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PARAMETRIC MODELLING OF INVOLUTE GEARS WITH CURVILINEAR SHAPED TEETH

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Abstract: *Gears are mechanical components used in various industrial applications. They ensure a constant transmission ratio and efficient operation at different speeds and power levels. The 3D modeling of gears with curved teeth allows design optimization, reducing noise and vibrations, as well as cost reduction through virtual testing and the minimization of manufacturing errors by integrating CNC processes and 3D printing. The article presents the parametric design of cylindrical gears with curved teeth with epicycloidal flanks. Using the parametric equations of the epicycloid and involute, calculation programs were developed to determine points on the flanks. These points were used in CAD software to obtain the 3D model, which allowed the observation of the theoretical position of the contact area.*
Keywords: *Involute gear, epicycloid, parametric modelling.*

1. INTRODUCTION

Gears are essential mechanical components with a wide range of applications in various industrial and technological fields. The primary advantages of gears include a constant transmission ratio and the ability to operate efficiently at varying speeds and powers, from low speeds in clocks to high speeds in heavy machinery with large peripheral speeds.

3D modeling of curved-tooth gears is essential for their design and optimization due to the complexity of their geometry and high-performance requirements. This technique allows for a precise representation of tooth surfaces, ensuring detailed analysis of contact, stress distribution, and gear stiffness. Advantages include simulating real-world operating conditions, reducing manufacturing errors through direct integration with CNC processes and 3D printing, and optimizing the design to reduce noise, vibration, and wear. Additionally, 3D modeling facilitates virtual testing of various configurations, saving time and costs associated with physical prototypes, making it indispensable in the development of modern gears. Continuous advancements in computer-aided design (CAD) software and finite element analysis (FEA) have led to a diverse range of approaches to modeling these components.

The approach of the work is supported by a series of relevant works, recent dates, which address 3D modeling, computer -aided design (CAD), structural analysis (FEA) and and manufacturing gears with curvilinear teeth using various mathematical curves.

Regarding mathematical modeling, simulation, and computer-aided design, article [1] presents mathematical models for generating the involute profile of cylindrical gears based on spatial gear theory. It has been demonstrated that the tooth profile in the mid-section is an involute curve. The coordinates of the characteristic points of the tooth profile have been calculated, and a practical example is presented. The manufacturing process of cylindrical gears with involute teeth using a CNC machine and a circular cutter is analyzed. The presented mathematical models can be utilized in computer-aided design software to optimize gear performance and reduce production costs.

Also, research [2] analyzes curved cylindrical gears with line contact, focusing on mathematical modeling, contact analysis, and stress analysis. Using fixed face cutters to generate tooth surfaces, researchers examined contact lines under ideal conditions and investigated transmission errors in the presence of assembly errors. Finite element analysis evaluated stresses for gears with and without tip relief. Results show that contact lines slightly exceed tooth width, certain assembly errors have a greater impact on transmission, tip relief reduces contact stresses at the edges, and maximum bending stress is distributed at the tooth ends.

All about this topic article [3] presents a new dynamic model for helical gear vibrations, combining a validated model for cylindrical gears with an innovative extension. The model addresses the challenge of calculating the time-varying stiffness of the helical gear. In this method, the width of the helical tooth is divided into infinitesimal slices, treated as straight teeth, thereby simplifying the complexity added by the helix angle. The innovation lies in the integration of manufacturing profile errors, generating data dispersion for a more realistic representation. The model has been experimentally validated by qualitatively comparing the RMS value of the vibration signal. The results show similar behavior between simulations and measurements at various speeds and loads, highlighting the model's potential for diagnosing gear faults.

Regarding the parameterized design and structural analysis (FEA) of the gears, some papers covering the relevant topic are offered.

Study [4] evaluate the mesh stiffness by developing the modified tooth surface equation for VH-CATT gears, followed by 3D reconstruction and the creation of a load tooth contact analysis (LTCA) model. This model, validated using the finite element method, enabled the calculation of mesh stiffness and the investigation of the influence of various variables on it. Results showed that the stiffness is significantly higher in the double tooth contact zone, increases with load and cutter inclination angle, but decreases with increasing parabolic coefficient in the double contact zone. These findings provide a solid foundation for optimizing the design and improving the load capacity of VH-CATT gears.

The authors [5] presented an innovative computer-aided design method for curved cylindrical gears utilizing involute profile functions based on parabolas to control the tooth shape. Researchers have experimented with various control points and curve combinations for the active tooth profile, analyzing meshing performance, mechanical behavior, and the influence of curve parameters. Compared to the reference gears generated by face cutters, five-control-point curved gears have shown significant advantages, including higher maximum contacts and lower bending stresses, as well as smaller loaded transmission errors, highlighting the potential of this approach in optimizing gear design.

Authors [6] focus on the design and analysis of involute helical gears, a highly efficient gearing system widely used for power and torque transmission. The research specifically examines the helical gear in the slag pot transfer car (SPTC) gearbox within the steel industry, identifying regions of high-stress concentration. To optimize performance, the gear was redesigned with a constant width of 45 mm, exploring various helix angles (between 14 and 30 degrees) while maintaining a pressure angle of 20 degrees. 3D modeling was conducted using AUTODESK FUSION 360, and numerical analysis was performed with ANSYS Workbench to calculate contact stresses. This study provides valuable insights for the safe and efficient design of helical gears in specific industrial applications such as the SPTC gearbox.

An important aspect is the manufacture of curved tooth gear. In tis context, study [7] analyzes existing production methods and proposes two innovative solutions: CNC machining, which overcomes the limitations of traditional techniques, and 3D printing, which is presented as a promising alternative for producing curved involute gears. These new approaches have the potential to facilitate the widespread adoption of high-performance gears for industrial applications. Paper [8] examines the impact properties of gear meshing for circular-arc tooth profile cylindrical gears (CATT), using a model based on contact dynamics theory and finite element analysis. Researchers created a precise 3D model to investigate the distribution of impact stresses and the relationship between impact velocity, position, and stress. Results identify critical areas for different impact types, such as the tooth tip and root on primary and secondary impact surfaces for root and tip impact, respectively, and the tooth root and pitch circle for pitch circle impact. The study underscores the significant influence of impact velocity and position of the resulting stress, providing a valuable theoretical foundation for the dynamic design and industrial applications of cylindrical gears.

The proposed work is to present a parametric design method for 3D modelling of cylindrical gears with epicycloidal arc-shaped teeth.

2. CURVILINEAR GUIDE OF FLANKS

Different cyclic curves can be used to design tooth flank profiles. An epicycloid is a curve generated by a point fixed to the circumference of a circle that rolls without slipping on the exterior of a fixed base circle. The resulting epicycloid is called a roulette, the rolling circle is called the rolling curve, and the fixed circle is called the base. Figure 1 is used to determine the parametric equations of the epicycloid.

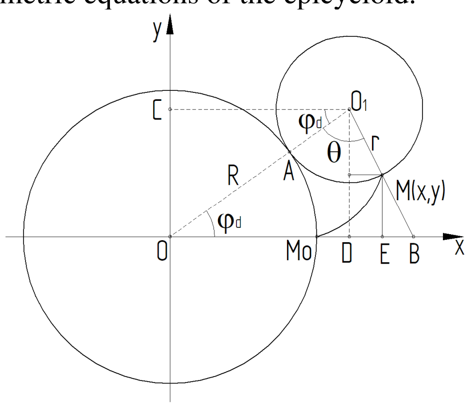

Fig. 1. Epicycloid generation.

The equations of the epicycloid can be expressed as a function of the rotation angle of the rolling circle (circle of radius r) over the base circle (roulette, of the radius R).

$$\begin{cases} x_M = (R+r)\cdot\cos\varphi_d - r\cdot\cos(\varphi_d\cdot\dfrac{r+R}{r}) \\ y_M = (R+r)\cdot\sin\varphi_d - r\cdot\sin(\varphi_d\cdot\dfrac{r+R}{r}) \end{cases}. \quad (1)$$

For the tooth guide (directrix), normal epicycloids are presented graphically, determined for various base circle and rolling circle radii (Fig. 2.).

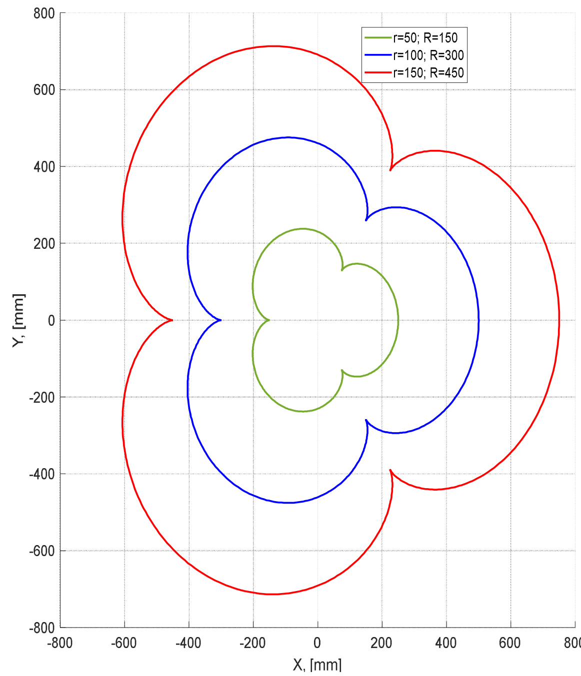

Fig. 2. Epicycloids with 3 loops ($\frac{R}{r}=3$).

3. TOOTH PROFILE

The involute profile is commonly used for gear teeth. An involute is a curve generated by a point on a straight line that rolls without slipping along a base curve. This base curve is often a circle in gear applications.

The involute is the preferred tooth profile in gear manufacturing due to its advantageous properties. These include the fact that its conjugate is also an involute, its ease of production using simple tooling, and the straightforward inspection of its profile.

In the case of the involute of a circle, the circle represents the base; the line that rolls without slipping on the base circle represents the generating line, and the trajectory (involute) described by any point on the generating line represents the roulette (Fig. 3).

The parametric equations of the involute curve described by point E are presented in:

$$\begin{cases} x_E = a[\cdot\cos(t)+(t-q)\cdot\sin(t)] \\ y_E = a[\sin(t)-(t-q)\cdot\cos(t)] \end{cases}. \quad (2)$$

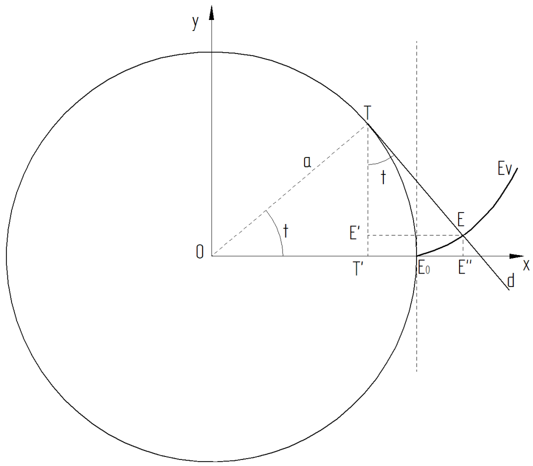

Fig. 3. Generating the involute of a circle.

where the term "q" is the angle that defines the starting point of the involute on the circle (Fig. 4).

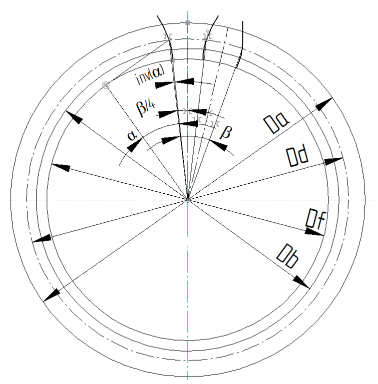

Fig. 4. Plotting the involute profile.

The angle q is determined with the relation:

$$q = \frac{\beta}{4} - \delta . \tag{3}$$

where β is the angle between two flanks of the tooth and is determined with the relation:

$$\beta = \frac{360}{z} . \tag{4}$$

where:

z is the number of teeth on the wheel;

δ is the angle formed by the radius of the pitch circle at the intersection with the involute and the radius of the base circle at the starting point of the involute. It is determined with the relation:

$$\delta = inv(\alpha) = tg(\alpha) - \alpha . \tag{5}$$

where α represents the pressure angle, α=20°. In the previous relationship, the angle α is entered in radians. By substitution we get:

$$q = \frac{\beta}{4} - [tg(\alpha) - \alpha] \cdot \tag{6}$$

With the notations in Fig. 4 the parametric equations of the involute flanks are:

$$\begin{cases} x_E = Rb \cdot \left(\cos(t) + \left(t - \left(\frac{\beta}{4} - (tg(\alpha) - \alpha) \right) \right) \cdot \sin(t) \right) \\ y_E = Rb \cdot \left(\sin(t) - \left(t - \left(\frac{\beta}{4} - (tg(\alpha) - \alpha) \right) \right) \cdot \cos(t) \right) \end{cases} . \tag{7}$$

in which t take values between 0 and 0.7.

Fig. 5 illustrate the involute profiles plotted for module 2 and tooth number 20 parameters.

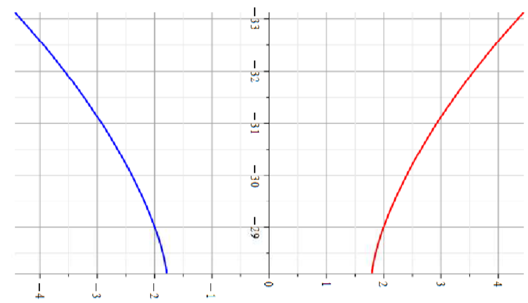

Fig. 5. Teeth profile in normal section for m = 3 z = 20.

4. MODELING OF THE GEAR

The calculation programs made in Matlab, previously developed for plotting the graphs of the flank line and profile, allow saving matrices in Excel containing the coordinates of the points belonging to the flank line (guide) (fig. 6) and the tooth profile (fig. 7). All values are in millimeters.

	A	B	C
89	-249.9492608	0	4.029102791
90	-249.9576489	0	3.681080515
91	-249.9652097	0	3.336396387
92	-249.972033	0	2.991416965
93	-249.9782259	0	2.639540316
94	-249.9839454	0	2.266538049
95	-249.9884253	0	1.924516703
96	-249.9921304	0	1.586885976

Fig. 6. The coordinates of the points of the tooth line (guide), in millimeters.

	A	B	C
49	-28.43530719	1.842000029	0
50	-28.4503379	1.845248118	0
51	-28.46425146	1.84830554	0
52	-28.47996735	1.851816475	0
53	-28.49535324	1.855311573	0
54	-28.5110254	1.858929423	0
55	-28.52773947	1.862850757	0

a)

	A	B	C
49	-28.43530719	-1.842000029	0
50	-28.4503379	-1.845248118	0
51	-28.46425146	-1.84830554	0
52	-28.47996735	-1.851816475	0
53	-28.49535324	-1.855311573	0
54	-28.5110254	-1.858929423	0
55	-28.52773947	-1.862850757	0

b)

Fig. 7. The matrices with the coordinates (in millimeters) of the points of the tooth profile: a) the left flank; b) the right flank.

A 3D model of a gear with an epicycloidal tooth profile is presented. The gear has the following characteristics: module m = 3 mm, number of teeth z = 20. The epicycloid is defined by a base circle radius R = 150 mm and a rolling circle radius r = 50 mm. The geometric parameters are determined using the calculation relationships outlined in reference.

In the Solid Edge application, these sets of points are imported with "Curve by Table" command and will be used to make the sketches for the 3D model of the wheel. The normal profile of the tooth is found in the median plane (Fig. 8).

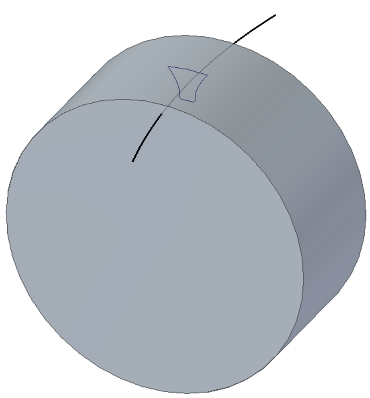

Fig. 8. The sketches obtained by imported sets of points.

Using the epicycloid as a guide and the closed contour defined by the involute curves and outer diameter and root diameter, the material is removed between two teeth using a boolean operation.

By applying circular pattern operation is obtained the 3D model of the gear (fig.9).

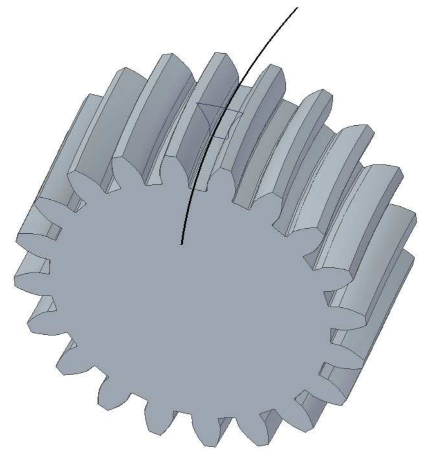

Fig. 9. The 3D model of the gear.

In Fig. 10, the gear train consisting of cylindrical gears with curved teeth is presented.

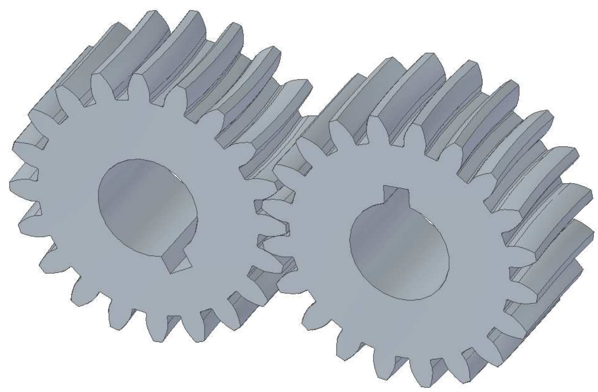

Fig. 10. The 3D model of the gear.

This study aims to locate the contact area between two engaged teeth. As shown in Fig. 11, the contact area is located in the central region of the tooth.

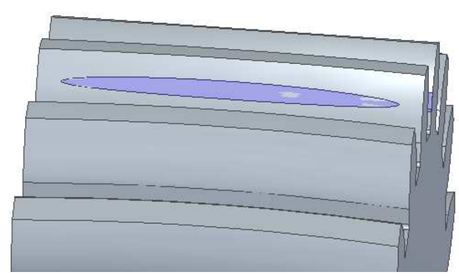

Fig. 11. Contact area.

5. CONCLUSION

Unlike conventional cylindrical gears with straight or helical teeth, cylindrical gears with curved teeth constitute a class of gears distinguished by a curved or arched tooth form.

This tooth type has benefits in terms of transmission performance and efficiency including more homogeneous load distribution, less noise and vibration, and less wear when running. Starting point for designing and modelling these gears were mathematical formulae specifying the directrix and generatrix of the tooth flanks.

Parametric equations were developed and then applied in computers calculation programs and graphical representation.

These equations were used in Matlab to obtain the points required for 3D model of gears with an epicycloidal flank.

By importing the generated points in Solid Edge, were produced 3D models that helped to visually ascertained the contact area.

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Modelarea parametrică a roților dințate evolventice cu dinți de formă curbilinie

Roțile dințate sunt componente mecanice utilizate în diverse domenii industriale, oferind avantaje precum raportul de transmitere constant și funcționarea eficientă la viteze și puteri variate. Modelarea 3D a acestor componente, în special a celor cu dinți curbi, permite o reprezentare precisă a suprafețelor dinților, facilitează analiza contactului și a distribuției tensiunilor, și permite optimizarea designului pentru reducerea zgomotului și vibrațiilor. Integrarea cu procese CNC și imprimarea 3D reduce erorile de fabricație, în timp ce testarea virtuală economisește timp și costuri. In articol, pentru roțile dințate cilindrice cu dinți curbi cu flanc epicicloidal, s-au dezvoltat programe de calcul bazate pe ecuații parametrice. Datele obținute au fost utilizate în aplicații CAD pentru generarea modelelor 3D și determinarea poziției teoretice a petei de contact.

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