 and angle β_3.

Therefore, along a kinematic cycle (considering the values of the angle φ_1 that increases by 45° each time), we have obtained the Cartesian coordinates of the point M.

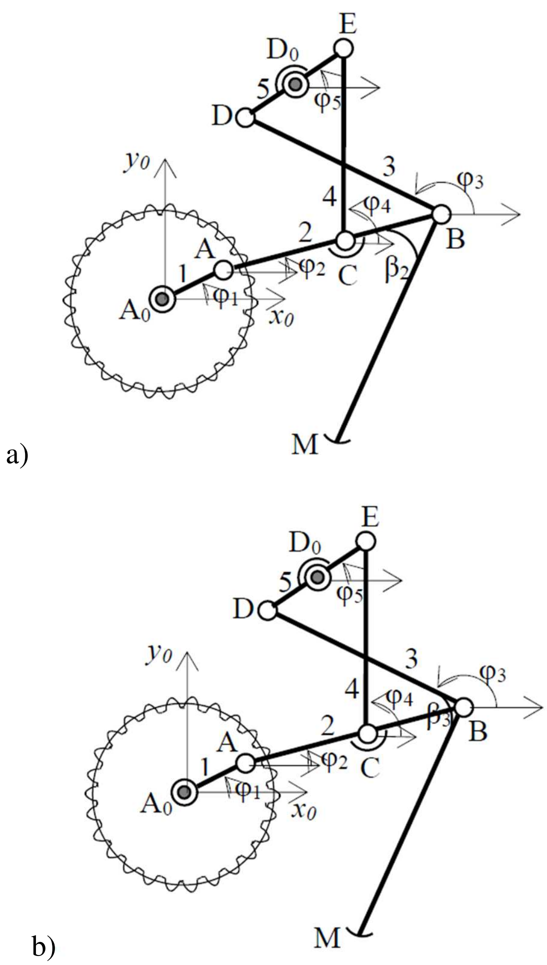


Fig. 8. Kinematic schemes of the tetradic crossed-bar mechanisms.

By means of the Excel software, we deduce the reciprocating coupler curves of the point M in the plane of the bar 2, for several values of the angle β_2 (fig. 9).

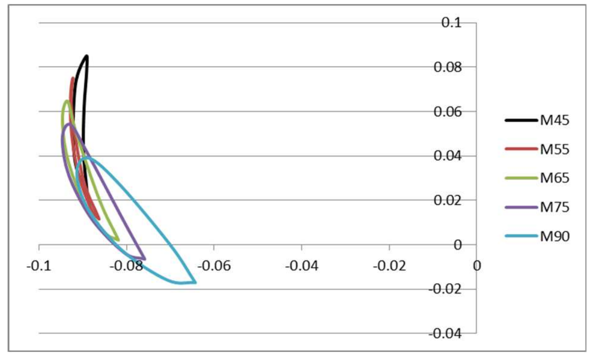


Fig. 9. The reciprocating coupler curves described by the point M

We notice that the last curve (point M90°) is the best solution for the leg mechanism of the mobile robot. We also mention that the solution of the mechanism in which the point M belongs to the bar 3 (fig. 8*b*) is not suitable for the conditions imposed by the movement of the quadruped robot.

5. CONCLUSIONS

The two solutions using a triad or a tetrad kinematic chain presented in the paper are suitable to be implemented in the structure of the legged mobile robots. The kinematic analysis applied on these two major solutions is the first step in designing the locomotion system of the quadruped robot.

Thus, in the case of the triad mechanism we have proposed two solutions, triangle-shape or bar, for the kinematic element that has three joints. Then, in the case of the tetrad mechanism we have proposed also two solutions: usual or crossed-bar deformable contour.

Further, we have also performed the mechanism synthesis on two tetradic solutions:

1. chain using a usual contour but with a slightly difference in its structure i.e. fixing together the point M with the bar 3 (fig. 5) instead of 2 (fig. 3*a*);
2. chain using a crossed-bar contour with the point M fixed together with the bar 2 (fig. 8*a*).

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Sinteza mecanismului complex al robotului mobil patruped

Lucrarea prezintă două noi scheme cinematice ale mecanismelor planare articulate utilizate ca picioare într-un robot mobil patruped. Aceste mecanisme sunt formate din 5 elemente cinematice mobile în comparație cu mecanismul Jansen, care este format din 7 elemente. Mecanismele celor patru picioare ale robotului patruped sunt acționate de un singur motor electric prin intermediul angrenajelor cilindrice. Structura schemelor de mecanisme este formată din lanțuri cinematice cu patru elemente articulate triadice sau tetradice. Pentru fiecare dintre schemele cinematice propuse, au fost luate în considerare mai multe opțiuni geometrice privind poziția articulațiilor și a punctului de sprijin. Mobilitatea fiecărei scheme cinematice este evaluată și este prezentată o metodă de sinteză geometrică pentru generarea punctului de sprijin al curbei cuplelor reciproce.

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