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MULTI-OBJECTIVE OPTIMIZATION OF LASER ENGRAVING PARAMETERS USING GREY RELATIONAL ANALYSIS

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Abstract Laser engraving is an advanced technique for precise application of markings on the surface of molds, offering advantages such as high accuracy, wear resistance, execution speed and the ability to mark complex surfaces. This environmentally friendly method ensures good long-term readability without the use of chemicals. The study presents multi-objective optimization of engraving parameters, using experimental design and gray relational analysis to simultaneously improve surface roughness, engraving depth and processing time. By analyzing seven input parameters, the research identified the optimal settings and the factors with the greatest influence on the process, demonstrating the effectiveness of the method and opening new perspectives for improving the process performance.

Keywords: Laser engraving, roughness, optimization, grey relational analysis.

1. INTRODUCTION

Laser Beam Machining (LBM) is an advanced, non-contact thermal processing technique applicable to a wide range of materials, utilizing focused laser beams to melt and vaporize unwanted material. This method is ideal for cutting complex geometric shapes and creating small holes. In recent years, researchers have made significant efforts to enhance the performance of LBM by analyzing various factors that influence the quality of the final product and exploring ways to optimize laser parameters, materials, and operating conditions. Experimental and theoretical studies have demonstrated that the proper selection of these parameters can lead to significant improvements in system performance. Article [1] provides a comprehensive review of research conducted in the field of LBM, presenting various modeling and optimization techniques for determining the best laser cutting conditions, and concludes with a discussion on future research directions in this domain.

Article [2] describes a study that employs the Taguchi method to optimize the manufacturing process of photolithographic masks for LCD screens. The research focuses on the laser

engraving of iron oxide-coated glass using a Q-switched Nd: YAG laser, analyzing the influence of five key parameters on the engraving line width: pulse repetition rate, focal distance, average laser power, beam expansion ratio, and engraving speed. The experiments [3] were conducted using an L16 orthogonal array, and the analysis of the results enabled the identification of optimal parameter values for achieving a minimal cutting line width. This contributes to improving precision in the fabrication of photolithographic masks for LCD screens.

The study [4] significantly advanced our understanding of the relationship between process parameters and the characteristics of items created using laser powder bed fusion by employing a multidisciplinary approach. Through the integration of artificial neural network modeling and the Taguchi experimental methodology, the authors created a reliable prediction model that can precisely estimate part qualities depending on process parameters.

The surface roughness in the stainless-steel cutting process is greatly influenced by process variables such as gas pressure, cutting speed, and laser power, according to a study [5]. A direct correlation was observed between the laser

power and roughness, and an inverse relationship was observed between the cutting speed, gas pressure, and roughness. These results highlight the importance of optimizing the process parameters to achieve superior cut surface quality.

The study [6] utilized the Taguchi approach and gray relational analysis for the multi-objective optimization of process parameters and cutting conditions in machining processes from an optimization standpoint, including machining with cutting fluid, cold compressed air, and dry machining. Experiments utilized an L18 orthogonal array comprising five input parameters: cutting speed, feed per tooth, radial depth, cutting duration, and cutting environment. The aim was the concurrent optimization of material removal rate, surface roughness, tool wear, and cutting forces. Grey relational analysis determined the ideal parameter configurations, but variance analysis indicated that cutting time, feed per tooth, and cutting environment are the most significant elements affecting performance.

To identify the relationship between multiple factors, a study [7] proposed a grey relational analysis (GRA) method based on variations in fluctuating and cross-sequences, both over time and across intervals. Relative angle modification was used to analyze the variations, and the optimal cycles were determined using time-delay models. The reliability of the method is validated through case studies on air quality, and the results show that GRA for cross sequences effectively identifies relationships between fluctuating sequences with minimal impact from the time lag.

Rapid prototyping methods, including 3D printing, have become increasingly popular owing to additive manufacturing, which creates objects by layering materials, thereby saving both time and cost. Research [8] employed grey relational analysis (GRA) to optimize the parameters influencing the tensile strength, flexural strength, and wear resistance in the FDM process. This study examines the effects of layer height, extruder temperature, infill percentage, and printing speed based on an L16 orthogonal array. GRA optimization identifies the ideal levels of these factors, whereas ANOVA determines the contribution of each

parameter. The results confirm that the regression analysis accurately predicts the grey relational grade (GRG), aligning well with the experimental data.

2. EXPERIMENTAL SETUP

Equipment performance influences increasing productivity in a competitive industrial environment. Identifying and understanding the factors that affect operations are a constant challenge. To fully exploit the potential of the equipment, an in-depth analysis of the internal processes and interaction of various parameters is required. This analysis can be conducted using analytical methods, experimental approaches, or a combination of both.

This study aims to determine the values of the influencing factors for optimizing three objective functions using Grey Relational Analysis (GRA). The objectives pursued were minimizing roughness (Ra) and time (time) while maximizing the penetration depth (depth).

The influencing factors and their variation ranges are presented in Table 1 [10].

Table 1

The influence factors of the engraving process.

Factors	Symbol	Units	Numerical values		Coded	
			min	max	min	max
Power	A	W	20	60	1	2
Speed	B	mm/s	300	1500	1	2
Frequency	C	Hz	20000	25000	1	2
No of cuts	D	-	20	80	1	2
Stepover	E	mm	0.01	0.03	1	2
Focal	F	mm	240	250	1	2
Strategy	G	-	circular	liniar	1	2

The XTL-FP50 fiber laser marking machine (Fig. 1), equipped with state-of-the-art IPG Raycus laser technology, offers an exceptional lifespan of over 100,000 h, without requiring maintenance or consumables. The intuitive EZCAD software enables easy operation and advanced customization of the marking parameters, ensuring high-quality results.

Figure 2 shows the engraving of the tests for the study.



Fig. 1. XTL-FP50 fiber laser marking machine.

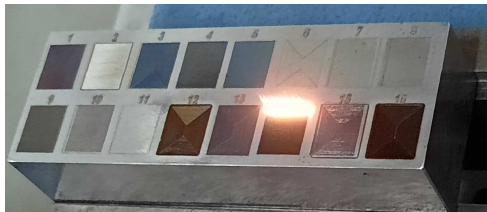


Fig. 2. Laser engraving of tests.

Penetration measurement was performed using Vertex 320 equipment (Fig. 3), which is a versatile and precise measurement system equipped with a high-resolution camera. It provides fast and detailed three-dimensional measurements for a wide range of parts, from the simplest to the most complex. Advanced features, such as programmable zoom and automatic edge detection, combined with the intuitive InSpec Metrology® software, make Vertex 320 an indispensable tool for ensuring quality control in production.

The SJ-210 Mitutoyo portable roughness tester (Fig. 4) is a measurement instrument designed to accurately assess surface roughness. It can measure a wide range of roughness parameters, making it suitable for various industrial applications.

Grey Relational Analysis (GRA) is a branch of systems theory that focuses on modeling and analyzing complex systems with incomplete information [9].

Based on the concept of a gray system," which lies between a white system (with complete information) and a black system (with completely unknown information), GRA employs a series of mathematical tools to handle uncertainty and extract useful insights from data.



Fig. 3. Determination of penetration depth.



Fig. 4. Roughness measurement.

One of the central concepts of GRA is the "grey relational grade," which measures the degree of association between two variables. This measure enables a quantitative evaluation of causal relationships and the identification of key factors influencing a system's behavior. If minimization is aimed at data normalization, use relation:

$$x_{ij} = \frac{\max(y_{ij}) - y_{ij}}{\max(y_{ij}) - \min(y_{ij})} \quad (1)$$

where y_{ij} represent the original values

The normalized value of the initial sequence for which maximization is desired is determined with relation:

$$x_{ij} = \frac{y_{ij} - \min(y_{ij})}{\max(y_{ij}) - \min(y_{ij})} \quad (2)$$

The grey relational coefficient for criterion j measures the similarity between the values x_{0j} (the reference sequence value for criterion j) and x_{ij} (the value of sequence x_i for criterion j). It is calculated separately for each criterion and reflects the local similarity between the reference and analyzed sequences.

$$\gamma(x_{0j}, x_{ij}) = \frac{\Delta_{\min} + \xi \Delta_{\max}}{\Delta_{ij} + \xi \Delta_{\max}}, \quad (3)$$

$i = 1, 2, \dots, m \text{ and } j = 1, 2, \dots, n$

$$\Delta_{ij} = |x_{0j} - x_{ij}|. \quad (4)$$

$$\Delta_{\min} = \min \{ \Delta_{ij}, i = 1, 2, \dots, m; j = 1, 2, \dots, n \}. \quad (5)$$

$$\Delta_{\max} = \max \{ \Delta_{ij}, i = 1, 2, \dots, m; j = 1, 2, \dots, n \} \quad (6)$$

$\gamma(x_{0j}, x_{ij})$ - gray relational coefficient between element and element;

Δ_{ij} - (the absolute difference between the reference value and the compared one);

$\Delta_{\min}$ - the minimum value among all Δ_{ij} ;

$\Delta_{\max}$ - the maximum value among all Δ_{ij}

ξ - the distinctive coefficient, takes values (0,1), usual 0.5. The smaller ξ it is, the greater the ability to distinguish.

In the grey relational analysis, to evaluate the relationship between a reference sequence (x_0) and other sequences (x_i) based on the grey relational coefficients ($\gamma(x_{0j}, x_{ij})$) and the weights assigned to each criterion (w_j), the grey relational grade ($\Gamma(X_0, X_i)$) is used.

The gray relational degree is an aggregate score that indicates how close the sequence X_i is to the reference sequence X_0 .

$$\Gamma(X_0, Y_0) = \sum_{j=1}^n w_j \gamma(x_{0j}, x_{ij}), \quad (7)$$

$i = 1, 2, \dots, m$

$$\sum_{j=1}^n w_j = 1. \quad (8)$$

w_j represents the weight associated with criterion j . These weights reflect the importance of each criterion in gray relational analysis and usually depend on the judgment of the decision maker. The sum of all weights is equal to 1;

n - total number of criteria (evaluation factors).

m - The total number of sequences (alternatives) that are compared to the reference sequence x_0 .

An important tool in Grey Relational Analysis for evaluating and comparing the Grey Relational Grade (GRG) obtained for different levels of experimental factors is the Response Table for Grade. The Response Table for Grade is used to calculate the average GRG for each level of every factor, helping to identify the optimal levels. The formula for calculating the average is as follows:

$$\bar{\Gamma}_k = \frac{\sum_{i \in k} \Gamma(X_0 X_i)}{n_k}. \quad (9)$$

The prediction of the optimal value of the Grey Relational Grade (GRG) after determining the optimal levels of the factors was calculated using the following

$$\hat{\gamma} = \gamma_t + \sum_{i=1}^p (\gamma_i - \gamma_t). \quad (10)$$

$\hat{\gamma}$ - predicted value of Gray Relational Grade (GRG) under optimal conditions;

γ_t - overall mean of GRG values for all experimental combinations;

γ_i - the GRG value for the optimal level of the factor i ;

p - the total number of factors included in the experiment.

After determining the optimal levels from the "Response Table for Grade," this formula helps verify whether the predicted performance aligns with the experimentally obtained values.

3. RESULTS

An experimental matrix with 16 experiments was created to investigate the impact of the seven factors mentioned earlier on the laser-engraving process (Table 2). Each factor was studied on two levels. The matrix was constructed using a fractional factorial design 2^{7-3} to ensure an efficient coverage of the experimental space [10].

Table 2

The experimental matrix.

Run	Power	Speed	Frequency	No of cuts	Stepover	Focal distance	Strategy	Ra	Depth	Time
	W	mm/s	Hz	-	mm	mm	-	μm	mm	min
	A	B	C	D	E	F	G	R1	R2	R3
1	60	1500	25000	80	0.03	250	liniar	1.533	0.0152	2.29
2	20	300	25000	20	0.03	250	liniar	0.946	0.0144	3.01
3	60	1500	25000	20	0.03	240	circular	1.467	0.0178	0.381
4	20	1500	25000	80	0.01	250	circular	0.526	0.003	7.379
5	20	1500	20000	20	0.03	250	circular	0.453	0.0138	0.38
6	20	300	20000	20	0.01	240	circular	0.504	0.0105	8.4
7	20	1500	20000	80	0.03	240	liniar	0.648	0.0098	2.29
8	20	300	25000	20	0.01	240	liniar	0.545	0.0027	8.56
9	60	300	20000	80	0.03	250	circular	2.214	0.212	11.3
10	60	300	20000	20	0.03	240	liniar	0.591	0.0198	3.01
11	20	300	20000	80	0.01	250	liniar	0.652	0.004	34.4
12	60	1500	20000	20	0.01	250	liniar	0.403	0.0113	1.49
13	60	300	25000	20	0.01	250	circular	4.53	0.1673	8.59
14	20	300	25000	80	0.03	240	circular	0.614	0.0151	11.22
15	60	1500	20000	80	0.01	240	circular	0.702	0.0302	7.38
16	60	300	25000	80	0.01	240	liniar	1.155	0.0919	35.45

Table 3 shows the normalized values for the three objective functions (Roughness, Depth and Time).

Table 4 shows the deviation sequence.

Table 3

Normalized values for objective functions.

Run	Roughness	Depth	Time
	μm	mm	min
1	0.7262	0.0597	0.9453
2	0.8684	0.0559	0.9247
3	0.7422	0.0721	1.0000
4	0.9702	0.0014	0.8005
5	0.9906	0.1161	0.9997
6	0.9755	0.0373	0.7711
7	0.9406	0.0339	0.9453
8	0.9656	0.0000	0.7665
9	0.5612	1.0000	0.6884
10	0.9544	0.0817	0.9247
11	0.9397	0.0062	0.0299
12	1.0000	0.0411	0.9681
13	0.0000	0.7864	0.7657
14	0.9489	0.0592	0.6907
15	0.9276	0.1314	0.8002
16	0.8178	0.4262	0.0000

Table 4

Deviation sequence.

Run	Roughness	Depth	Time
	μm	mm	min
1	0.2738	0.9403	0.0547
2	0.1316	0.9441	0.0753
3	0.2578	0.9279	0.0000
4	0.0298	0.9986	0.0003
5	0.0094	0.8839	0.0006
6	0.0245	0.9627	0.2289
7	0.0594	0.9661	0.0547
8	0.0344	1.0000	0.2335
9	0.4388	0.0000	0.3116
10	0.0456	0.9183	0.0753
11	0.0603	0.9938	0.9701
12	0.0000	0.9589	0.0319
13	1.0000	0.2136	0.2343
14	0.0511	0.9408	0.3093
15	0.0724	0.8686	0.1998
16	0.1822	0.5738	1.0000

Table 5 shows the relational gray and rank coefficients for each experiment. It can be seen that the highest gray relational degree corresponds to the experiment.

Table 5

Grey relational coefficients and grey relational grade for experiments.

Run	Roughness	Depth	Time	Grade	Rank
	μm	mm	min		
1	0.6462	0.3472	0.9013	0.6315	12
2	0.7917	0.3462	0.8692	0.6690	7
3	0.6598	0.3502	1.0000	0.6700	6
4	0.9437	0.3337	0.7148	0.6640	8
5	0.9763	0.3456	0.9994	0.7737	1
6	0.9533	0.3418	0.6860	0.6604	9
7	0.8939	0.3410	0.9013	0.7121	5
8	0.9356	0.3333	0.6817	0.6502	11
9	0.5326	1.0000	0.6161	0.7162	3
10	0.9165	0.3525	0.8692	0.7127	4
11	0.8923	0.3347	0.3401	0.5224	15
12	1.0000	0.3427	0.9400	0.7609	2
13	0.3333	0.7007	0.6809	0.5716	14
14	0.9072	0.3470	0.6178	0.6240	13
15	0.8734	0.3653	0.7145	0.6511	10
16	0.7329	0.4656	0.3333	0.5106	16

According to Table 5, the highest gray relational degree corresponds to experiment 5, which has the rank of 1. It ensures the optimal combination of the factors of the laser engraving process to achieve the proposed objectives: power 20 W; speed 1500 mm/s, Frequency 20000Hz; Stepover 0.03 mm; Focal distance 250 mm; Circular strategy.

The Table 6 shows the average GRG values for each level of each factor, providing a clear view of the impact of each factor on overall performance. Its main role is to identify the optimal levels of the factors, with the level with the highest average value of GRG being considered optimal. The table also allows the evaluation of the influence of the factors by analyzing the differences between the GRG values for different levels, a large difference indicating a significant impact on the process performance. In addition, this table simplifies the interpretation of experimental results, facilitating the identification of factors and

levels that contribute most to process optimization.

Table 6

Response table for Grade.

	Factors	Units	Level 1	Level2
A	Power	W	0.6883	0.6859
B	Speed	mm/s	0.6520	0.6947
C	Frequency	Hz	0.6650	0.6527
D	No of cuts	-	0.7124	0.6290
E	Stepover	mm	0.6239	0.7175
F	Focal distance	mm	0.6488	0.6925
G	Strategy	-	0.6663	0.6750

The values written in bold represent the optimal levels of the factors that ensure the best gray relational degree

Figures 5-11 shows the dependence of the gray relational degree on the etching parameters.

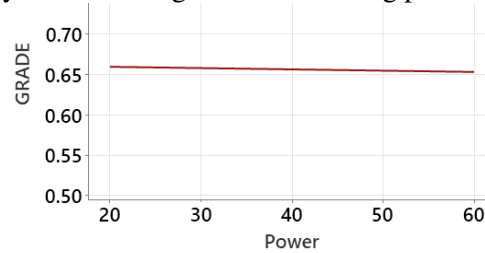


Fig. 5. Grade vs. Power.

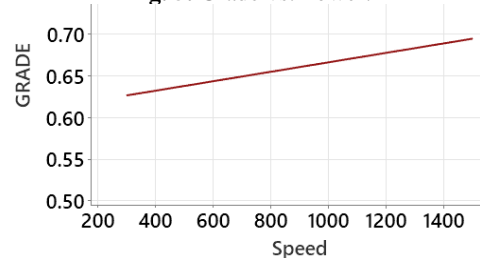


Fig. 6. Grade vs. Speed.

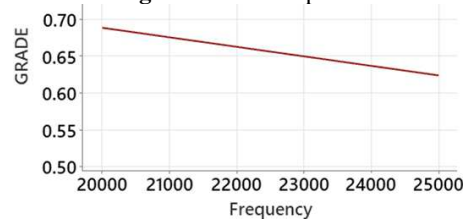


Fig. 7. Grade vs. Frequency.

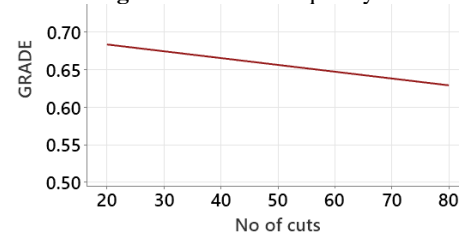


Fig. 8. Grade vs. No. of cuts.

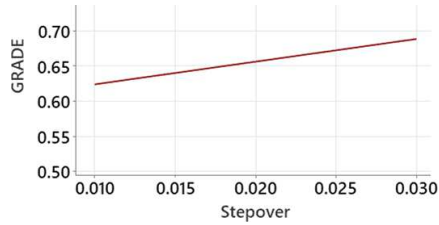


Fig. 9. Grade vs. Stepover.

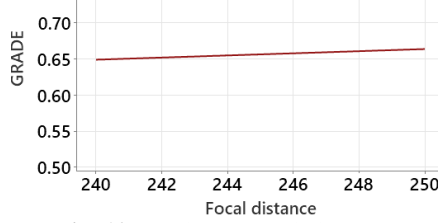


Fig. 10. Grade vs. Focal distance.

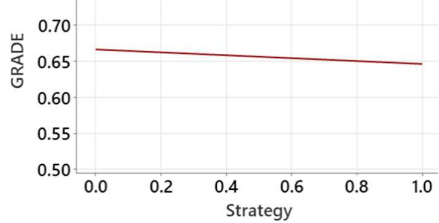


Fig. 11. Grade vs. Strategy.

Table 7

Analysis of Variance.

Source	U	D	Ad	A	F-	p-	Obs.	Con
A-	W	1	0.0	0.	0.	0.	unsi	0
B-	m	1	0.0	0.	4.	0.	signi	1
C-	H	1	0.0	0.	4.	0.	signi	1
D-No	-	1	0.0	0.	5.	0.	signi	1
E-	m	1	0.0	0.	4.	0.	signi	1
F-	m	1	0.0	0.	0.	0.	unsi	0
G-	-	1	0.0	0.	0.	0.	unsi	0
Error		8	0.0					
Total		1	0.0					

For the confirmation test, the parameters corresponding to the optimal combination were adjusted as mentioned: Power (A)-60 W; Speed (B) 1500 mm/s; Frequency (C) 20000Hz; No of cuts (D) 20; Stepover (E) 0.03 mm; Focal distance (F) 250 mm; Strategy (G) linear.

4. CONCLUSION

This work integrated experimental design with grey relational analysis to enhance the laser engraving process, focusing on enhancing performance metrics like surface roughness, penetration depth, and duration.

After determining the Grey Relational Grade (GRG), ANOVA analysis of variance (Table 7) is used to identify the significant factors that influence process performance and to determine the contribution of each to the total response variation. Although Grey Relational Grade provides an overall measure of performance, ANOVA allows the individual influence of each factor to be separated and quantified, helping to validate the model and optimize the process. By analyzing variation, one can identify critical factors that need to be adjusted to improve performance, while eliminating insignificant factors, which simplifies the process and increases efficiency. The significant factors identified from the ANOVA are Speed, Frequency, No of cuts, and Stepover.

Taking into account the results of the ANOVA test, the levels identified as optimal (Table 6) and the relationship (10) are: A1 B2 C1 D1 E2 F2 G2, where 1 and 2 means the level of the factor.

The predicted and confirmed values for the objective functions are presented in Table 8

From the analysis of the obtained results and the comparison with the initial values, very small differences can be observed between the values predicted by calculation and the measured ones. which confirms that the statistical model and optimization are correct and valid. Small differences for all parameters show that the process is well-controlled and robust.

Table 8

The predicted and confirmed values for the functions.

Objective function	Units	Predicted value	Confirmed value
Ra	μm	0.453	0.442
Depth	mm	0.0138	0.016
Time	min	0.38	0.39
Grade		0.7737	0.7738

Multi-objective optimization enabled the identification of an optimal combination of parameters (A1 B2 C1 D1 E2 F2 G2) that maximizes process performance. ANOVA analysis confirmed the statistical significance of the studied factors, and validation experiments demonstrated the robustness of the proposed model. The results led to practical

recommendations for improving the laser engraving process.

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Optimizarea multiobiectiv a parametrilor de gravare cu laser folosind analiza relațională cu gri

Gravarea cu laser este o tehnică la aplicarea precisă a marcajelor pe suprafața matrițelor, oferind avantaje precum precizie ridicată, rezistență la uzură, viteză de execuție și capacitatea de a marca suprafețe complexe. În articol se prezintă optimizarea multiobiectiv a parametrilor de gravare, utilizând design experimental și analiza relațională gri, pentru a îmbunătăți simultan rugozitatea suprafeței, adâncimea de gravare și timpul de procesare.

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