TECHNICAL UNIVERSITY OF CLUJ-NAPOCA

ACTA TECHNICA NAPOCENSIS

Series: Applied Mathematics, Mechanics, and Engineering
Vol. 69, Issue Special I, February, 2026

NUMERICAL AND ANALYTICAL INVESTIGATION OF THE ORTHOGONAL CUTTING PROCESS OF C45 MEDIUM CARBON STEEL

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Abstract: *This study builds on previously published experimental research of the machining process by applying analytical and numerical modeling to better interpret the observed results. The analytical Oxley model and a 2D orthogonal cutting simulation using FEM DEFORM software are employed to investigate the cutting zone behavior for C45 medium carbon steel, a commonly used reference material. The FEM analysis reveals that the distribution of deformations in the shear plane is not constant, in contrast to the assumption of the simplified Oxley machining model. The FEM predictions show the closest agreement with the experimental findings and provide additional insight into cutting phenomena that are difficult to access experimentally. The combined use of validated analytical and numerical models complements the earlier experimental work, deepens the understanding of machining mechanics, and supports the development of improved machining strategies, cutting tool design, and enhanced production efficiency.*

Key words: Orthogonal Cutting Process, Analytical Model, FEM (Finite Element Method), Cutting Zone, Chip Formation, Johnson-Cook Flow Law, Shear Plane

1. INTRODUCTION

Understanding the mechanisms of chip formation, shear plane behavior, and plastic deformation in the cutting zone is fundamental to optimizing machining processes and enhancing tool performance. Examination of chip formation not only provides insight into the material removal process but also supports tool design to improve the machining process, tool life, and surface integrity and roughness of the workpiece surfaces. Several researchers have employed experimental work, analytical, and numerical calculations to study these phenomena in various machining conditions. Some experimental methods are often employed to measure deformation and friction at the tool-chip interface, as well as to observe microstructural changes in the cutting zone. Ortiz-de-Zarate et al. [1] introduced a methodology for determining the friction and normal stress distributions on the rake face of tools during orthogonal cutting and demonstrated the distinct regions of sticking and sliding contact. Similarly, Necpal and Martinkovič [2] analyzed chip compression and plastic deformation in steel C45 through metallographic techniques, focusing on grain orientation and stress distribution. These experimental setups provide a direct understanding of chip morphology and cutting zone behavior.

Analytical models are another powerful approach, offering a simplified yet effective means of predicting cutting forces, chip thickness, and temperatures. Early works by Merchant [3], and Oxley [4] laid the foundation for understanding chip formation mechanics, with Oxley's model providing detailed insights into shear plane behavior and chip formation based on elastic-plastic material deformation. However, analytical models often simplify thermal effects and strain rate sensitivity, which limits their applicability to specific materials and special cutting processes. Extensions of Oxley's model have been made to address these limitations. For instance, Adibi et al. [5] incorporated the Johnson–Cook flow law to account for temperature-dependent strain hardening, enabling more accurate predictions for materials like AISI 1045 and C45 steel. Lalwani [6] further extended Oxley's model.

Numerical simulations, particularly Finite Element Method (FEM), provide a detailed analysis of stress, strain, and thermal fields in the cutting zone, overcoming many limitations of analytical approaches. FEM has been widely adopted to simulate chip formation and cutting forces. For instance, Pimenov and Guzeev [7] developed a FME-based model to predict stress distributions on the tool flank during orthogonal cutting and explored how these stresses correlate with cutting forces. Afrasiabi et al. [8] combined FEM and Smoothed Particle Hydrodynamics (SPH) to simulate thermo-mechanical chip formation in AISI 1045 steel, offering insights into strain localization and temperature gradients at the tool-chip interface. Similarly, Denkena et al. [9] utilized in-situ machining oscillation analysis to monitor chip formation in hybrid materials.

This study focuses on comparing chip compression, shear plane angle, and strain values obtained experimentally for C45 steel with predictions from analytical models, particularly Oxley's model, and orthogonal 2D FEM simulations using DEFORM software. By integrating these approaches, the research aims to evaluate their accuracy and applicability for modern machining conditions, contributing to a deeper understanding of chip formation mechanics and cutting zone behavior.

2. MATERIALS AND METHODS

2.1 Experimental setup and result

The numerical model of orthogonal cutting and FEM simulation is set up so that the results can be compared with the experiment provided by Kuruc [10] in which C45 material was used. The cutting parameters, with the fitted results for the shear angle of the chip compression plane and the deformation of the cutting zone, are shown in table 1. Where ϕ is the shear plane angle, ε is the deformation in primary zone and K is the chip compression ratio. The local strain in the analysed area of the sample was achieved by stereological measurement of the degree of grain boundaries' orientation Martinkovič [2].

Table 1

Cutting parameter and cutting zone deformation

	Cutting par.		Cutting zone		
Exp. No.	**v_c (m/min)**	**f (mm)**	**ϕ (°)**	**ε (-)**	**K (-)**
1	110	0.2	37	0.680685	2.010
2		0.4	39	0.637729	1.880
3		0.6	49	0.602918	1.156
4	145	0.2	37	0.650248	1.560
5		0.4	39	0.585835	1.345
6		0.6	49	0.698620	1.120
7	180	0.2	37	0.330676	1.348
8		0.4	39	0.293802	1.260
9		0.6	49	0.214603	1.000

2.2 Extended-Oxley Orthogonal Cutting Analytical model

In the cutting process the mass of chip m_{chip} ($kg.min^{-1}$) produced over the time is given by equation (1).

$$m_{chip}=\rho \cdot v_c \cdot t_1 \cdot a_p \quad (\mathrm{g.min^{-1}}) \qquad (1)$$

Where ρ is the density of material ($g.mm^{-3}$), v_c is main cutting speed ($\mathrm{m.min^{-1}}$), t_1 is the uncut chip thickness (mm), and a_p is the depth of cut (mm). Figure 1 shows a schematic representation of the orthogonal cutting process and highlighting aspects of chip formation and material flow.

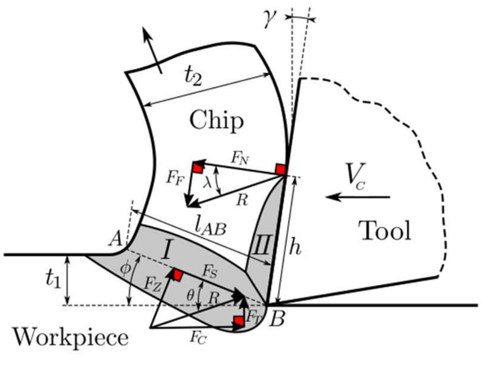

Fig. 1. Oxley Orthogonal Cutting Model

The diagram shows the interaction between workpiece and cutting tool and key geometric parameters, including the cutting rake angle(α) and cuttin gangle(shear). It also identifies the force system that worksin the cutting area, including the cutting force (FC), thrust force (FT) and friction (FF) and normal (FN) components at the tool-chip interface.

The basic principle of the Oxley analytical cutting model is to correctly determine three key parameters: the ratio of the length to the width of the primary shear zone C_0, the ratio of the secondary shear zone to the chip thickness δ, and the shear angle φ. Precise determination of these internal parameters enables obtaining comprehensive model results. According to Oxley's theory, the flow stress is one of the most important principles for analytical modeling and numerical simulation, conforming to material deformation behavior. Using the Von Mises criterion and according to the notations defined in Figure 1, the equivalent shear flow stress in the primary shear zone can be expressed from the Johnson-Cook flow law, as detailed in the equations provided in the original work [11].

The equivalent plastic strain, plastic strain rate, and average temperature along the shear plane are considered constant according to Oxley [4]. The maximum value of the normal stress at the tool-tip point is determined by the tool rake angle α (illustrated in Figure 1) and the equivalent strain hardening coefficient defined from the constitutive flow law.

Lalwani [6] extended the original Oxley model by introducing a modified form of the equation for determining the angle between the resultant forces and the primary shear plane. According to Lalwani [6] and the Johnson-Cook flow law, this parameter can be calculated using the relationships presented in the referenced equations.

Conforming to Oxley's theory, the cutting and advancing forces are calculated based on the geometric and material parameters of the cutting process. By assuming uniform normal stress at the tool-chip interface, the normal and tangential stresses along the interface can be determined, where lc represents the tool-chip contact length as illustrated in Figure 1. The equilibrium of internal stresses imposes conditions to be ensured along both the primary and secondary shear zones.

In the secondary shear zone, the flow stress at the tool-chip interface is computed from the Johnson-Cook flow law. Equilibrium in this zone imposes additional conditions between the calculated stress values.

The analytical Oxley model was implemented according to [11] using the Python programming language with the LMFIT library. The results obtained by this algorithm are summarized in Table 3.

2.3 The Johnson-Cook Constitutive Flow Law Material Model

For the Oxley analytical model, but also for numerical FEM summary, material properties need to be defined. In both cases the Johnson-Cook material flow law model can be used. This flow law, originally developed to characterize the response of materials subjected to impact loading from explosives, is probably the most widely used for the simulation of high strain rate deformation processes, taking into account the effects of plastic strain, plastic strain rate, and temperature. The equation (11) gives the general formulation of the yield strength of the material $\sigma^y(\varepsilon^p, \dot{\varepsilon}^p, T)$ defined.

$$\sigma^y = (A + B\varepsilon^{p^n})\left[1 + C\ln\left(\frac{\dot{\varepsilon}^p}{\dot{\varepsilon}_0}\right)\right]\left[1-\left(\frac{T-T_w}{T_m-T_w}\right)^m\right] \tag{11}$$

Where $\dot{\varepsilon}_0$ is the reference strain rate, T_w and T_m are the initial and the melting temperatures of the material, respectively, and $A, B, C, n,$ and m are the five constitutive flow law parameters. Johnson-Cook material properties of the C45 medium carbon steel are reported in Table 2.

Table 2

C45 Johnson Cook Law Model

$A\ (MPa)$	$B\ (MPa)$	$C\ (-)$	$n\ (-)$	m $(-)$
553.1	600.8	0.0134	0.234	1
$\dot{\varepsilon}_0\ (s^{-1})$	$T_w\ (°C)$	$T_m\ (°C)$	$\rho (kg/m^3)$	
1	25	1460	8000	

Thermal conductivity K and heat capacity C_p are dependent on the temperature T through the following equations (12):

$$K(T) = 52.61 - 0.0281T,\ C_p(T) = 420 + 0.504T \quad (12)$$

2.4 Numerical orthogonal Finite Element Method (FEM) of Chip formation

The orthogonal cutting model was developed in the DEFORM 2D software environment. The FEM calculation of the chip formation was based on the principle of time incremental implicit Lagrangian formulation. The tool was modeled as an ideally rigid body, while its thermal properties were defined by equation (12). The workpiece material is visco-plastic as defined by the Johnson-Cook model equation (11), which is equivalent to steel C45. This constitutive equation takes the form of a multiplicative relationship between stress and strain and depends on five parameters lit [5].

The parameters of the model definition are shown in table 2. The kinematics of the tool and workpiece objects are such that the workpiece is fixedly anchored in space on the face side. The tool object moves in the y-axis direction with a working motion that has the magnitude of the cutting speed. The workpiece mesh is refined in the cutting zone, since the parameters of the variables of interest are changed in this part. Chip separation criterion is ensured by remesh functionality. The geometric model with initial and boundary conditions is shown in Figure 2. The interaction between the workpiece and the tool was set within the mechanical contacts to a coefficient of friction of 0.5, and a thermal transfer coefficient 45 KWm^{-2}C^{-1}.

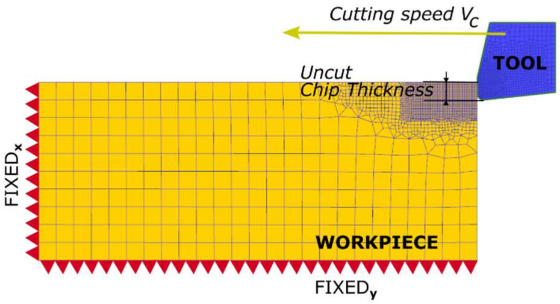

Fig. 2. Deform Orthogonal Cutting Model preparation

3. RESULT AND DISCUSSION

A summary of the analytical Oxley model and FEM numerical simulation were presented in table 3, which includes the shear plane angle, shear zone deformation, and chip thickness for the machining process of C45 steel.

Table 3

Oxley Analytical and DEFORM FEM result

	Analytical Oxley			FEM Simulation		
Exp. No.	ϕ (°)	ε (-)	t_2 (mm)	ϕ (°)	ε (-)	t_2 (mm)
1	31.8	0.56	0.36	29.7	Fig.3	0.32
2	35.3	0.53	0.64	32.9		0.60
3	37.3	0.51	0.90	37.3		0.92
4	33.3	0.55	0.34	30.0		0.32
5	36.7	0.51	0.61	35.1		0.60
6	38.6	0.50	0.87	37.2		0.90
7	34.5	0.53	0.33	32.1		0.32
8	37.8	0.51	0.59	35.0		0.60
9	39.0	0.50	0.85	38.4		0.88

FEM visualizations of the cutting zone provide clear, realistic insight into chip formation, deformation zones, and tool–workpiece interactions. These visual tools are valuable for education, helping students understand machining phenomena that are otherwise hidden.

The FEM numerical simulation reveals that the shear plane strain distribution within the cutting zone is not constant, as assumed in the simplified Oxley machining model. To represent this strain distribution, a specific methodology was employed, as illustrated in Figure 1. Along the shear plane, represented by line AB, strain values were extracted and plotted as a function of the distance from point A and the corresponding strain.

The results indicate that the strain value progressively increases towards the cutting edge, as shown by the relationship in Figure 4. This finding suggests that the theory of constant strain along the shear plane would only hold if the shear plane were not flat, as assumed in the Oxley model, but instead curved. This curvature is represented by the thick orange line in figure 3, which aligns with the observed distribution of strain.

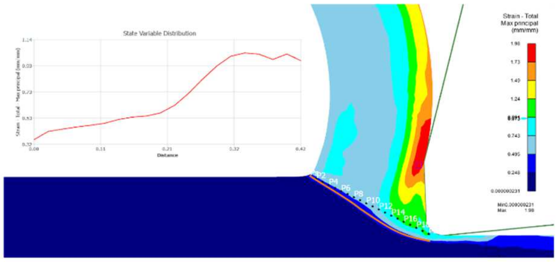

Fig. 3. Strain distribution in cutting zone obtained by DEFORM 2D orthogonal FEM simulation

The strain dependencies for each machining operation are presented in Figure 4. The figure clearly demonstrates that the length of the shear plane is primarily influenced by the feed rate during the machining process. In contrast, the cutting speed has only a minor effect on the magnitude and distribution of strain within the cutting zone.

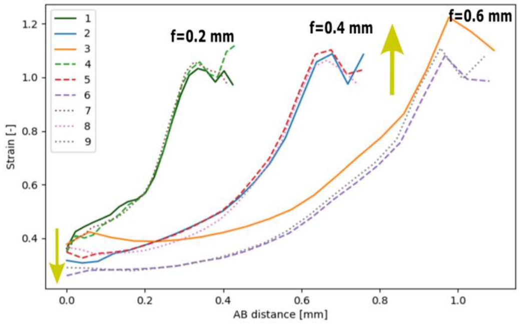

Fig. 4. Strain Dependencies on shear plane in cutting zone

Regarding the influence of feed rate, an interesting phenomenon is observed: at higher feed rates, the strain near the cutting edge (associated with tool bluntness) increases, whereas the strain at point A on the shear plane decreases. This behavior can likely be attributed to the increased length of the shear plane at higher feed rates. Additionally, when machining different materials, this phenomenon may vary in magnitude depending on the material's specific properties.

4. CONCLUSION

The investigation of strain properties in the cutting zone is essential for understanding the mechanisms of chip formation and improving machining operations. This study focuses on the importance of integrating experimental observations, analytical modeling, and numerical simulations to gain better understanding of material behavior during cutting. Although the analytical Oxley model is not entirely accurate or capable of capturing all complexities of the cutting process, it remains a valuable tool for providing insights into the stress-strain states and material deformation in the cutting zone.

On the other hand, FEM simulation offers the most reliable and detailed representation of material behavior and the influence of machining parameters in the cutting zone. When correctly set up, FEM results closely align with experimental findings, providing valuable predictions about the cutting process. However, FEM simulations pose challenges in terms of preprocessing, setup, and computational requirements, making them resource-intensive and complex. Despite these challenges, FEM simulations remain indispensable for accurately studying and optimizing machining operations, especially for advanced materials and processes.

This combined approach confirms the significance of analytical and numerical methods in complementing experimental data, facilitating a deeper understanding of machining processes, and driving advancements in cutting tool design and operational efficiency.

Acknowledgement

This work was supported by the Science Grant Agency - project VEGA 1/0391/24 and KEGA 0007STU-4/2025.

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Investigație numerică și analitică a procesului ortogonal de tăiere a oțelului carbon mediu C45

(Numerické a analytické skúmanie ortogonálneho procesu rezanie stredne uhlíkovej ocele C45)

Táto štúdia nadväzuje na predtým publikovaný experimentálny výskum procesu obrábania. Na lepšiu interpretáciu pozorovaných experimentálnych výsledkov sa využíva analytické a numerické modelovanie. Pre skúmanie zóny rezu sa využíva Oxley analytický model a 2D ortogonálna simulácia rezania pomocou softvéru FEM DEFORM. Skúmaný materiál je stredne uhlíkovú oceľ C45 čo je bežne používaný referenčný materiál. Analýza FEM odhaľuje, že rozloženie deformácií v strihovej rovine nie je konštantné, na rozdiel od predpokladu zjednodušeného Oxley modelu obrábania. Výsledky FEM simulácie sa najviac zhodujú s experimentálnymi zisteniami a poskytujú rozšírený pohľad na proces rezania, ktorý je experimentálne ťažko dostupný. Kombinované použitie overených analytických a numerických modelov dopĺňa skoršiu experimentálnu prácu, prehlbuje pochopenie mechaniky obrábania a podporuje vývoj vylepšených stratégií obrábania, konštrukcie rezných nástrojov a zvýšenie efektívnosti výroby.

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