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MECHANICAL VIBRATIONS STUDY PRODUCED BY MACHINE-TOOLS OVER A HUMAN OPERATOR, WHO SERVES IT. PART IIb: INTEGRATED SOLUTION OF DIFFERENTIAL EQUATIONS SYSTEM FOR FEMALE OPERATOR

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Abstract: *The work is part of the doctoral thesis, in which the action of vibrations on an operator serving a machine tool during the production process is studied. The paper carries out the theoretical study of the transmission of vibrations from the machine tool through the operator's feet, which is placed on a pedestal on which there is a rubber mattress. Consider vertical vibrations, through both legs, up to the abdomen. In the second part of the paper noted IIb, we start from the system of differential equations, which characterizes the real system with 13 degrees of freedom, of the feet of the female operator, which is integrated through Matlab and is graphically represented with the mechanical characteristics of the female operator, considered a right-handed user*

Key words: *vibrations simulation, female operator, integration of differential equations system.*

1. INTRODUCTION

The present paper, marked IIb, together with papers I and Iia, form a set of three papers, through which the theoretical study on the action of vibrations on the human operator, who serves a machine tool, with a numerical command, is carried out.

In the first part of this study (Part I) the mechanical system of the operator is established, which is in the vicinity of the machine tool, placed on a rubber mat, and the vibrations propagate through both legs, up to the abdomen which, together with the upper part of the body, is considered to form a rigid solid [6]. Here the mechanical characteristics are highlighted, the mechanical scheme of each constituent element is drawn up, the differential equations are written and the system of differential equations denoted 13DEGOFR is formed, being a system of 13 differential equations of the order, with 13 unknowns – the generalized coordinates of the 13 component parts, into which the material system assimilated to the human operator is considered divided.

The study will be carried out on two human operators, of medium age, who have experience in servicing machine tools with numerical control. They are:

a. A 46-year-old male operator, weighing 83 kg and 187cm tall;
b. A 45-year-old female operator, weighing 78 kg and 170 cm tall.

In this part, noted IIb, the study contains only the female integrated differential equations system. The "woman" operator is considered right-handed.

2.THE NOTATIONS FOR THE FEMALE BODY VIBRATION

A human operator, who serves a machine tool with numerical control, is required to vibrate, to supervise the processing process, placed on an elastic rubber support, positioned in front of the machine at a distance appropriate to the supervision and in which he can perform his tasks [1], [2].

She is positioned on both legs, and the vibration is transmitted evenly through them, the body is assimilated to a rigid solid [3], [4], [5].

The body segments referred to in this theoretical study are given in the generalized

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coordinates: the foot ($z_{1S/D}$), ankle ($z_{2S/D}$), calf ($z_{3S/D}$), knee joint ($z_{4S/D}$), femur ($z_{5S/D}$), hip joint ($z_{6S/D}$), abdomen – which together with the upper body form an assembly assimilated to a rigid solid (z_7).

The general characteristics of the material system are given by:

m – the mass of the element obtained by discretizing the real system;

k – the elasticity constant of the element;

c – the damping constant of the element;

M – is he mass of the upper body of legs;

S - refers to the left leg;

D – refers to the right leg of the human operator.

The external values are [6]:

k_0 = 310,000 N/m, elasticity constant of rubber;

c_0 = 3,900 Ns/m, the damping constant of the rubber.

The law of motion of the point of stress to vibrations of the organism is symmetrical with respect to the two legs and presents the law:

$$u = u(t) = M\omega^2 u_0 \sin(\omega t) \quad (1)$$

Each bodily element of the 'male' operator is isolated, the active forces, the connection, the inertia, which act on the selected element are considered, then the principle of action and reaction is applied to the passage from one element to another. The forces are considered positive if they act in the direction of the coordinate and negative otherwise.

3. THE INTEGRATION CONDITIONS FOR THE FEMALE OPERATOR

The 13 segments of the "woman" operator, highlighted in the paper [6], of the 13 body segments, in which the body of the "female" operator subjected to the oscillations of the numerically controlled machine tools that she serves has been segmented. The system of differential equations is found in [6] the work preceding it, in the sequence of relations related to body segments.

The mechanical system, assimilated to the "female" operator, is requested from the outside by a harmonic force, due to the rotational movement of the main shaft of the machine tool, shown in the relation (1). This force can be easily represented graphically and was made in the complementary work, noted IIa.

3.1. MATLAB script

The integration of the system of differential equations with 13 unknowns, for the 'female' operator was done through the MATLAB script, which is presented in detail in Figure 1.

```
%% System parameters
n = 10000;        % rpm, spindle speed
u0 = -5e-3;       % m, Initial deformation
of the rubber mat
k0 = 310000;      % N/m, elasticity
constant of rubber mat
c0 = 3900;        % Ns/m, Rubber mat
damping constant
% Angular Frequency Calculation
omega = 2 * pi * n / 60; % rad/s
% Body Segment Masses
m1F = 0.67;
m2F = 0.46;
m3F = 3.63;
m4F = 1.56;
m5F = 6.24;
m6F = 5.54;
m7F = 41.8;
MF = 2 * (m1F + m2F + m3F + m4F + m5F +
m6F) + m7F; % Total weight of the operator
% Elasticity and damping constants
k1FS = 90000;
k2FS = 105000;
k3FS = 100000;
k4FS = 110000;
k5FS = 60000;
k6FS = 100000;
k1FD = 110000;
k2FD = 120000;
k3FD = 120000;
k4FD = 120000;
k5FD = 50000;
k6FD = 105000;
k7F  = 105000;
c1FS = 910;
c2FS = 1100;
c3FS = 11500;
c4FS = 2320;
c5FS = 1250;
c6FS = 2000;
c1FD = 850;
c2FD = 1000;
c3FD = 10000;
c4FD = 2200;
c5FD = 1100;
c6FD = 1800;
c7F  = 1600;
```

Fig. 1. MATLAB Input Data for the Female' Operator

4. THE SOLUTION OF INTEGRATING THE 13DEGOFR DIFFERENTIAL EQUATION SYSTEM FOR THE "FEMALE" OPERATOR

The result of the integration of the system of 13 differential equations with 13 unknowns – the generalized coordinates of the bodily sequences, is presented for each segment, through accelerations, velocities and displacements.

The integration was done in the time interval 0 – 5 seconds, to highlight the damping of the free vibrations of the material system and the continuity of the forced vibrations, under the action of the operator's request to the vibrations produced by the machine tool during operation, when idling.

4.1. The accelerations of the "female" operator

The accelerations of the vibrations simulated by MATLAB are presented below, which enter the legs of the "female" operator and propagate to the hip, where they are summed up in the abdomen for both legs. It is worth noting how there are differences between the two legs, because the operator is considered right-handed.

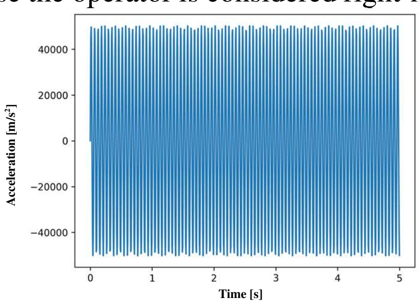

Fig. 2. Acceleration of the left foot

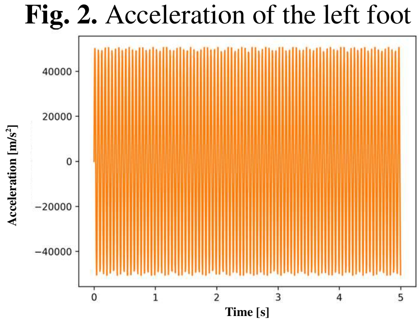

Fig. 3. Acceleration of the right foot

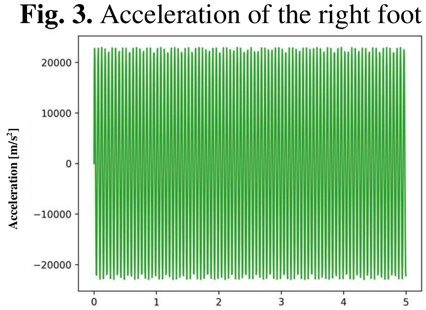

Fig. 4. Acceleration of the left ankle

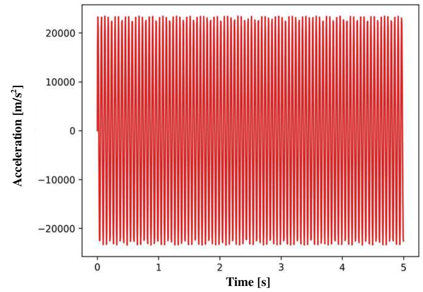

Fig. 5. Acceleration of the right ankle

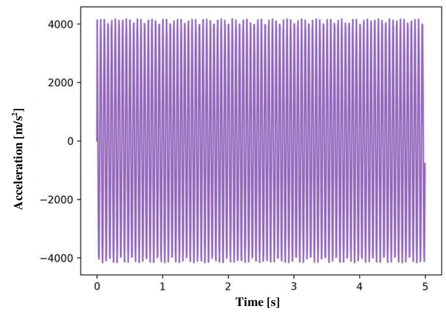

Fig. 6. Acceleration of the left leg

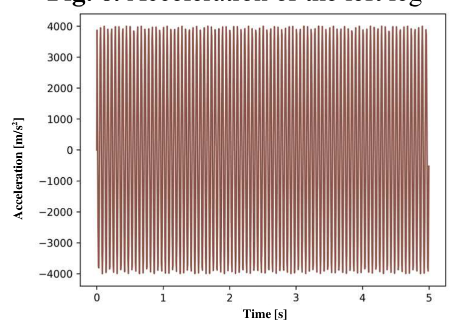

Fig. 7. Acceleration of the right leg

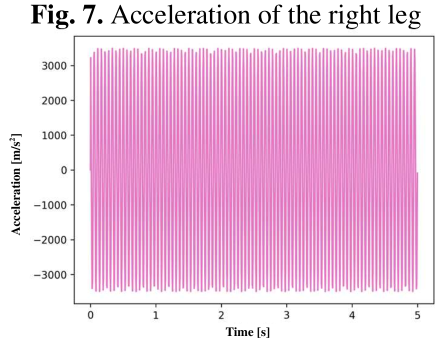

Fig. 8. Acceleration of the left knee

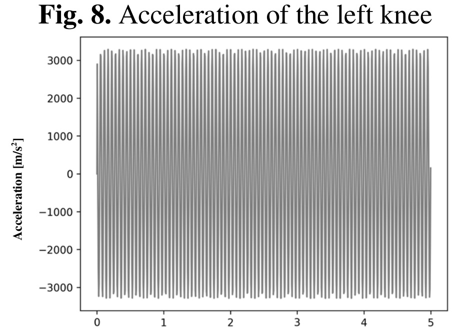

Fig. 9. Acceleration of the right knee

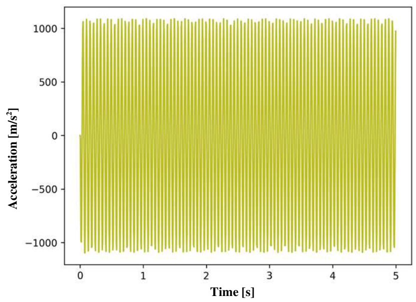

Fig. 10. Acceleration of the left femur

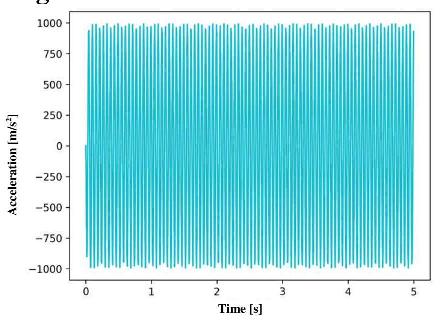

Fig. 11. Acceleration of the right femur

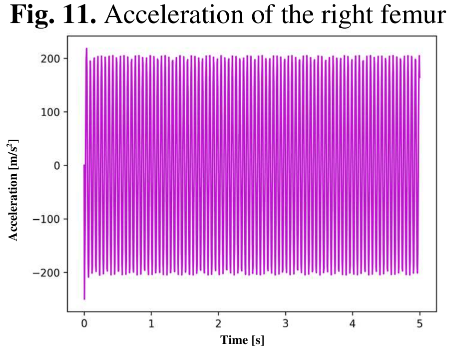

Fig. 12. Acceleration of the left shalt/hip

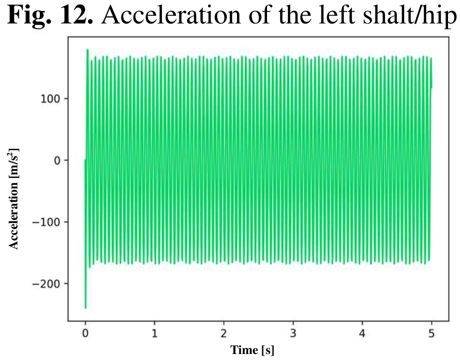

Fig. 13. Acceleration of the right shalt/hip

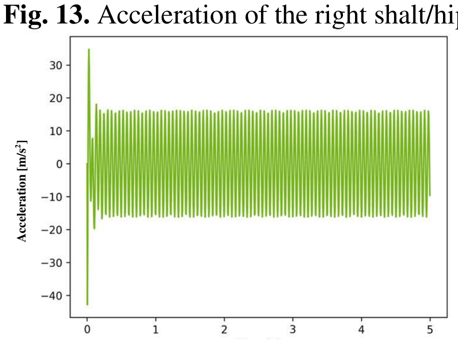

Fig. 14. Acceleration of the operator's body

4.2. The velocities of the "female" operator

The MATLAB programming environment, by integration in the interval 0 - 5 seconds, gives the simulation of the speed required by the segments of the feet of the 'female' operator, requested to vibrations by a numerical control machine tool, during operation, when idling.

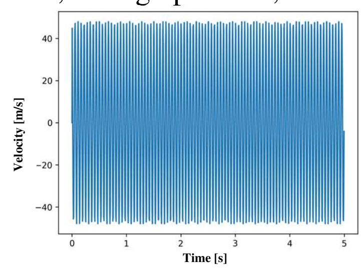

Fig. 15. Velocity of the left foot

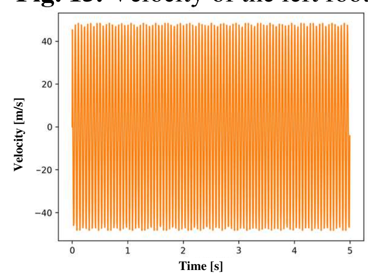

Fig. 16. Velocity of the right foot

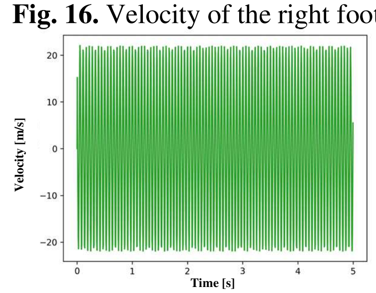

Fig. 17. Velocity of the left ankle

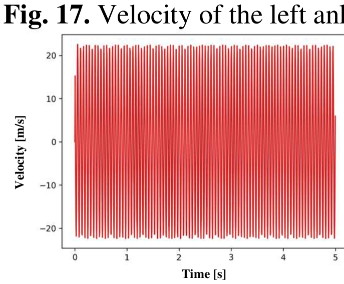

Fig. 18. Velocity of the right ankle

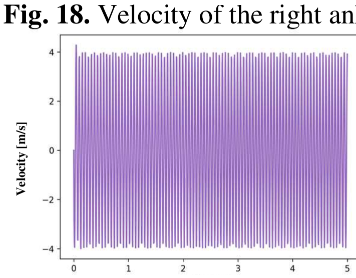

Fig. 19. Velocity of the left leg

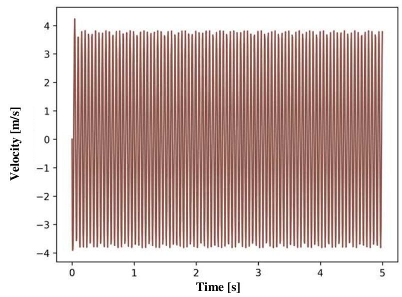

Fig. 20. Velocity of the right leg

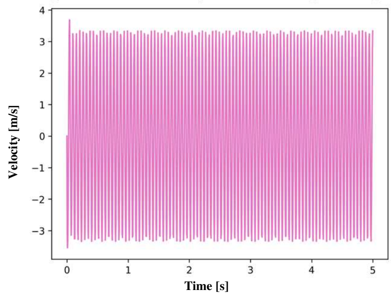

Fig. 21. Velocity of the left knee

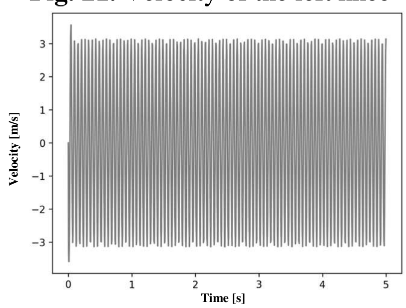

Fig. 22. Velocity of the right knee

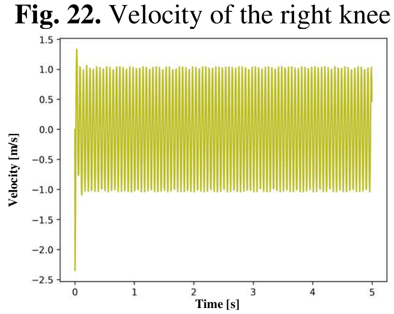

Fig. 23. Velocity of the left femur

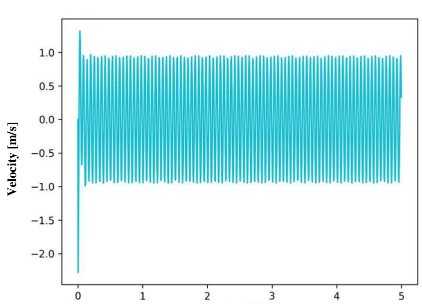

Fig. 24. Velocity of the right femur

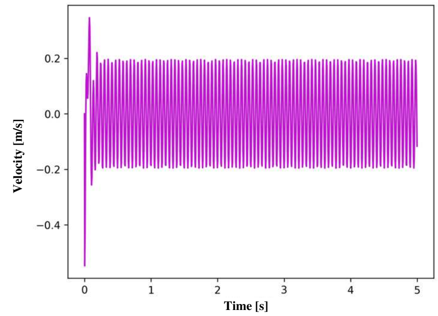

Fig. 25. Velocity of the left shalt/hip

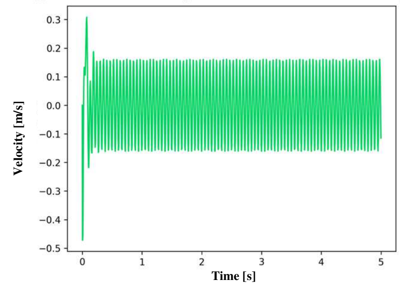

Fig. 26. Velocity of the right shalt/hip

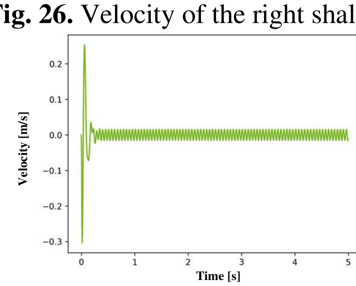

Fig. 27. Velocity of the operator’s body

4.3. The displacement of the "female" operator

The action of vibrations on the human operator has a plausible explanation by analyzing the displacements of the constituent elements, by simulating them produced by the machine tool, when idling. It is the possible moment when the operator is in the vicinity of the machine tool.

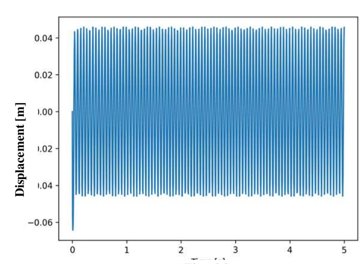

Fig. 28. Displacement of the left foot

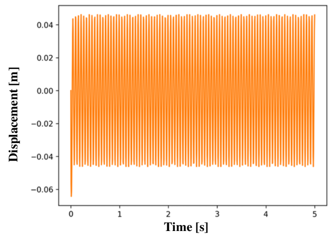

Fig. 29. Displacement of the right foot

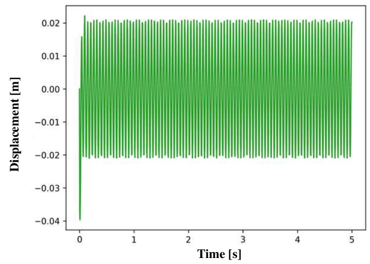

Fig. 30. Displacement of the left ankle

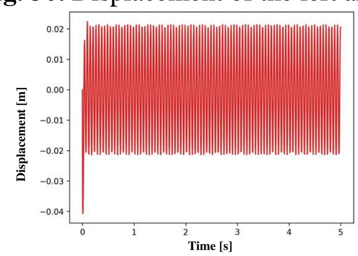

Fig. 31. Displacement of the right ankle

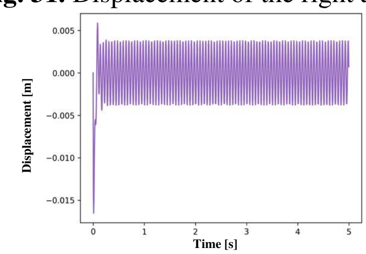

Fig. 32. Displacement of the left leg

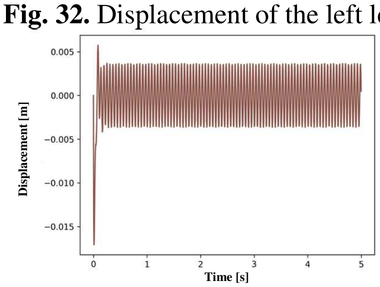

Fig. 33. Displacement of the right leg

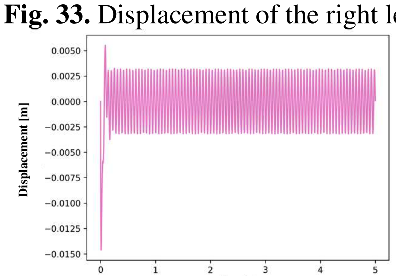

Fig. 34. Displacement of the left knee

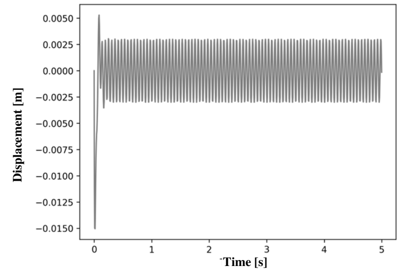

Fig. 35. Displacement of the right knee

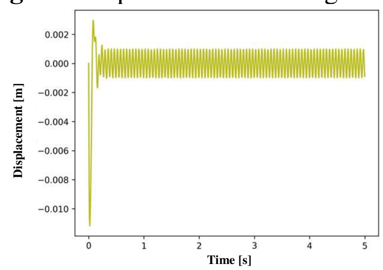

Fig. 36. Displacement of the left femur

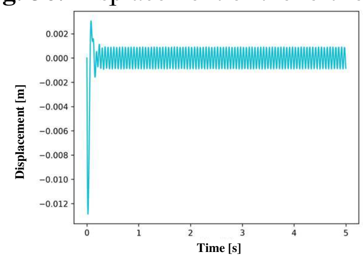

Fig. 37. Displacement of the right femur

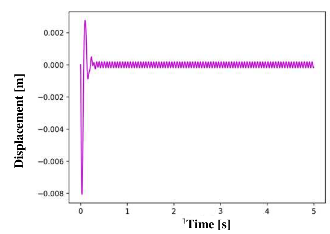

Fig. 38. Displacement of the left shalt/hip

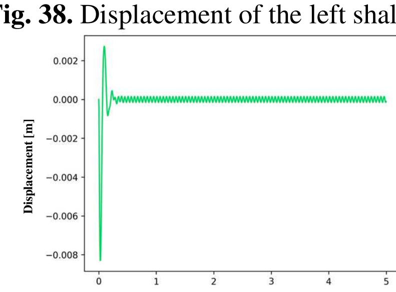

Fig. 39. Displacement of the right shalt/hip

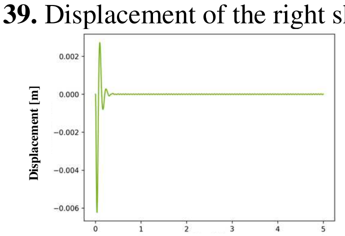

Fig. 40. Displacement of the operator’s body

5. CONCLUSIONS

This paper studies the action of vibrations, which act on the operator, who serves a machine tool, with numerical control during the production process, without removing the material, when idle.

The resulting diagrams for the "female" operator are successively analyzed differentiated by acceleration, speed and displacement

A. The ordinal magnitude for accelerations is amplified by 10^{-4}, compared to the real values. This method was applied to measure the decrease on the component elements of the human operator's legs.
   1. For the right-handed operator – in this case – the simulated values of the right foot are noticeably lower than those of the left foot.
   2. The acceleration of the vibration accumulated in the abdomen is negligible.
   3. At the acceleration of the cumulative vibration, the free vibrations of the system can be observed in the first fractions of a second, after which the forced vibration is dated to the vibrations produced by the machine tool.

B. For speeds it can be said that:
   1. The velocities simulated by MATLAP on the legs of the 'female' operator have a superunit order of magnitude in the lower parts and subunit starting with her knee.
   2. Starting from the femur, the free vibrations are highlighted in the first fractions of a second, which are damped and only the forced vibration remains, due to the operation of the machine tool.

C. The graphic representations of the displacements of the legs segments of the "female" operator are conclusive, because at high values they can cause organ dislocations, or even changes in their functioning. The same order of magnitude of the vibrations formed by amplification by 10^{-4} is maintained, due to the transmission through the foundation, which diminishes the vibrations of the MU in the construction. From the analysis of the representatives, the following can be found:
   1. The mechanical diagram of the material system assimilated to the operator's feet, which serves the machine tool, is drawn up in sequence: foot, ankle, calf, knee, femur, hip to the abdomen.
   2. The displacements of the fragmentary components are subunit from the soles of the feet.
   1. The same differentiation of the response to the forced request is maintained, for the "right-handed" operator, with low values on the right leg. Compared to the left one.

D. The movement is transmitted through both legs, from the sole of the foot to the hip / thigh.

E. The upper part of the body sums up the vibrations transmitted through the legs; the entrance being made in the abdomen.

F. The abdomen together with the upper part of the body is considered to form a rigid solid, which reacts to the vibrations.

G. The paper contributes to the study of vibrations acting on the operator, who serves a numerically controlled machine tool, without causing health damage to the human body during processing, when the machine tool is idling.

1. It is not recommended for the position of human operator to be in the vicinity of the machine, during the machining process, under load.

6. REFERENCES

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[6] Adela Bianca CRACIUN (HSU), Ovidiu Aurelian DETESAN, Mariana ARGHIR, Mechanical Vibrations Study Produced by Machine-Tools Over Human Operator, Who Serves It. Part I:Operator Mechanical Characteristics and the System of Differential Equations, Acta Technica Napocensis, Series: Applied mathematics, Mechanics, and Engineering, Vol. 68, Issue I, pag. 173-180, March, 2025.

Studiul vibrațiilor mecanice produse de o mașină-unealtă asupra unui operator uman, care o servește. Partea IIb: integrarea sistemul ecuatiilor diferentiale pentru operatorul femeie

Rezumat*: Lucrarea este parte a tezei de doctorat, in care se studieaza actiunea vibratiilor asupra unui operator, ce deserveste o masina-unelta in timpul procesului de productie. In lucrare se realizeaza studiul teoretic al transmiterii vibratiilor de la masina-unealta prin picioarele operatorului, care este asezat la local de nunca pe un postament, pe care se gaseste o saltea de cauciuc. Se considera vibratiile verticale, prin ambele picioare, pana la abdomen. In cea de a doua parte a lucrarii notata IIb, se porneste de la sistemul ecuatiilor diferentiale, care caracterizeaza sistemul real cu 13 grade de libertate, ale picioarelor operatorului femeie, care se integreaza prin Mathlab si se reprezinta grafic cu caracteriasticile mecanice ale operatorului femeie, considerat utilizator dreptaci*

Cuvinte cheie: *simularea vibratiilor, operator uman femeie, integrarea sistemul ecuatiilor diferentiale.*

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