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THE INVARIANTS OF AN ARTICULATED BEAM STRUCTURES UNDETERMINED STATICALLY IN THE FORCES METHOD

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Abstract: In the paper there are presented the invariants of an articulated beam structures undetermined statically in the forces method, invariants through which the fact that any infinite like variation of an effort in a beam r of the structure produces a variation of the same order of the dimensions am the section of the beam s of that same structure.

Key words: invariants, undetermined structure, force method

1. INTRODUCTION

It is known that in the force method (effort) the statically undetermined structure is transformed through the deletion of the supplementary links in an undetermined structure called base system. Instead of the suppressed links the forces that constitute the variables of the problem. Suppressing the links of a statically undetermined structure for it to get to the form (system) base can be done in many ways, but they all have to take in consideration a condition: the base system must not be a critical form. This way, in the force method the base system is not unique like in the displacement method therefore more base systems exist. In the following it is presented the identification of the invariants of a structure of bars articulated in nodes, statically undetermined, and its analysis is done through the effort method in classical and matricial way.

2. THEORETICAL STUFF

The force method [1] uses as variables the forces in the supplementary links the number of this variables being equal to the degree of statically indetermination. The statically determined structure obtained by removing the number of simple links equal to the degree of

statically indetermination can be solved for any external load. The general condition imposed to the base system is that, the system, loaded with the exterior given forces and the unknown ones, representing the mechanical equivalent of the removed links, should act the same way as a real structure. This means that the all the displacements on all directions of the variables are null, $\Delta_i=0=(i=1, \dots, n)$, because in reality in these directions there are links and there are no displacements.

In the classical formulation of the forces method, these conditions are given through the linear system:

$$\sum_{j=0}^n \delta_{ij} \cdot X_j + \Delta_{ip} = 0 \quad (1)$$

in which:

X_i, \dots, X_n are the forces of unknown values from the suppressed links;

δ_{ij} is the displacement on the direction of the X_i variable, when the base system is loaded only with the variable $X_j = I$;

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Δ_{ip} is the displacement on the direction of the X_i variable when the base system is loaded only with the exterior given forces.

The displacements δ_{ij} and δ_{ji} are called unitary displacements, and in the (1) system they are coefficients of the variables.

After the conceiving and the solving of the equations system values for the variable X_i are obtained, values who can help for determining the final efforts in the bars by using the superposition of effects with the help of the following:

$$N_i = N_i^0 + \sum_{j=1}^n n_{ij} \cdot X_j \quad (2)$$

In matricial formulation the analysis of the statically undetermined structure through the force method [2] leads to the following operation sequence:

The vector of exterior actions is created $A_s = \begin{Bmatrix} A_s^D \\ A_s^R \end{Bmatrix}$ and the equilibrium matrix $H_s = \begin{bmatrix} H_s^D \\ H_s^R \end{bmatrix}$, who determine the relation:

$$A_s = \begin{Bmatrix} A_s^D \\ A_s^R \end{Bmatrix} = \begin{bmatrix} H_s^D \\ H_s^R \end{bmatrix} \bar{P}_s \quad (3)$$

where: A_s is the action vector on the structure nodes; \bar{P}_s the vector of independent forces on the two ends of the bars; H_s the equilibrium matrix of the bar.

The statically determined system is adopted, and the forces from the suppressed links chosen as variables statically undetermined will be collected in the variable vector X and the equilibrium matrix H_s^X is created, this leads to:

$$X = H_s^X \cdot P_s \quad (4)$$

By combining (3) and (4) yields:

$$\begin{Bmatrix} A_s^D \\ A_s^R \end{Bmatrix} = \begin{bmatrix} H_s^D \\ H_s^R \end{bmatrix} \bar{P}_s = H_s^* \cdot \bar{P}_s \quad (5)$$

The matrices B_0 and B are obtained by inverting the matrix H_s^* or directly, by expressing the equilibrium of the nodes of the base system thus obtaining:

$$\bar{P}_s = [H_s^*]^{-1} \cdot \begin{Bmatrix} A_s^D \\ X \end{Bmatrix} = \begin{bmatrix} B_0 \\ B \end{bmatrix} \cdot \begin{Bmatrix} A_s^D \\ X \end{Bmatrix} \quad (6)$$

The flexibility matrices of the of the bars are calculated and are assembled into the quasidiagonal matrix \bar{F}_s :

$$\bar{F}_s = \begin{bmatrix} \bar{F}_a & & & \\ & \bar{F}_b & & \\ & & \ddots & \\ & & & \bar{F}_i & \\ & & & & \ddots \end{bmatrix} \quad (7)$$

The matrices are calculated:

$$F_{XX} = B^T \cdot \bar{F}_s \cdot B \quad (8)$$

$$F_{XD} = B^T \cdot \bar{F}_s \cdot B_0 \quad (9)$$

The F_{xx} matrix is inverted and the vector of the variables statically undetermined is determined with:

$$X = -F_{XX}^{-1} \cdot F_{XD} \cdot A_s^D \quad (10)$$

The vector of the variables X given in the relation (10) is introduced in (6) and the forces \bar{P}_s resulted on the end of the bars, or the equilibrium matrix of the statically undetermined structure is determined with:

$$B' = B_0 \cdot B \cdot F_{XX}^{-1} \cdot F_{XD} \quad (11)$$

this yields:

$$\bar{P}_s = B' \cdot A_s^D \quad (12)$$

If the nodes displacements are required the flexibility matrix of the statically undetermined structure is calculated:

$$F_s = B_0^T \cdot \bar{F}_s \cdot B' \quad (13)$$

and from here the displacements are obtained with:

$$D_s^D = F_s \cdot A_s^D \quad (14)$$

The exterior action vector is determined with:

$$A_s = \begin{Bmatrix} A_s^D \\ H_s^R \cdot B' \cdot A_s^D \end{Bmatrix} \quad (15)$$

and then the reaction vector is calculated:

$$A_s = A_s - A_e \quad (16)$$

or with:

$$A_r = H_s^R \cdot \bar{P}_s \quad (17)$$

3. DETERMINATION OF THE INVARIANTS

A structure of articulated bars in nodes, statically indeterminate (fig. 1) is considered. The bars have the same stiffness EA , their lengths in meters and $P = 10 \text{ kN}$. Two base systems are considered [3] R_1 (fig. 2) and R_2 (fig. 3) of the same statically undetermined structure to which the following variables correspond $X_i, Y_i, (i = 1, \dots, n)$

The expression of the effort in a bar of the structure is:

$$N_i = N_i^0 + \sum_{j=1}^n n_{ij} \cdot X_j \quad (18)$$

where N_i^0 represents the effort from the exterior load and n_{ij} from the load given by $X_j (j = 1, \dots, n)$. Considering the structure R_1 , it is obtained:

$$N_i = N_i^{(1)} + \sum_{j=1}^n n_{ij}^{(1)} \cdot X_j \quad (19)$$

and for the R_2 structure it is obtained:

$$N_i = N_i^{(2)} + \sum_{j=1}^n n_{ij}^{(2)} \cdot Y_j \quad (20)$$

The structure from fig. 1 is twice statically undetermined once on the inside and once on the outside

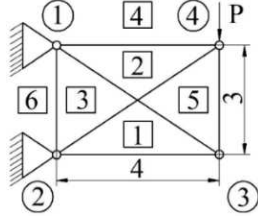


Fig. 1. Statically undetermined structure

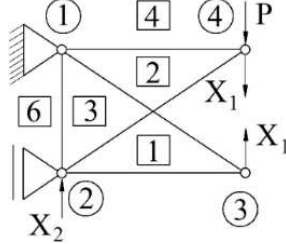


Fig. 2. Base system R_1

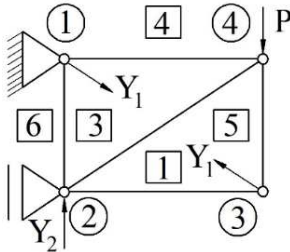


Fig. 3. Base system R_2

To ease the calculations and their following, they have been organized in tables table 1 for the base system R_1 and table 2 for the base system R_2 .

The coefficients of the variables and the free terms are calculated through:

$$\delta_{ij} = \sum \frac{n_i \cdot n_j \cdot l}{EA} \quad (21)$$

$$\Delta_{ij} = \sum \frac{n_i \cdot N_i^0 \cdot l}{EA} \quad (22)$$

The systems are for the base system R_1 :

$$\begin{cases} 48 \cdot X_1 - 3 \cdot X_2 = -24P \\ -3 \cdot X_1 + 3 \cdot X_2 = 3P \end{cases}$$

and for the system R_2 :

$$X = -F_{XX}^1 \cdot F_{XD} \cdot A_s^D = \begin{bmatrix} -\frac{1}{45} & -\frac{1}{45} \\ -\frac{1}{45} & -\frac{16}{45} \\ \frac{16}{45} & -\frac{1}{45} \end{bmatrix} \cdot \begin{bmatrix} \frac{16}{3} & 21 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \cdot \begin{pmatrix} 0 \\ 0 \\ 0 \\ -10 \end{pmatrix} = \begin{pmatrix} -\frac{14}{3} \\ -\frac{16}{3} \\ \frac{16}{3} \end{pmatrix}$$

for the base system R_2 (fig.3).

$$\begin{cases} \frac{432}{25} \cdot Y_1 - \frac{9}{5} \cdot Y_2 = \frac{216}{15} P \\ \frac{9}{5} \cdot Y_1 + 3 \cdot Y_2 = 3P \end{cases}$$

The values of the variables for the two systems are:

$$\begin{aligned} X_1 &= \frac{-7P}{15} & X_2 &= \frac{8P}{15} \\ Y_1 &= \frac{7P}{15} & Y_2 &= \frac{8P}{15} \end{aligned}$$

The final efforts calculated are within the last column of the 1 tables and table 2.

The effort in the bars being the same this it results:

$$N_i^{0(1)} + \sum_{j=1}^n n_{ij}^{(1)} \cdot X_j = N_i^{0(2)} + \sum_{j=1}^n n_{ij}^{(2)} \cdot X_j \quad (23)$$

The initial structure is n times undetermined, the axial effort in the bar i is depending only on independent parameters:

$$n_{ij}^{(2)} = \lambda_{j1} \cdot n_{i1}^{(1)} + \dots + \lambda_{jk} \cdot n_{ik}^{(1)} + \dots + \lambda_{jn} \cdot n_{in}^{(1)} \quad (24)$$

and

$$N_{ij}^{0(2)} = N_{ij}^{0(1)} + \mu_{j1} \cdot n_{i1}^{(1)} + \dots + \mu_{jn} \cdot n_{in}^{(1)}$$

The condition equation systems for the variables X , and Y , are:

$$\begin{cases} \sum_{j=1}^n \delta_{ij}^{(1)} \cdot X_j + \Delta_i^{0(1)} = 0 \quad (i=1, \dots, n) \\ \sum_{j=1}^n \delta_{ij}^{(2)} \cdot Y_j + \Delta_i^{0(2)} = 0 \quad (i=1, \dots, n) \end{cases} \quad (25)$$

where Δ_p are the displacements on the direction of the variable X , or Y , produced by applying to the structure the given load, and δ_{ij} are the unitary on the direction of the variable X_i produced by applying successively on the base structure of the unitary loads $X_i=1$, ($i=1, \dots, n$).

The matrices of the displacements are:

$$\begin{aligned} U_1 &= |\delta_{ij}^{(1)}|, & U_2 &= |\delta_{ij}^{(2)}| \\ U_3 &= |\Delta_{ij}^{(1)}|, & U_4 &= |\Delta_{ij}^{(2)}| \end{aligned} \quad (26)$$

This way, the statically undetermined variables become:

$$X_j = -U_1^{-1} \cdot U_3, \quad Y_j = -U_2^{-1} \cdot U_4 \quad (27)$$

where X , Y are the column vectors of the statically undetermined variables.

For the structure in fig 1 the statically undetermined variables are according to ..., for the base system R_1 (fig.2).

$$Y = -F_{YY}^{-1} \cdot F_{YD} \cdot A_s^D = \begin{bmatrix} \frac{5}{81} & \frac{1}{27} \\ \frac{1}{27} & -\frac{45}{27} \end{bmatrix} \cdot \begin{bmatrix} -\frac{16}{3} & \frac{81}{5} & -\frac{16}{3} & \frac{72}{5} \\ 0 & 3 & 0 & 3 \end{bmatrix} \cdot \begin{Bmatrix} 0 \\ 0 \\ 0 \\ -10 \end{Bmatrix} = \begin{Bmatrix} \frac{79}{9} \\ \frac{16}{3} \end{Bmatrix}$$

The effort in the bar r is called S^0 and we have:

$$S^0 = S_p^0 + \bar{S}^0 \cdot X = S_p^0 - \bar{S}^0 \cdot U_1^1 \cdot U_3 \quad (28)$$

for the base structure R_1 and

$$S^0 = R_p^0 - \bar{R}^0 \cdot U_2^1 \cdot U_4 \quad (29)$$

for the base structure R_2 where S_p^0 represents the effort corresponding to the load, and \bar{S}^0 the effort corresponding to the unitary variables.

$$\bar{P}_s = B_0^X \cdot B^X \cdot A_s^D = \begin{pmatrix} \begin{bmatrix} 1 & \frac{16}{3} & 0 & 0 \\ 0 & \frac{5}{3} & 0 & 0 \\ 0 & -\frac{5}{3} & 0 & 0 \\ 0 & 0 & 0 & \frac{5}{3} \\ 0 & 0 & 1 & -\frac{4}{3} \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} \frac{4}{3} & 0 \\ -\frac{5}{3} & 0 \\ -\frac{5}{3} & 0 \\ -\frac{4}{3} & 0 \\ 1 & 0 \\ 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} \frac{16}{45} & \frac{1}{45} \\ \frac{1}{45} & \frac{16}{45} \\ \frac{16}{3} & 0 \\ -24 & 3 \end{bmatrix}^T \cdot \begin{Bmatrix} 0 \\ 0 \\ 0 \\ -10 \end{Bmatrix} = \begin{Bmatrix} \frac{56}{9} \\ \frac{70}{9} \\ \frac{80}{9} \\ -\frac{9}{14} \\ \frac{14}{3} \\ 0 \end{Bmatrix}$$

and for the base systems R_2 .

$$\begin{pmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & \frac{5}{3} & 0 & \frac{5}{3} \\ 0 & \frac{4}{3} & 1 & -\frac{4}{3} \\ 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & -1 \end{bmatrix} & \begin{bmatrix} -\frac{4}{5} & 0 \\ 1 & 0 \\ 1 & 0 \\ -\frac{4}{5} & 0 \\ -\frac{5}{3} & 0 \\ 3 & -1 \end{bmatrix} \cdot \begin{bmatrix} \frac{5}{81} & -\frac{1}{27} \\ -\frac{1}{27} & \frac{16}{45} \end{bmatrix} \cdot \begin{bmatrix} -\frac{16}{3} & 0 \\ \frac{81}{5} & 3 \\ \frac{16}{3} & 0 \\ \frac{72}{5} & 3 \end{bmatrix}^T \cdot \begin{Bmatrix} 0 \\ 0 \\ 0 \\ -10 \end{Bmatrix} = \begin{Bmatrix} -\frac{56}{9} \\ \frac{70}{9} \\ \frac{80}{9} \\ -\frac{9}{14} \\ \frac{14}{3} \\ 0 \end{Bmatrix}$$

By stating that $Y^{(1)}$ are the reactions from the structure R_1 because of the variables Y and $X^{(2)}$ the reactions in R_2 because of X , we obtain:

$$Y = Y^{(1)} + A_1 \cdot X \quad (39)$$

for the R_1 structure and:

$$X = X^{(1)} + A_2 \cdot Y \quad (40)$$

for the R_2 structure.

therefore, $Y = Y^{(1)} + A_1 \cdot X^{(2)} + Y$, which yields:

$$Y^{(1)} = -A_1 \cdot X^{(2)}, \quad X^{(2)} = -A_2 \cdot Y^{(1)} \quad (41)$$

The reactions in the base structures R_1 and R_2 are obtained from each other through a linear transformation.

\bar{S} is known as the effort in the bars because of the unitary variables we obtain:

$$S = S_p + \bar{S} \cdot X \quad (42)$$

for the base structure R_1 and:

$$S = R_p + \bar{R} \cdot X \quad (43)$$

The matrices and R_p^0 have the same meaning as S_p^0 and \bar{S}^0 .

For the structure in fig. 1, taken as a calculation example the independent forces from the ends of the bars, are using the (12) for the base systems R_1 .

for the base structure R_2 .

Having:

$$Y = A_1 \cdot (X - X^{(2)}), \quad X = A_2 \cdot (Y - Y^{(1)}) \quad (44)$$

the expression (42) becomes:

$$S = S_p + \bar{S} \cdot A_2 \cdot Y - \bar{S} \cdot A_2 \cdot Y^{(1)} \quad (45)$$

by comparing (45) and (43) we notice that:

$$\bar{R} \cdot Y = \bar{S} \cdot A_2 \cdot Y \quad (46)$$

or

$$\bar{R} = A_2^T \cdot \bar{S}_2$$

Where A_2^T is the transposed matrix of A_2 .

The same way,

$$\bar{S} = A_2^T \cdot \bar{R} \quad (47)$$

where A_2^T is the transposed matrix of A_2 .

From (45) and (43) we obtain:

$$S_p - R_p = \bar{S} \cdot X^{(2)} \quad (48)$$

$$S_p - R_p = \bar{S} \cdot Y^{(2)} \quad (49)$$

the expressions (48) and (49) can also be written:

$$\begin{aligned} N^{0(2)} - N^{0(1)} &= \sum_{i=1}^n n_i^{(1)} \cdot X_i^{(2)} \\ N^{0(1)} - N^{0(2)} &= \sum_{i=1}^n n_i^{(2)} \cdot Y_i^{(2)} \end{aligned} \quad (50)$$

The displacement:

$$U_4 = \sum \int \bar{R} \cdot R_p \cdot \Omega ds \quad (51)$$

and by taking into consideration (49)

$$U_4 = \sum \int \bar{R} \cdot (S_p \cdot \bar{R} \cdot Y^{(1)}) \Omega ds \quad (52)$$

where $\Omega = \frac{1}{EA}$

By using (46) we obtain:

$$U_4 = A_2^T \cdot U_3 - U_2 \cdot Y^{(1)} \quad (53)$$

relation, which describes the linear transform of the displacements according to a base structure into the displacements according to the other base system.

For calculating the coefficients of the variables statically undetermined:

$$\delta_{ij} = \sum \frac{\bar{n}_i \cdot \bar{n}_j \cdot l}{EA} \quad (54)$$

we use (46) and we obtain:

$$U_2 = A_2^T \cdot U_1 \cdot A_2, \quad U_1 = A_1^T \cdot U_2 \cdot A_1 \quad (55)$$

by using (41) we obtain:

$$U_3 = A_i^T \cdot (U_4 + U_2 \cdot Y^{(1)}) \quad (56)$$

$$U_4 = A_2^T \cdot (U_3 + U_1 \cdot X^{(2)}) \quad (57)$$

From the relations (27), (28) and (48) yields:

$$\begin{aligned} S_p^0 \cdot R_p^0 &= \bar{R}^0 \cdot Y^{(1)} \\ \bar{R}^0 \cdot Y^{(1)} + \bar{R}^0 \cdot U_2^1 \cdot U_4 \cdot \bar{S}^0 \cdot U_1^1 \cdot U_3 &= 0 \end{aligned} \quad (58)$$

or by using (51) and (52) we obtain:

$$\int \bar{R}^0 \cdot U_2^1 \cdot \bar{R} \cdot S_p \cdot \Omega ds = \int \bar{S}^0 \cdot U_1^1 \cdot \bar{S} \cdot S_p \cdot \Omega ds \quad (59)$$

The expression (59) is true for any given bar r and for any value of the effort S_p in the bar s , if only, for a pair of s and r bars:

$$\bar{R}^0 \cdot U_2^1 \cdot \bar{R} = \bar{S}^0 \cdot U_1^1 \cdot \bar{S} \quad (60)$$

The expressions $\bar{R}^0 \cdot U_2^1 \cdot \bar{R}$, $\bar{S}^0 \cdot U_1^1 \cdot \bar{S}$ represent the invariants of the statically undetermined structure these invariants, defined by:

$$Z = \bar{R} \cdot U^{-1} \cdot \bar{S} \quad (61)$$

do not depend by the chosen structure or the load.

In the relation (61), \bar{S} represents the effort in the bar s and \bar{R} in the bar r .

For the base systems considered in the example:

$$\bar{R}^0 \cdot U_2^1 \cdot \bar{R}$$

for the base system R_2 and:

$$\bar{S}^0 \cdot U_1^1 \cdot \bar{S}$$

for the base system R_1 .

For $n = 2$, the invariants of the axial forces are:

Where $\Delta_1 = \delta_{11} \cdot \delta_{22} - \delta_{12}^2$ is the determinative of the displacements.

4. CONCLUSIONS

The present study demonstrates that the invariants of a statically indeterminate structure with articulated nodes are independent of both the chosen base system and the external loading conditions. The core findings are summarized as follows:

- intrinsic properties: the invariants $Z = R \cdot U \cdot I \cdot S$ are strictly a function of the structural topology and the mechanical properties (stiffness EA) of the component bars.

- analytical consistency: regardless of the transformation between different base systems (e.g., R_1 to R_2), the linear transformations ensure that the physical response of the structure remains invariant.

- practical application: a critical property identified is that any variation in the internal efforts of a specific member r correlates directly

with a variation of the same order in the stiffness characteristics $1/EA$ of member s .

- methodological efficiency: the invariant-based approach provides a robust framework for verifying the correctness of matrix force method computations, offering a unique "signature" of the statically indeterminate structure.

From the expression of the invariants it is observed that they depend only on the geometrical configuration of the statically undetermined structure and of the mechanical characteristics of the articulated bars that make the structure.

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Invarianții unei structuri articulate de grinzi, static nedeterminate, Prin metoda forțelor

Rezumat: În lucrare sunt prezentați invarianții structurilor static nedeterminate de grinzi articulate în metoda forțelor, invarianți prin care se demonstrează că orice variație infinit de mică a unui efort într-o grindă r a structurii produce o variație de același ordin de mărime a secțiunii grinzii s din aceeași structură.

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