TECHNICAL UNIVERSITY OF CLUJ-NAPOCA

ACTA TECHNICA NAPOCENSIS

Series: Applied Mathematics, Mechanics, and Engineering
Vol. 68, Issue III, September, 2025

A STATISTICAL-COMPUTATIONAL FRAMEWORK FOR TRANSIENT AVAILABILITY ANALYSIS IN A COMPLEX SYSTEM

Zeenat ZAIDI

Abstract: *This study proposes a statistical-computational framework to analyze the transient availability of a complex system. A Markov-based model is developed using state transition diagram, with governing differential equations derived to capture dynamic system behavior. The solution is implemented computationally via matrix methods in C++, enabling efficient evaluation of availability under continuous failure and repair processes. To statistically characterize transient-state dependencies, time-availability relationships are investigated using correlation and regression analysis in SPSS. The integrated methodology combines techniques from reliability engineering, applied mathematics, and data analytics, providing a rigorous approach for assessing and optimizing system performance during transient operations.*
Key words: *Availability, differential equations, markov-based model and applied mathematics.*

1. INTRODUCTION

Modern engineering systems are meticulously designed to deliver optimal performance throughout their expected service life. Given the potential for system degradation, understanding the time-dependent availability of these systems is crucial. Traditional reliability studies, which often rely on a binary (two-state) model where a system is either fully operational or completely failed, may not sufficiently capture the complexities of contemporary systems. These complexities encompass intricate system architectures, environmental influences, operational conditions, and dynamic efficiency variations. As a result, the conventional two-state correlation theory falls short for real-world engineering applications. Systems or units can undergo multiple phases of transition, each with a distinct probability of shifting from normal operation to total failure.

In the domain of multi-state systems (MSS), complex systems and their components can operate at varying efficiency levels, requiring advanced analytical and numerical methods for evaluation. Singh [1] pioneered this field by analyzing multi-channel systems through matrix differential equations, while Blischke and Murthy [2] expanded the discussion by linking maintenance and maintainability to reliability, highlighting the economic consequences of system failures. Further advancing this research, Castro and Cavalca [3] utilized genetic algorithms to optimize availability in industrial systems, demonstrating the critical role of reliability, availability, and maintainability (RAM) throughout equipment life cycles. Probabilistic approaches were introduced by Noortwijk et al. [4], who compared maintenance models to balance reliability and life-cycle costs, whereas Srinivasan and Subramanian [5] explored stochastic analysis in systems with multiple standby units. To mitigate failures, Eti et al. [6] integrated RAMS (Reliability, Availability, Maintainability, and Safety) with risk analysis, while Dhillon and Liu [7] examined how human error during maintenance impacts system performance. Practical applications were demonstrated by Herder et al. [8], who developed a RAM simulation model for an industrial plastics plant, showcasing its feasibility in real-world settings.

The process industry, with its interconnected production networks, has also been a focal point

Received: 30.06.25; **Similarities**: 01.09.25: **Reviewed**: 28.08./28.08.25: **Accepted**:11.09.25.

for RAM studies. Sharma and Kumar [9] applied RAM analysis to evaluate system performance, while Rajiv and Poja [10] introduced methodologies for reliability analysts to estimate key metrics like Mean Time Between Failures (MTBF) and Mean Time To Repair (MTTR). Case studies, such as Xie and Li's [11] analysis of a meat processing line, further validated these approaches. Singh et al. [12] provided a comprehensive review of analytical reliability methods, and Zaidi and Goyal [13] employed Markov models to assess availability in the paper industry. Similarly, Zaidi [14] evaluated time-dependent availability in container manufacturing, and Tsarouhas et al. [15] conducted RAM analysis in wine packaging, calculating reliability indices for production lines. Notably, Tsarouhas [16] developed a reliability model for food machinery, emphasizing design optimization, while Mishra et al. [17] and Mathews et al. [18] applied RAM modeling to coal handling systems and power plants, respectively. Patil et al. [19] further advanced the field by combining failure-repair data analysis with Markov chains to estimate steady-state availability in CNC machine tools.
Extending these principles, Kumar et al. [20] used stochastic partial differential equations to analyze paint manufacturing systems, and Taj and Rizwan [21] modeled reliability for complex industrial units. Mathematical modeling has proven indispensable across disciplines, offering insights into system behavior through equations and simulations. For instance, Janmanit and Pochai [22] applied numerical methods to assess airborne transmission risks, while Alburaikan et al. [23] compared probabilistic and fuzzy-based reliability techniques. In aerospace, Elshoubary et al. [24] derived reliability indicators for fuel control systems, and Klankaew and Pochai [25] simulated groundwater pollution models. These diverse applications underscore the versatility of RAM analysis in optimizing system performance across industries, from food production to energy and environmental management.

This research utilizes the matrix method to solve a mathematical model for estimating system availability under various constant failure and repair rates. This paper is structured systematically to present the proposed methodology and its contributions. Following this introductory section (Section 1), which includes the literature review, Section 2 develops the mathematical framework, including the Markov state transition model formulation and derivation of the governing differential equations for transient availability analysis, along with its computational implementation using matrix exponential methods in C++. Section 3 conducts empirical validation through statistical characterization of time-availability dependencies using correlation analysis and regression modeling in SPSS. The final sections provide critical discussion of results, with Section 4 presenting concluding remarks and Section 5 examining the practical implications of this integrated reliability analysis approach for system performance optimization during transient operational phases. The combined application of stochastic modeling, numerical computation, and statistical analytics establishes a rigorous foundation for dynamic system behavior assessment.

2. METHODOLOGY

2.1 The System

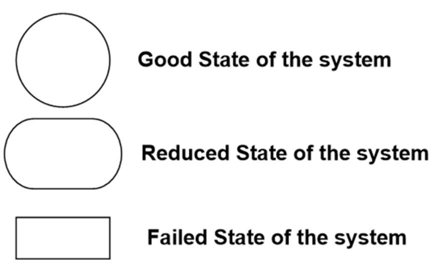
The printing industry comprises four primary subsystems. The printing process requires meticulous attention to various steps, including layout specifications such as size and paper thickness, color selection typically involving four colors, and the inclusion of photos as needed. Once the design is finalized, it undergoes digital processing and is then transferred to an aluminum plate via a scanner machine. The designed material is then loaded into a printing machine for the actual printing process. Finally, the printed material is bound. This industry relies on four key subsystems: computers for processing material, scanner machines, printing machines, and binding machines. Contingency plans are in place for the computer and binding machine subsystems in case of failure. The complexity of the system necessitates a comprehensive mathematical analysis involving numerous differential-difference equations.

2.2 Assumptions and Notations

This research is founded on several

assumptions and notations that guide the construction of the model, as outlined below:
The symbols A, B, C, and D represent the operational states of the computer, scanner machine, printing machine, and binding machine, respectively. The symbols A_1 & D_1 indicate reduced capacity for subsystems A and D.

- Repair rates and failure rates are unrelated and occur independently of each other.
- Only one failure can happen at any given time.
- Systems that have been repaired are restored to a condition similar to new systems.
- Failures in subsystems A and D occur exclusively through reduced states.
- Repairs are conducted solely when the subsystems are in a reduced or failed state.
- a, b, c, d represents the failure states of the subsystems
- λ_i: $(1 \le i \le 6)$ represents failure rate of the subsystems
- μ_i: $(1 \le i \le 6)$ represents repair rate of the subsystems
- P_i (t) = P (i, t) denotes probability that the system is in i-th state at time t, $(0 \le i \le 15)$.



Based on the aforementioned notations and assumptions, the transition diagram is constructed as illustrated in Figure 1.

2.3 Formulation of the System

A transition diagram graphically represents a system's states and the transitions between them. Each state is depicted as a node, while transitions between states are shown as directed arrows labeled with the corresponding rates or probabilities. A model describes the probabilities of various system states over time, effectively analyzing the time-dependent availability of subsystems, particularly when failure and repair rates are constant. The system's state behavior follows a Markov process, where the probability of transitioning to a future state depends solely on the current state, not the path taken to reach that state. The two key variables in this process are state and time. Differential equations are derived using the mnemonic rule, which states that the derivative of the probability at a given state equals the sum of all probability flows entering that state from other states minus the sum of all probability flows leaving that state to other states.

The differential-difference equations (1) – (11) related to the birth-death process are expressed as follows:

$$P_0'(t) + A_1 P_0(t) = \mu_1 P_1(t) + \mu_3 P_5(t) + \mu_4 P_4(t) + \mu_5 P_2(t) \quad (1)$$

$$P_1'(t) + A_2 P_1(t) = \mu_2 P_6(t) + \mu_3 P_7(t) + \mu_4 P_8(t) + \mu_5 P_3(t) + \lambda_1 P_0(t) \quad (2)$$

$$P_2'(t) + A_3 P_2(t) = \mu_1 P_3(t) + \mu_3 P_9(t) + \mu_4 P_{10}(t) + \mu_6 P_{11}(t) + \lambda_5 P_0(t) \quad (3)$$

$$P_3'(t) + A_4 P_3(t) = \mu_2 P_{12}(t) + \mu_3 P_{13}(t) + \mu_4 P_{14}(t) + \mu_6 P_{15}(t) + \lambda_1 P_2(t) + \lambda_5 P_1(t) \quad (4)$$

$$P_4'(t) + \mu_4 P_4(t) = \lambda_4 P_0(t) \quad (5)$$

$$P_5'(t) + \mu_3 P_5(t) = \lambda_3 P_0(t) \quad (6)$$

$$P_{4+i}'(t) + \mu_i P_{4+i}(t) = \lambda_i P_j(t), i = 2, 3, 4, \; j = 1 \quad (7)$$

$$P_{6+i}'(t) + \mu_i P_{6+i}(t) = \lambda_i P_j(t), i = 3, 4, \; j = 2 \quad (8)$$

$$P_{11}'(t) + \mu_6 P_{11}(t) = \lambda_6 P_2(t) \quad (9)$$

$$P_{10+i}'(t) + \mu_i P_{10+i}(t) = \lambda_i P_j(t), i = 2, 3, 4, \; j = 3 \quad (10)$$

$$P'_{15}(t) + \mu_6 P_{15}(t) = \lambda_6 P_3(t) \qquad (11)$$

where

$$A_1 = \sum_{i=1,\neq 2}^{5} \lambda_i$$

$$A_2 = \sum_{i=2}^{5} \lambda_i + \mu_1$$

Let R be the matrix such that $R^{-1}UR = D$, where $D = (d_0, d_1 \ldots\ldots, d_{15})$ be the diagonal matrix of eigen values of the matrix U.

$R^{-1}(\theta I - U)\overline{P}(s,t) = R^{-1}\overline{0} = \overline{0}$ gives

$R^{-1}(\theta I - U)RR^{-1}\overline{P}(s,t) = \overline{0}$ i.e.

$(\theta I - R^{-1}UR)R^{-1}\overline{P}(s,t) = \overline{0}$

$(\theta I - D)V(s,t) = \overline{0}$, where $V(s,t) = R^{-1}\overline{P}(s,t)$

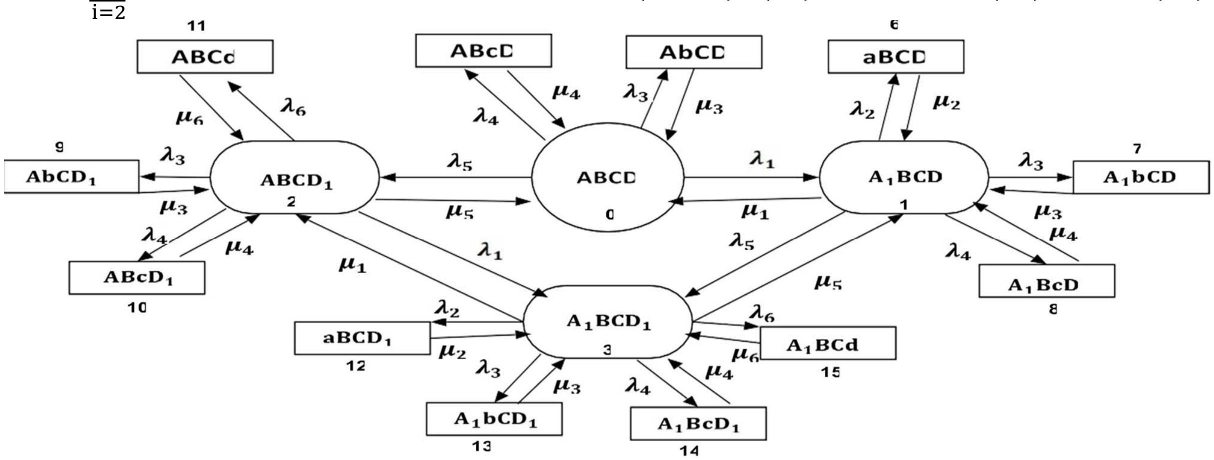


Fig. 1. Transition Diagram

$$A_3 = \sum_{i=1,\neq 2,\neq 5}^{6} \lambda_i + \mu_5$$

$$A_4 = \sum_{i=2,\neq 5}^{6} \lambda_i + \mu_1 + \mu_5$$

With initial conditions at time $t = 0$,

$$P_i(t) = 1 \text{ for } i = 0$$
$$= 0 \text{ for } i \neq 0$$

Taking matrix, U, the matrix of coefficients of probabilistic states, the differential equations may be written as;

$$(\theta I - U)\overline{P}(s,t) = \overline{0},$$

where $\theta = \frac{d}{dt}$, $\bar{0}$ is the null matrix.

$\overline{P}(s,t) = [P_0(t)P_1(t) \ldots \ldots \ldots P_{15}(t)]^T$ and $I_{16\times 16}$ is the identity matrix.

$$\frac{d}{dt}V(s,t) - D\,V(s,t) = \overline{0}$$

Equation is a matrix linear differential equation in variable $V(s,t)$.

Solution of the equation is

$V(s,t)e^{-Dt} = m$………. (i), for some constant m.

Taking particular value $t = 0$ as the initial condition, we get

$m = R^{-1}\overline{P}(s,0)e^{-Dt}$… (ii).

From (i) and (ii), $V(s,t) = R^{-1}\overline{P}(s,0)$.

As $V(s,t) = R^{-1}\overline{P}(s,t)$,

It follows $R^{-1}\overline{P}(s,t) = e^{Dt}R^{-1}\overline{P}(s,0)$

$$\overline{P}(s,t) = R\left(1 + Dt + \frac{D^2t^2}{2!} + \cdots \ldots.\right)R^{-1}\overline{P}(s,0)$$

$$= \left(1 + Ut + \frac{U^2t^2}{2!} + \cdots \ldots\right)R^{-1}\overline{P}(s,0)$$

$$= 1 + M_1 t + \frac{M_2 t^2}{2!} + \cdots \ldots + \frac{M_n t^n}{n!} \ldots \text{(iii)},$$

where $M_n = U^n \overline{P}(s,0)$

The initial conditions make it clear that $\overline{\mathbf{P}}(\mathbf{s},\mathbf{0})$ is the column matrix $(1\ 0\ 0 \ldots\ldots 0)^T$ of order 16×1.

Note that $U\overline{P}(s,0)$ is just the 1st column of the matrix U. Let us denote this column matrix by

$U_1 = (u_{1\,1}\ \ u_{1\,2} \ldots \ldots \ldots \ldots u_{1\,16})^T$

$U^2\overline{P}(s,0) = U.U\overline{P}(s,0) = U.U$, is again a column matrix, let us denote it by,

Matrix U

$$= \begin{bmatrix}
-A_1 & \mu_1 & \mu_5 & 0 & \mu_4 & \mu_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\lambda_1 & -A_2 & 0 & \mu_5 & 0 & 0 & \mu_2 & \mu_3 & \mu_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\lambda_5 & 0 & -A_3 & \mu_1 & 0 & 0 & 0 & 0 & 0 & \mu_3 & \mu_4 & \mu_6 & 0 & 0 & 0 & 0 \\
0 & \lambda_5 & \lambda_1 & -A_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu_2 & \mu_3 & \mu_4 & \mu_6 \\
\lambda_4 & 0 & 0 & 0 & -\mu_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\lambda_3 & 0 & 0 & 0 & 0 & -\mu_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \lambda_2 & 0 & 0 & 0 & 0 & -\mu_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \lambda_3 & 0 & 0 & 0 & 0 & 0 & -\mu_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \lambda_4 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \lambda_3 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu_3 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \lambda_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu_4 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \lambda_6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu_6 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \lambda_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu_2 & 0 & 0 & 0 \\
0 & 0 & 0 & \lambda_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu_3 & 0 & 0 \\
0 & 0 & 0 & \lambda_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu_4 & 0 \\
0 & 0 & 0 & \lambda_6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu_6
\end{bmatrix}$$

$U_2 = (v_{1\,1}\ \ v_{1\,2} \ldots \ldots \ldots \ldots v_{1\,16})^T$.

Let $U^{r-1}\overline{P}(s,0) = U_{r-1} = (P_{1\,1}\ \ P_{1\,2} \ldots \ldots \ldots \ldots P_{1\,16})^T$

It follows from mathematical induction that

$U^r\overline{P}(s,0) = UU_{r-1} = (l_{1\,1}\ \ l_{1\,2} \ldots \ldots \ldots \ldots l_{1\,16})^T$ for some $l_{i\,j}$'s.

Note that the identity (iii) is a column matrix,

Probability of different stages are;

$$\mathbf{P_0(t) = P(0,t) = 1 + u_{11}t + \frac{v_{11}t^2}{2!} +}\ \ldots\ldots$$

$$\mathbf{P_1(t) = P(1,t) = u_{21}t + \frac{v_{21}t^2}{2!} +}\ \ldots\ldots$$

$$P_i(t) = P(i,t) = u_{i1}t + \frac{v_{i1}t^2}{2!} +\ \ldots\ldots$$

Since P (0, t), P (1, t), P (2, t) and P (3, t) are the only working states of the system, so Availability is

Av (t) = P (0, t) + P (1, t) + P (2, t) + P (3, t)

$$= 1 + (u_{11} + u_{21} + u_{31} + u_{41})t + (v_{11} + v_{21} + v_{31} + v_{41})\frac{t^2}{2!} + \ \ldots\ldots$$

3. AVAILABILITY ANALYSIS

3.1 Availability Analysis in Transient State

Availability of the system at time t is

Av (t) = P (0, t) + P (1, t) + P (2, t) + P (3, t)

Results are generated using the computer program, and the table and graph illustrating the transient state availability are presented below. Take the failure and repair rates as $\lambda_1 = 0.002$, $\lambda_2 = 0.001$, $\lambda_3 = 0.001$, $\lambda_4 = 0.0015$, $\lambda_5 = 0.0015$, $\lambda_6 = 0.001$, $\mu_1 = 0.015$, $\mu_2 = 0.001$, $\mu_3 = 0.001$, $\mu_4 = 0.02$, $\mu_5 = 0.015$ and, $\mu_6 = 0.001$.

Table 1

Variation of availability with time

Time	**10**	**20**	**30**	**40**	**50**
Availability	0.9766	0.9559	0.9376	0.9211	0.9061
Time	**60**	**70**	**80**	**90**	**100**
Availability	0.8924	0.8799	0.8684	0.8581	0.8498

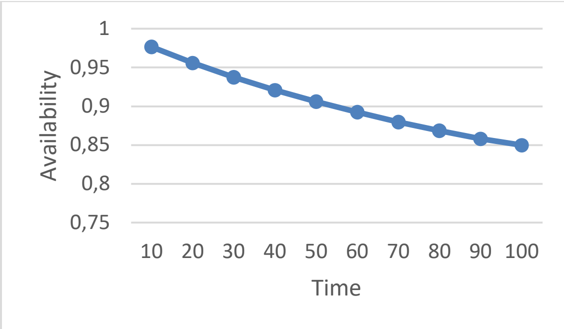


Fig. 2. Variation of availability with time

The data demonstrates a clear trend of decreasing availability over time, with each 10-unit increment resulting in a consistent decline in the system's performance. Initially, availability is high at 97.66%, but over 100 units, it falls to 84.98%. This trend suggests that the system experiences gradual wear and tear or other factors that reduce its operational efficiency over time. The steady decline underscores the importance of maintenance and potential interventions to mitigate the reduction in availability and ensure the system remains as functional as possible.

3.2 Correlation and Regression Analysis

A pearson correlation analysis is conducted to assess the strength of the relationship between time and availability in the system using SPSS software. The findings in Table II showed a strong, statistically significant negative correlation between time and availability, with a correlation coefficient: $R = -0.992,\ p < 0.001$.

Table 2

Correlation analysis

		Time	Availability
Time	**Pearson Correlation**	1	-.992**
	Sig. (2-tailed)		<.001
	N	10	10
Availabi-lity	**Pearson Correlation**	-.992**	1
	Sig. (2-tailed)	<.001	
	N	10	10

**. Correlation is significant at the 0.01 level (2-tailed).

Simple linear regression analysis is conducted to assess the impact of time on predicted availability using SPSS software.

Table 3

Regression Statistics

Multiple R	0.992
R Square	0.984
Adjusted R Square	0.983
Standard Error	0.005636
Observations	10

Table 4

Anova[a]

Model	Sum of Squares	df	Mean Square	F	Sig.
Regression	.016	1	.016	522.023	<.001[b]
Residual	.000	8	.000		
Total	.016	9			

a. Dependent Variable: Availability

b. Predictors: (Constant), Time

Table 5

Coefficients[a]

	Unstandardized Coefficients		Standard Coefficients		
Model	**B**	**Std. Error**	**Beta**	**t**	**Sig.**
Constant	.9815	.004		255.01	<.001
Time	-.0014	.000	-.992	-22.84	<.001

a. Dependent Variable: Availability

Table 3 shows a R-square value $(R^2) = 0.984$, indicating that 98.4% of the variability in availability can be explained by changes over time.

The substantial R-square value indicates a strong and significant association between the variables, suggesting that as time advances, there is a noteworthy effect on the system's availability in the printing plant. Table 4 presents the ANOVA results which illustrates the overall regression is statistically significant $(R^2 = 0.984, F(1,8) = 522.023, p < 0.001)$, which indicates that time has a significant impact on availability.

Table 5 presents the coefficients, with a beta value of -0.992, indicating that a one-unit increase in time corresponds to a 0.992-unit decrease in availability.

Estimated Regression line:

$$Availability = 0.9815 - 0.0014(Time)$$

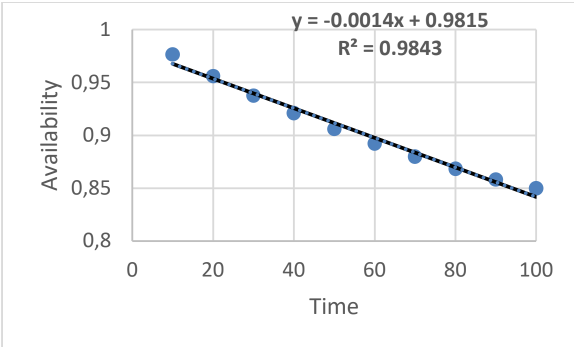


Fig. 3. Estimated regression line

Figure 3 illustrates the estimated regression line, which represents the statistical model describing the relationship between the independent variable (time) and the dependent variable (availability) in a simple linear regression analysis.

4. CONCLUSION

This research presents a systematic evaluation of transient-state system availability through an integrated Markov modeling and statistical analysis approach. The investigation reveals a progressive deterioration in operational reliability, with availability metrics decreasing from 97.66% at initial operation to 84.98% after extended runtime. Statistical evaluation demonstrates a significant inverse relationship between operational duration and system performance ($r = -0.992$, $p < 0.001$), with temporal factors accounting for 98.4% of observed variability ($R^2 = 0.984$). The developed predictive equation (Availability = 0.9815 - 0.0014 × Time) provides a reliable tool for forecasting system performance. These outcomes emphasize the necessity for strategic maintenance planning to preserve system functionality in industrial applications where continuous operation is critical.

5. DISCUSSION

The progressive decline in system performance corresponds with theoretical expectations of component degradation under sustained operational demands. While the Markov model successfully characterizes the transient behavior, incorporating alternative failure distributions could enhance model precision for specific operational contexts. The exceptionally strong temporal correlation ($r = -0.992$) establishes time as the primary determinant of availability reduction, though this linear relationship may require reassessment for prolonged operational periods. The combined computational-statistical methodology offers a comprehensive analytical framework for reliability assessment, though its dependence on constant failure and repair parameters may necessitate adjustments for dynamic operating conditions. Practical applications of these findings suggest implementing preemptive maintenance protocols and optimizing repair resource allocation, particularly for high-failure components. Future research directions should include empirical validation across diverse operational environments and investigation of additional performance-influencing variables. This study provides both a methodological framework for transient-state analysis and practical insights for reliability optimization in complex operational systems.

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UN CADRU STATISTICO-COMPUTAȚIONAL PENTRU ANALIZA DISPONIBILITĂȚII TRANZITORII ÎNTR-UN SISTEM COMPLEX

Rezumat: Acest studiu propune un cadru statistico-computațional pentru a analiza disponibilitatea tranzitorie a unui sistem complex. Un model bazat pe Markov este dezvoltat folosind diagrama de tranziție de stare, cu ecuații diferențiale de guvernare derivate pentru a capta comportamentul dinamic al sistemului. Soluția este implementată din punct de vedere computațional prin metode matriceale în C++, permițând evaluarea eficientă a disponibilității în cadrul proceselor continue de defectare și reparare. Pentru a caracteriza statistic dependențele dintre stările tranzitorii, relațiile timp-disponibilitate sunt investigate utilizând analiza de corelație și regresie în SPSS. Metodologia integrată combină tehnici din ingineria fiabilității, matematica aplicată și analiza datelor, oferind o abordare riguroasă pentru evaluarea și optimizarea performanței sistemului în timpul operațiunilor tranzitorii.

Zeenat ZAIDI, Department of Mathematics, College of Science, Qassim University, Buraidah 51452, Qassim, Kingdom of Saudi Arabia (Email: z.hasain@qu.edu.sa), Home Address: 2146, AlFaiziah, Al AAkhdar, Buraidah, Qassim, Saudi Arabia, 52391, Mobile no. +966 503936490.