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INFLUENCE OF THE U-JOINTS ARRANGEMENT OF THE CARDAN SHAFTS ON VIBRATIONS AND MOVEMENT UNSYNCHRONIZATION

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Abstract: *The paper explains through the presented mathematical model, the importance of cardan shaft phasing. In the case of the cardan shaft with perpendicular U-joints (compared to the one with coplanar U-joints), a high degree of non-uniformity of movement occurs which will lead to vibrations with negative implications on premature wear of the transmission as well as high discomfort. For construction with perpendicular U-joints, synchronism is recommended and the inclination angle of cardan shaft should not exceed 6 degrees, in which case the degree of non-uniformity of the movement would not exceed 3%.*
Key words: *cardan shaft, vibrations, out of phase, u-joints, non-uniformity.*

1. INTRODUCTION

Existance of the vibrations in the drivelines with cardan shafts are a real practical issue due to their potential to cause noise and vibrations which are transmitted to the vehicle chassis, which have negatively impact vehicle performance and passenger comfort [1], [2], increasing in this way also the premature wear of components. The idea of this work and this calculation arose from the real fact that there are many aftermarket cardan shaft construction variants that do not respect the coplanar arrangement of the U-joints ends, and many users complain of strong vibrations that propagate into the vehicle chassis. These aftermarket cardan shaft variants perform their general function of transmitting movement, are suitable in terms of dimensions, but due to faulty design they will lead to strong vibrations in operation, vibrations that are all the stronger the greater the angle of inclination, vibrations that lead to rapid damage to the cardan joints and discomfort for vehicles passengers. Many of the papers that address vibrations of drivelines with cardan shafts are based mainly on operating simulations using only existing softwares, without contributions to the calculation model, and anyway I haven't come across any functioning simulations for the case of out-of-phase U-joints of cardan shafts.

2. THE INFLUENCE OF THE ARRANGEMENT OF THE U-JOINTS OF THE CARDAN SHAFT ON THE OCCURRENCE OF VIBRATIONS IN THE TRANSMISSION DRIVE LINE

In a superficial analysis of the influence of the arrangement of the U-joints relative to the driving and driven shafts, their arrangement would not matter, but in reality (as will be demonstrated mathematically in this paper) the non-coplanar arrangement of the U-joints will cause a variations of the driven cardan shaft speed which will lead to vibrations in transmission. Correctly oriented U-joints are said to be in phase (as illustrated below, Figure 1, Figure 2, [4], [5]). When assembled out of phase, the U-joints will cause vibration in the cardan shaft drivetrain even at small cardan tilt angles. The vibration may not be very noticeable, but it’s definitely stressful on the driveshafts.

In the Figure 1 and Figure 2 we can see how the driveshaft and U-joints are in correct phase

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and respectively how they are "out of phase" so in an incorrect phase. The angle of the output shaft and stub axles, [6], [7] is important; ideally, they should be parallel, as in the picture below, Figure 3.

Fig. 1. Cardan shaft U-joints in phase and out of phase.

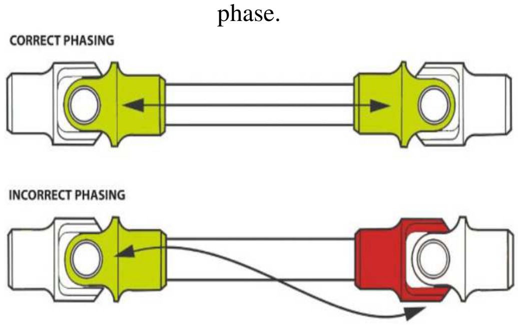
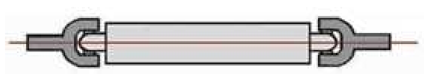
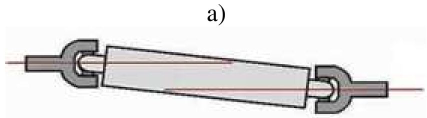


Fig. 2. Correct phasing and incorrect phasing of U-joints.



a)



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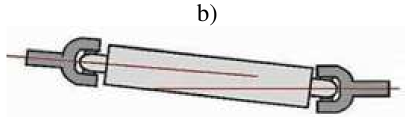

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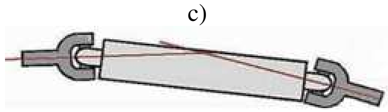


d)

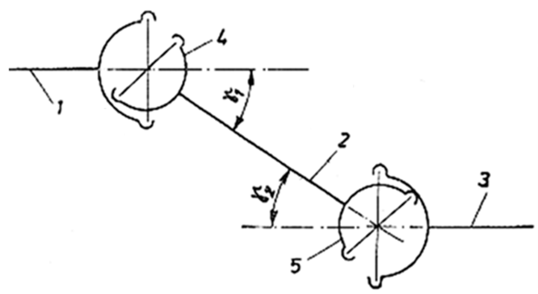
Fig. 3. Constructive variants of the layout of the input and output shaft of the cardan driveline
(a) The perfect option - without vibrations; **(b)** Parallel alignment - low to medium vibrations; **(c)** Not recommended option (different angles) - high vibrations; **(d)** The option to be completely avoided - very high vibrations that will affect the joints and pinions in the transmission.

Even the U-joints are in phase, the option with different angles between input and output should be avoided, as it leads to high vibrations. When there is an angular misalignment between the cardan shaft components, the driveline connected by universal u-joints can determine fluctuations in the driven shaft speed [2], [3]. The fluctuations can cause undesirable vibrations in torsional and lateral directions. These vibrations can excite the system's natural frequencies and lead to instability [3]. According [1] when the static angular misalignment is increased, the vibration amplitudes for both vibrations (torsional and lateral vibrations) are increased and because of this reason, restriction of this aspect in practical implementation is significant [1].

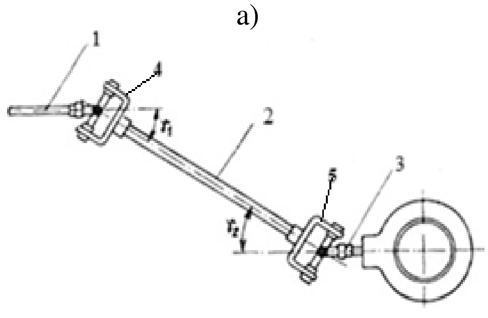
Driveshaft phasing refers to aligning the driveshafts and the U-joints, to balance and prevent unwanted vibration from the vehicle’s driveline system, [7]. Often are two or even three drive and driving shafts that need to be aligned properly and also the U-joints must be put in the same plane to ensure that are in phase and are not generated vibrations which will be transmitted throughout the vehicle [8], [9]. If the cardan shaft driveline is left out of phase will be generated vibrations and these will wear on the driveline components and this way will occur failure, [8]. Because of vibrations will appear premature wear of the input and output seals of transmission of the transmission support bearings and the U-joints.

3. MATHEMATICAL MODEL TO CALCULATE THE SPEED OF DRIVELINE WITH U-JOINTS IN INCORRECT PHASING

The mathematical model is presented that allows the calculation of the output cardan shaft speed depending on the input shaft speed for the constructive variant with U-joints in incorrect phassing, depending on the inclination angle of the cardan shaft. Also it can be calculate the degree of non-uniformity of the driven shaft movement depends of the tilt angle of the cardan shaft. The justification for presenting this mathematical calculation model is that I have not come across works that present such an mathematical model which would allow to calculate the speed of driven cardan shaft in case of out-phase of U-joints, many papers that address vibrations of drivelines with cardan shafts are based mainly on operating simulations using only existing computer software and only in case of inphase U-joints.



a)



b)

Fig. 4. Cardan shaft with U-joints in phase: **(a)** cardan shaft diagram **(b)** cardan shaft construction variant; 1-input shaft; 2-cardan shaft; 3-output shaft; 4-input U-joint; 5-output U-joint;

For the cardan shaft with the U-joints arranged in the same plane (U-joints 4 and 5 in the Figure 4 are coplanar), [6], [9], [10], [11], [12], we can write the relations:

$$\omega_1 = \frac{d\alpha}{dt}; \omega_2 = \frac{d\beta}{dt} \quad (1)$$

$$tg\alpha = tg\varphi_1 \cdot \cos\gamma_1 ; \; tg\beta = tg\varphi_1 \cdot \cos\gamma_2 \quad (2)$$

Result:

$$tg\beta = tg\alpha \cdot \frac{\cos\gamma_2}{\cos\gamma_1} \Rightarrow \frac{d\beta}{dt} = \frac{d\alpha}{dt} \cdot \frac{\cos^2\beta}{\cos^2\alpha} \cdot \frac{\cos\gamma_2}{\cos\gamma_1} \Rightarrow$$
$$\Rightarrow \omega_2 = \omega_1 \cdot \frac{\cos^2\beta}{\cos^2\alpha} \cdot \frac{\cos\gamma_2}{\cos\gamma_1} \quad (3)$$

From the condition $\gamma_1 = \gamma_2 = \gamma$ result $\beta = \alpha \Rightarrow$ synchronism.

For the cardan shaft with the U-joints arranged at 90 degrees (U-joints 4 and 5 in the Figure 4 are perpendicular), the previous relations become:

$$tg\alpha = tg\varphi_1 \cdot \cos\gamma_1 \quad (4)$$

$$tg\beta = tg\left(\varphi_1 + \frac{\pi}{2}\right) \cdot \cos\gamma_2 \quad (5)$$

Result:

$$tg\beta = tg\alpha \cdot \frac{\cos\gamma_2}{\cos\gamma_1} \cdot \frac{tg\left(\varphi_1 + \frac{\pi}{2}\right)}{tg\varphi_1} = tg\alpha \cdot \frac{\cos\gamma_2}{\cos\gamma_1} \cdot \left(-\frac{1}{tg^2\varphi_1}\right) \quad (6)$$

Under the conditions $\gamma_1 = \gamma_2 = \gamma$, and how $tg\varphi_1 = \frac{tg\alpha}{\cos\gamma_1}$, result:

$$tg\beta = \frac{\cos^2\gamma}{tg\alpha} \Rightarrow$$
$$\frac{d\beta}{\cos^2\beta} = \frac{d\alpha \cdot \cos^2\gamma}{\cos^2\alpha \cdot tg^2\alpha} \Rightarrow \frac{\omega_2}{\cos^2\beta} = \frac{\omega_1 \cdot \cos^2\gamma}{\cos^2\alpha \cdot tg^2\alpha} \quad (7)$$

Result:

$$\frac{\omega_2}{\omega_1} = \frac{\cos^2\beta \cdot \cos^2\gamma}{\cos^2\alpha \cdot tg^2\alpha} \quad (8)$$

As we know:

$$\cos^2\beta = \frac{1}{1 + tg^2\beta} = \frac{1}{1 + tg^2\left(\varphi_1 + \frac{\pi}{2}\right) \cdot \cos^2\gamma} =$$
$$= \frac{1}{1 + \cos^2\gamma \cdot \frac{\cos^2\varphi_1}{\sin^2\varphi_1}} = \frac{1}{1 + \cos^2\gamma \cdot \frac{\cos^2\gamma}{tg^2\alpha}} \quad (9)$$

Using the previous relation it will result:

$$\frac{\omega_2}{\omega_1} = \frac{\cos^2\gamma}{\sin^2\alpha + \cos^2\alpha \cdot \cos^4\gamma} \quad (10)$$

For:

$$\alpha = 0 \Rightarrow \frac{\omega_2}{\omega_1} = \frac{1}{\cos^2\gamma} \Rightarrow \omega_{2,\max} = \frac{\omega_1}{\cos^2\gamma} \quad (11)$$

For:

$$\alpha = \frac{\pi}{2} \Rightarrow \omega_{2,\min} = \omega_1 \cdot \cos^2\gamma \quad (12)$$

The degree of non-uniformity of the movement will be:

$$\delta = \frac{\omega_{2,\max} - \omega_{2,\min}}{\omega_1} = \frac{1 - \cos^4\gamma}{\cos^2\gamma} \quad (13)$$

Using relation (10) for construction with perpendicular U-joints (Figure 8), results the variation (Figure 5) of driven cardan shaft speed for 1000 rpm driving shaft speed and tilt angle between 0-30 degrees.

Using relation (10) for construction with perpendicular U-joints (Figure 8), results the variation of driven cardan shaft speed for 2000 rpm driving shaft speed and tilt angle between 0-30 degrees, Figure 6.

According to the variation in the Figure 5 and Figure 6, regardless of the operating speed, results the degree of non-uniformity of the driven shaft movement, Figure 7:

- for 2 -degrees tilt of the cardan shaft results in a 0,24 % speed deviation;
- for 6 -degrees tilt of the cardan shaft results in a 2,2 % speed deviation;
- for 10 -degrees tilt of the cardan shaft results in a 6,12 % speed deviation;
- for 15 -degrees tilt of the cardan shaft results in a 13,88 % speed deviation;
- for 20 -degrees tilt of the cardan shaft results in a 25 % speed deviation;
- for 25 -degrees tilt of the cardan shaft results in a 39,6 % speed deviation;
- for 30 -degrees tilt of the cardan shaft results in a 58,33 % speed deviation.

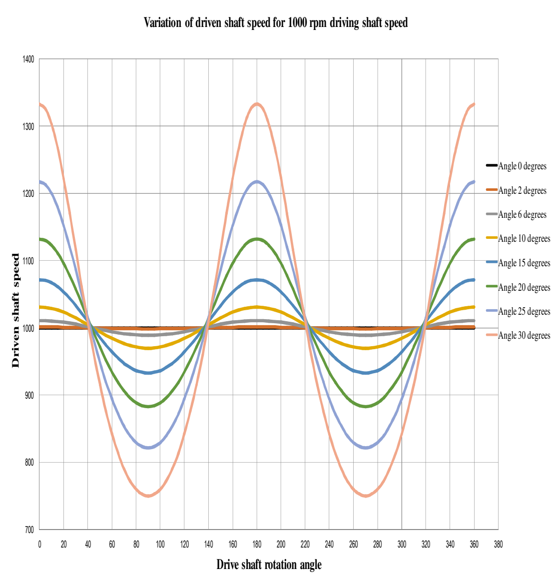


Fig. 5. Variation of driven cardan shaft speed for 1000 rpm driving shaft speed and tilt angle between 0-30 degrees

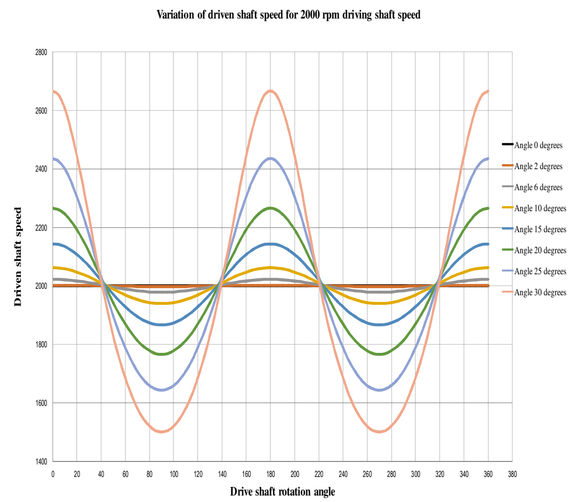


Fig. 6. Variation of driven cardan shaft speed for 2000 rpm driving shaft speed and tilt angle between 0-30 degrees

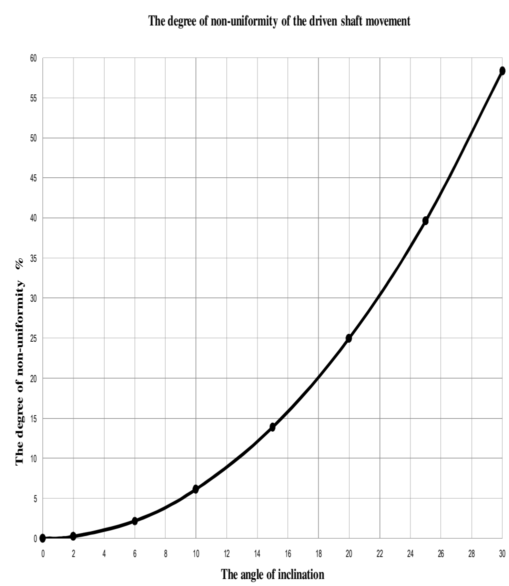


Fig. 7. The degree of non-uniformity of the driven cardan shaft speed

4. DISCUSSION

According to what has been demonstrated and indicated in this study, it is recommended to use cardan shafts (as a component of the vehicle transmission) that have coplanar U-joints, because otherwise (in the case of end U-joints arranged perpendicularly) vibrations will occur (even if the input and output angle of cardan shaft are the same) that propagate into the vehicle chassis, vibrations that are harmful and that amplify with the increase in the angle of the cardan shaft.

For the cardan shaft construction variants that do not respect the coplanar arrangement of the U-joints ends, it turns out why many users complain of strong vibrations that propagate into the vehicle chassis. Even the U-joints are in phase, the option with different angles between input and output should be avoided, as it leads to high vibrations.

However, if we are forced to use construction with perpendicular U-joints (Figure 8), synchronism is recommended and the inclination angle of cardan shaft should not exceed 6 degrees, in which case the degree of non-uniformity of the movement would not exceed 3%, which is a very important observation for transmissions variants with cardan shaft.

5. CONCLUSIONS

U-Joints of cardan shaft must be kept in phase, because else small changes in phase produce medium to big vibrations.

Fig. 8. Construction with perpendicular U-joints, out of phase

If the cardan shaft driveline is left out of phase (as we see in Figure 8 the U-joints are not in the same plane) will appear vibrations and changes of output speeds of the driven cardan shaft and all of this will be lead to failure of transmissions components.

Even the U-joints are in phase, the option with different angles between input and output should be avoided, as it leads to high vibrations.

According to what is established in this work, for construction with perpendicular U-joints (Figure 8), to avoid harmful vibrations, first time the input and output angles of cardan shaft must be the same and then the inclination angle of cardan shaft should not exceed 6 degrees, in which case the degree of non-uniformity of the movement would not exceed 3%, which is a very important observation for transmissions variants with cardan shaft.

The author's personal contribution to this work refers to the achievement of mathematical model which permite calculate the speed of driven shaft of driveling transmissions with U-joints in incorrect phasing and the calculus of the degree of non-uniformity of the movement.

Also the important conclusion: if we are forced to use construction with perpendicular U-joints, to avoid harmful vibrations, first time the input and output angles of cardan shaft must be the same and then the inclination angle of cardan shaft should not exceed 6 degrees, in which case the degree of non-uniformity of the movement would not exceed 3%, which is a very important observation for transmissions variants with cardan shaft.

As the tilt angle of the cardan shaft increases, the degree of non-uniformity of the output shaft movement will increase (according to Figure 7) and the vibrations transmitted to the chassis will be greater.

Through the content of this work, author answer also to many users complain of strong vibrations that propagate into the vehicle chassis when they are forced to used aftermarket cardan shafts with perpendicular U-joints (in incorrect phasing), because they didn’t find any other constructive variants.

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Influența aranjamentului articulațiilor în U a arborilor cardanici asupra vibrațiilor și desincronizării mișcării

Abstract:Lucrarea explică, prin intermediul modelului matematic prezentat, importanța fazării arborelui cardanic. În cazul arborelui cardanic cu articulații cardanice perpendiculare (comparativ cu cel cu articulații cardanice coplanare), apare un grad ridicat de neuniformitate a mișcării, ceea ce va duce la vibrații cu implicații negative asupra uzurii premature a transmisiei, precum și la un disconfort ridicat. Pentru construcția cu articulații cardanice perpendiculare, se recomandă sincronizarea, iar unghiul de înclinare al arborelui cardanic nu trebuie să depășească 6 grade, caz în care gradul de neuniformitate al mișcării nu ar trebui să depășească 3%.

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