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CARDINALITY CONSTRAINT FOR TRUSS SIZING OPTIMIZATION OF PLANAR TRUSS STRUCTURES

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Abstract: *The optimization goal function of minimal weight generally produces results which use many different cross-sections. In practice, using such a large number of different bar stocks is irrational, impractical and expensive. This research explores the influence of implementing cardinality constraints in sizing optimization by limiting the number of possible different cross-sections that are used in any iteration of the optimization process. The example chosen to showcase the difference in using such an approach is the typical 47-bar planar problem. Results using the cardinality constraint are compared to results from an analytical solution where the entire structure was sized according to the structure's most stressed bar, as well as to comparable results from literature, which do not limit the number of different cross-section dimensions.*

Keywords: *truss optimization, genetic algorithm, Euler buckling, dynamic constraints, cardinality.*

1. INTRODUCTION

The use of structural optimization is widespread in the research fields related to truss construction. However, this approach has not been widely adopted. One of the issues with adopting this design approach is the still inadequate formulation of optimization constraints and a range of different objectives which vary.

A multi-objective optimization (MOO) of steel trusses, considering total weight and displacement as objectives and employing direct analysis, was presented by researchers in [1]. They used six different metaheuristic algorithms to solve the MOO problem and presented their results for 10-,72-,47-, and 113-bar truss examples. In [2], the authors used a modified GreyWolf Optimizer (GWOM) using three different mutation operators for sizing optimization in truss structures. Their results were evaluated using several benchmark examples showing the competitiveness of the method compared to other modern methods.

Current optimized models mainly focus on improving optimization methods' speed and the possibility of providing marginally better results than other unmodified or older methods.

Some researchers approach truss optimization without a starting geometry. Rather than traditional methods, the truss layout problem is formulated as a Markov Decision Process (MDP) model, as demonstrated in [3], which introduced AlphaTruss—a Monte Carlo Tree Search approach for optimal truss layout design. This model significantly expands the solution space through three sequential action sets: adding nodes, adding bars, and selecting sectional areas. The reward function provides feedback on actions based on geometric stability and structural simulation.

In the past, some attempts have been made to include ways of implementing cardinality as a means of approaching the applicability threshold for optimization to be the leading factor in the design choices of truss construction [4, 5]. This idea has resurfaced in recent years and has been implemented in [6, 7], where researchers used a two-step approach where variables are reassigned to newly selected sets, which has been applied to the optimization process of this research as well. In [8], researchers applied the ant colony optimization method for multi-objective structural optimization, incorporating the cardinality constraint directly into the algorithm. By employing a two-step construction process to assign design variables based on the defined cardinality, they achieved minimization of both weight and nodal displacement. An encoding method for automatic variable linking was used by authors in [9] for sizing optimization and in [10] for simultaneously optimizing sizing and layout, thereby not allowing solutions which do not meet the cardinality conditions. The penalty function is avoided for this constraint in such an approach. This research explores the influence of implementing cardinality constraints in sizing optimization by limiting the number of possible cross-section diameters used in any given iteration. The difference in using such an approach is presented on a typical 47-bar planar problem where results are compared to those from an analytical solution sized according to the bar experiencing the largest stress, as well as to comparable results from literature.

2. TRUSS STRUCTURAL OPTIMIZATION

In structural optimization, sizing treats cross-sections as variables. Cross-section variables should be considered as a discrete set of values to ensure practical applicability. The objective is to identify the combination of cross-section diameters assigned to specific bars which minimizes weight while satisfying constraints for both stress and displacement. The majority of truss sizing problems found in literature approach the minimal weight optimization challenge in the following way:

$$\begin{cases} \min W(A) = \sum_{i=1}^{i=n} \rho_i A_i l_i \text{ with } A = (A_1, \ldots, A_n) \\ \text{subjected to} \begin{cases} A_{min} \le A_i \le A_{max} & \text{for } i = 1, \ldots, n \\ \sigma_{min} \le \sigma_i \le \sigma_{max} & \text{for } i = 1, \ldots, n \\ u_{min} \le u_j \le u_{max} & \text{for } j = 1, \ldots, k \\ \left| F_{Ai}^{comp} \right| \le F_{Ki} & \text{for } i = 1, \ldots, n \\ \text{where } F_{Ki} = \dfrac{\pi^2 \cdot E_i \cdot I_i}{l_i^2} \end{cases} \end{cases} \quad (1)$$

where W is the mass of the truss, n is the number of elements the truss consists of, A_i is the cross-section area of the i^{th} element, l_i is i^{th} element's length, σ_i is the i^{th} element's stress, u_j is the j^{th} node displacement, and k is the total number of nodes.

Euler buckling is incorporated to achieve results that are practically applicable. The iterative changes in the moment of inertia, caused by variations in cross-sections, also alter the Euler critical buckling constraint during each iteration (2). Consequently, this constraint is treated as dynamic. Its inclusion substantially increases the complexity of the optimization problem. Since the stress comparison utilizes the same area on both sides of the equation, Euler critical load is employed as the constraint.

$$\begin{aligned} &\sigma_{Ai}^{comp} \le \sigma_{Ki} \\ &\text{where } \sigma_{Ai}^{comp} = \frac{F_{Ai}^{comp}}{A_i} \text{ and } \sigma_{Ki} = \frac{F_{Ki}}{A_i} \\ &F_{Ki} = \frac{\pi^2 \cdot E_i \cdot I_i}{l_i^2} \\ &\left| F_{Ai}^{comp} \right| \le F_{Ki} \text{ for } i = 1, \ldots, n \end{aligned} \quad (2)$$

where the compression axial stress of the i^{th} bar element is σ_{Ai}, and σ_{Ki} is the critical buckling stress of the i^{th} element ; the compression force in the axial direction is F_{Ai}^{comp}, the critical load of the i^{th} element according to Euler is F_{Ki}, the minimum moment of inertia of the i^{th} element's cross-section is I_i, and the i^{th} element's modulus of elasticity is E_i. The length between nodes of the i^{th} element is given as l_i. The constraint from expression (2) is appended to the constraints given in expression (1).

3. CARDINALITY CONSTRAINT

This research further proves the benefits of using cardinality constraints presented by [6]. In order to implement the limitation of the number of different cross-section diameters an optimal solution can have, a cardinality constraint given in expression (3) has been used.

$$\begin{aligned} &\frac{m}{m_{\max}} - 1 \le 0 \\ &\text{where } m = |\{A_1^G, A_2^G, A_3^G, \ldots, A_n^G\}| \\ &\text{and } m \le m_{\max} \le n \end{aligned} \quad (3)$$

The variables and their indexes from this relation are further explained in Fig. 1. The cross-section assignment module is used to complete several tasks to ensure

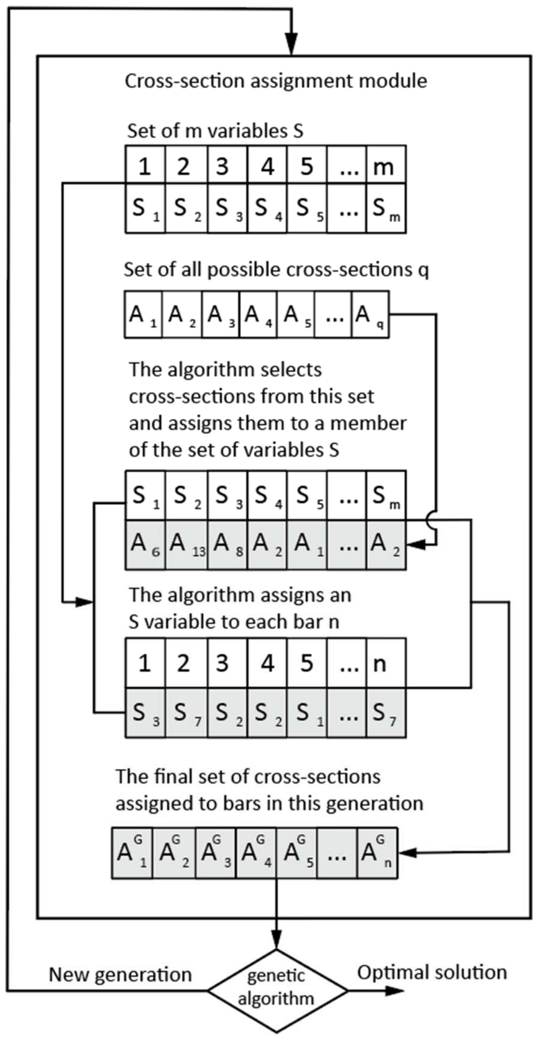

Fig. 1. Schematic overview of cross-section assignment in genetic algorithm operation, using the cardinality constraint.

4. EXAMPLE

An example of a frame with 47 rods [11-13] represents a planar problem where the node positions are symmetric about the y-axis. The cross-sections of the elements are grouped into 27 groups, maintaining symmetry about the y-axis. The configuration is arranged in 22 nodes, as shown in Figure 2. For this example, profiles with solid circular cross-sections made of structural steel were used, characterized by the following properties: an elasticity modulus of 206,842.719 MPa and a density of 7.4 g/cm³.

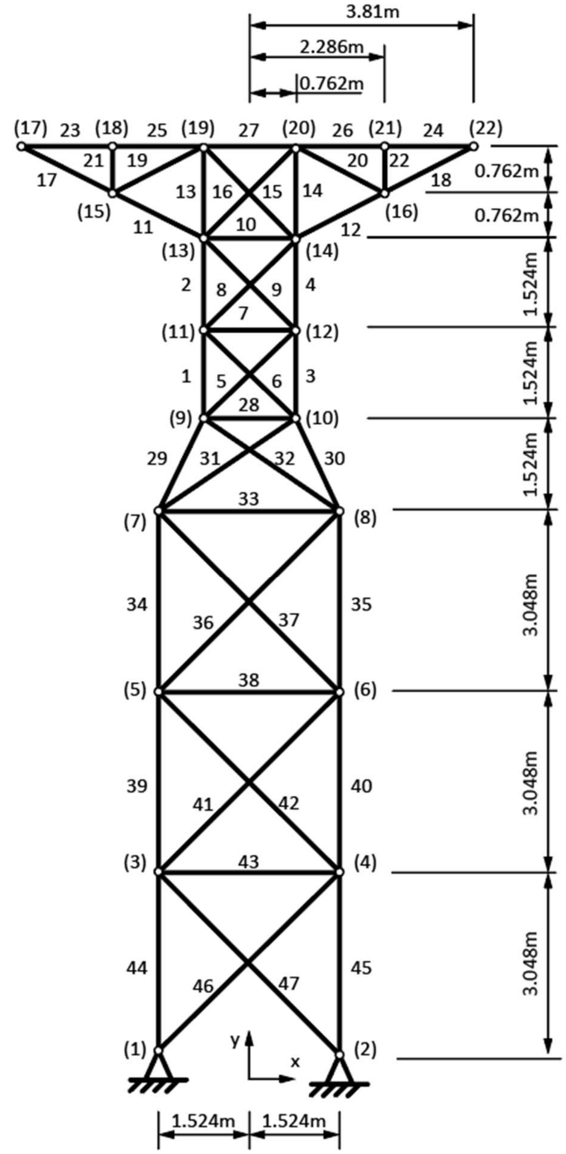

Fig.2. The 47-bar truss example layout.

Regarding discrete variables, the cross-sections were selected from catalogs of various manufacturers, and a list of possible rod diameters was compiled as follows: 6, 8, 12, 12, 14, 15, 16, 17, 18, 20, 22, 24, 25, 28, 30, 32, 35, 36, 38, 40, 45, 50, 55, 56, 60, 63, 65, 70, 75, 80, 85, 90, 95, 100, 105, 110, 115, 120, 125, 130, 140, 150, 160, 170, 180, 190, 200, 220, and 250, with values given in millimeters.

The structure is required to resist three distinct load cases (LC1, LC2, and LC3). In

LC1, concentrated forces of 26.689 kN in the +x direction and 66.275 kN in the –y direction are applied at both nodes 17 and 22. LC2 applies the same set of forces solely at node 17, while LC3 applies them solely at node 22.

The example with 47 rods has specified stress limits of 103.421 MPa in compression and 137.895 MPa in tension. This example does not include a specified limit for the maximum allowable nodal displacement due to deformation. The genetic algorithm was used as the optimization method due to its capabilities and availability. All constraints in this research are subject to a uniform penalty function. If one or more constraints are violated, the invalid result are multiplied by a significant factor.

5. RESULTS

The cross-sectional optimization involves 27 variables. Table 1 compares optimal weights for different limited numbers of cross-sections (cardinalities) with the general optimal solution and the optimal solution using only one cross-section diameter (the analytical solution) for the whole model for the sizing optimization.

Table 1

Comparison of masses according to the cardinality constraint.

Cardinality	Weight [*kg*]	Difference from the analytical solution	Difference from overall optimum
1	3969.870	-	-121.999%
2	2510.117	-36.771%	-40.368%
3	2260.859	-43.050%	-26.429%
4	2246.385	-43.414%	-25.620%
5	2194.342	-44.725%	-22.710%
6	2157.790	-45.646%	-20.666%
7	2018.764	-49.148%	-12.891%
...			
14	1788.238	-54.955%	-

The analytical solution is sized according to the element under the highest stress, which is compression and subjected to buckling constraints, according to Euler. This is the same solution as the one where the cardinality is set to one, since all bars are sized to use only one cross-section. Only examples with cardinality constraints from 1 to 7 were run since it was unreasonable to assume that, in practice, more than four or five different cross-section diameters would be used, at most. The decision to go up to seven different cross-sections was made in order to show a trend of weight increase with the increased limitation of cardinality.

Cross-sectional areas for optimal sizing solutions for all cardinality-constrained solutions between 1 and 7, and the overall optimum with 14 different cross-sections are provided in Table 2.

6. CONCLUSION

The broad application of optimization in the design process of truss structures is closer than ever. Faster and faster processing speeds, coupled with heuristic optimization methods, have in recent years shown that it is possible to use these tools in a variety of other fields, and their acceptance as an alternative to engineer experience and conventional design methods is expanding significantly.

The specific problem of using optimization to design trusses has to cover a broad range of constraints that are frequently case-dependent. Over the years, many constraints have been tried and tested, and an all-encompassing solution is still some years away. However, the implementation of practical constraints, the use of producible cross-section sets, and other steps have ensured the right path for development.

This paper shows the difference in optimal weight when comparing a more traditional optimization approach to one which is mindful of the number of different cross-sections used to produce the complete structure.

The limitation of the number of varying cross-section diameters inevitably gives results which are heavier than those without the cardinality constraint, but their advantage is their simplicity, the possibility of limiting and comparing different maximal numbers of different stock and assessing the solutions to come up with the best use case.

Table 2

Comparison of cross-section areas by element of cardinality solution.

Element number	Cross-section area [cm^2] for each cardinality solution							
	1	**2**	**3**	**4**	**5**	**6**	**7**	**14**
1	44.179	24.630	44.179	28.274	28.274	28.274	44.179	24.630
2	44.179	24.630	23.758	23.758	23.758	23.758	23.758	23.758
3	44.179	24.630	23.758	23.758	23.758	15.904	11.341	12.566
4	44.179	24.630	1.310	0.503	0.503	0.503	1.131	7.069
5	44.179	24.630	23.758	23.758	23.758	23.758	15.904	15.904
6	44.179	24.630	23.758	23.758	15.904	15.904	19.635	12.566
7	44.179	24.630	23.758	23.758	23.758	23.758	15.904	15.904
8	44.179	24.630	23.758	23.758	15.904	15.904	11.341	11.341
9	44.179	24.630	23.758	23.758	23.758	23.758	19.635	19.635
10	44.179	24.630	23.758	23.758	15.904	15.904	15.904	15.904
11	44.179	24.630	1.310	0.503	0.503	0.503	1.131	0.283
12	44.179	24.630	1.310	0.503	0.503	0.503	1.131	0.503
13	44.179	24.630	23.758	23.758	23.758	23.758	11.341	11.341
14	44.179	24.630	23.758	23.758	23.758	23.758	23.758	11.341
15	44.179	24.630	23.758	23.758	23.758	15.904	11.341	9.621
16	44.179	24.630	23.758	23.758	23.758	23.758	11.341	7.069
17	44.179	24.630	23.758	23.758	23.758	23.758	23.758	24.630
18	44.179	24.630	23.758	23.758	23.758	31.172	28.274	23.758
19	44.179	24.630	23.758	23.758	23.758	23.758	11.341	1.131
20	44.179	44.179	44.179	44.179	44.179	44.179	44.179	38.485
21	44.179	24.630	23.758	23.758	23.758	23.758	23.758	15.904
22	44.179	24.630	1.310	0.503	0.503	0.503	1.131	7.069
23	44.179	24.630	44.179	44.179	44.179	44.179	44.179	38.485
24	44.179	24.630	23.758	23.758	23.758	23.758	23.758	23.758
25	44.179	24.630	1.310	0.503	0.503	0.503	1.131	1.539
26	44.179	44.179	44.179	44.179	44.179	44.179	44.179	44.179
27	44.179	24.630	23.758	28.274	28.274	23.758	23.758	23.758
Weight [kg]	3969.87	2510.117	2260.859	2246.385	2194.342	2157.79	2018.764	1788.238

The results of the 47-bar test example show that for sizing optimization, a ~55% lighter solution can be achieved using the typical optimization approach with a structure using 14 different cross-sections, compared to the solution using just one (analytical solution). While solutions with three or four different cross-sections making up the same structure are ~43% lighter than the analytical solution. Compared to the typical optimization approach, these simpler solutions are only ~26% heavier but require the acquisition of a far smaller number of different cross-sections to produce. More work is still needed to bring truss optimization to mass adoption in truss design.

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Constrângere de cardinalitate pentru optimizarea dimensiunii elementelor în structuri de tip grindă plană

Funcția obiectiv de optimizare a greutății minime produce, în general, rezultate care utilizează multe secțiuni transversale diferite. În practică, utilizarea unui număr atât de mare de tipuri diferite de bare este irațională, nepractică și costisitoare. Această cercetare explorează influența implementării unor constrângeri de tip cardinalitate în optimizarea dimensionării, prin limitarea numărului de secțiuni transversale diferite care pot fi folosite într-o anumită iterație. Exemplul ales pentru a evidenția diferența produsă de această abordare este problema planară tipică cu 47 de bare. Rezultatele obținute folosind constrângerea de cardinalitate sunt comparate cu rezultatele unei soluții analitice în care întreaga structură este dimensionată în funcție de elementul cel mai solicitat, precum și cu rezultate comparabile din literatura de specialitate, care nu limitează numărul de secțiuni transversale diferite.

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