TECHNICAL UNIVERSITY OF CLUJ-NAPOCA

ACTA TECHNICA NAPOCENSIS

Series: Applied Mathematics, Mechanics, and Engineering
Vol. 69, Issue 69, February, 2026

DIGITAL TWIN FOR INDUSTRIAL ROBOTS USED IN PRODUCTION SYSTEMS

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Abstract: *The paper aims to address, in a structured way, the necessary steps in order to develop a Digital Twin for a complex process that involves industrial robots used in manufacturing applications. The main goal is to facilitate the design, optimization, and digital validation of robotic cells, and the emphasis is on the integration of kinematic and dynamic modeling of robots, simulation of technological forces, and functional validation. Furthermore, the paper integrates the kinematic and dynamic equations of an ABB IRB 1200 7/0.7 robot and includes the desired trajectories to be followed by the end effector. The application of external forces is correlated with the dynamic responses of the robot (resistant torques in joints or energy consumption). The dynamic model developed thus allows the testing and optimization of work configurations without the need for physical intervention on the real system. The approach presented by the paper allows a thorough understanding concerning the interaction between the parameters of the process and the dynamic behavior of the industrial robotic structure. The paper contributes to the development of a robust, energy-efficient solution that can be adapted to the requirements of modern production.*

Key words: *Digital twin, Dynamic modeling, Industrial robots, Mechatronic simulation, Production system*

1. INTRODUCTION

In recent years and in recent times were everything is focused on digital transformation, the development of digital twins has become a fundamental pillar of the Industry 4.0 concept. This paradigm is increasingly found recently, in the Industry 5.0 concept. Digital twins one can say that are dynamic cyber-physical systems that can provide a real-time link, considered bidirectional, between the physical entity and its digital model. All of this can be used to facilitate the simulation, analysis and optimization of the behavior of a system that can be found in real applications [1].

The use of digital twins in the domain of industrial robotics have intensified in the last years, especially due to their potential to reduce the time and costs associated with the design, validation and maintenance processes of robotic manufacturing cells [2]. The use of a comprehensive digital model allows the integration of kinematic and dynamic models of a robot, along with the modeling of technological forces, and creates the premises for testing work configurations in virtual environments, without requiring interventions on physical systems [3].

Recent research has demonstrated the successful development of digital twins in the MATLAB/Simulink environment – in particular with the help of the Simscape Multibody library – that allow the simulation of the dynamic behavior of robots with six degrees of freedom, including joint friction, transmissions and drive patterns [4]. These models have been validated by comparing them with real torque, position and energy consumption data obtained from sensors integrated in robotic systems [5].

Another essential advantage of using digital twins is that they allow the integration of data from physical environments in real time. This feature allows for online monitoring and adjustment of industrial processes, as well as for predictive analysis of component wear and energy consumption [6], [7]. Also, integrating AI (artificial intelligence) and ML (machine learning) algorithms gives the possibility for adapting the behavior of the robotic system to dynamic operating conditions within digital twins [8].

In manufacturing processes, DT offer major benefits in terms of flexibility, energy efficiency and quality control. A notable example is the application of the concept in the processes of incremental forming of metal sheets, where the correct modeling of external forces and robot reactions allows the definition of the optimal working configuration, with a significant reduction of the resisting moments in the joints [9], [10].

At the same time, digital twins contribute to increasing the safety and ergonomics in collaborative robotic cells, by integrating advanced monitoring and control systems of the workspace [11], [12], as well as to the development of hybrid simulation environments (virtual and augmented reality), useful in the design and training process [13], [14].

However, the development of a DT for small industrial robots, for manufacturing applications, involves considerable challenges in terms of model fidelity, real-time synchronization, integration of trajectories and the application of external forces in a manner coherent with the physical behavior of the system [15], [16], [17].

In conclusion the main porpoise of the research is to develop a digital twin for a small industrial robot, for production applications, taking into account the following:

- kinematic and dynamic modeling of the robot, using 3D design tools and the MATLAB/Simulink environment;
- integration of desired trajectories and simulated external forces;
- analysis of resistive moments and energy consumption in dynamic mode;
- functional validation of the digital model through simulations, without intervention on the physical system;
- optimization of the work configuration for maximum efficiency and minimum wear.

2. ROBOT KINEMATICS

To formulate the mathematical equations for the inverse kinematics of the ABB IRB 1200 robot (7 kg / 0.7 m), it is necessary to follow the typical steps of kinematic analysis for a serial robot with six degrees of freedom. The referenced article covers the kinematics and dynamics of the ABB IRB 1200. One, can provide the general formulas and a geometric approach applicable to the ABB IRB 1200, based on Denavit-Hartenberg (D-H) parameters, if one has those parameters.

2.1. D-H (Denavit-Hartenberg) parameters of the structure

Table 1, presents the D-H parameters for the ABB IRB1200 robotic structure

Table1

i	**θ_i (variabilă)**	**d_i (mm)**	**a_i (mm)**	**α_i (rad)**
1	θ_1	0	0	0
2	θ_2	0	a_1	$-\pi/2$
3	θ_3	0	a_2	0
4	θ_4	0	a_3	$-\pi/2$
5	θ_5	d_4	0	$\pi/2$
6	θ_6	d_5	0	0

The D-H parameters are based on the robot diagram showed in Figure 1.

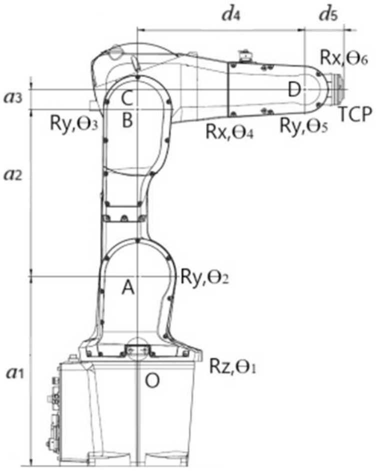

Fig. 1. Simplified view of ABB IRB 1200 7/07 robot

2.2. Inverse kinematics equations

To formulate the inverse kinematics of a 6-DOF industrial robot such as the ABB IRB 1200, one can start from the imposed pose of the end-effector. This includes both position and orientation in 3D space. The goal is to compute and solve the six joint angles (θ_1 through θ_6) that will place the robot's end-effector at the desired/imposed pose.

The pose of the end-effector is defined by a homogeneous transformation matrix H_{06}, which consists of an orientation matrix R_{06} (3x3) and a position matrix $P = [x\ y\ z]^T$:

$$\begin{aligned} H06 = {} & [\, R_{06} \mid P \,] \\ & [\ 0 \mid 1] \end{aligned} \tag{1}$$

This matrix is given in the absolute coordinate system of the robot base and represents the target orientation and position of the end-effector.

The first step is to determine the position of the wrist center of movement.

Because the last three joints of a 6-DOF robot form a spherical wrist, the orientation of the end-effector can be decoupled from its position. To isolate the first three joints (which determine the position of the EE), one can compute the position of the wrist center *Pwc*:

$$P_{wc} = P - d^6 \cdot R^{06} \cdot \hat{z} \tag{2}$$

Where:

- P is the position of the end-effector from H_{06}.
- d_6 is the distance from the wrist center to the end-effector (along z-axis of the tool).
- $\hat{z}$ is the unit vector $[0\ 0\ 1]^T$ in the end-effector frame.

The next step is to compute θ_1. This is the angle of the first joint and is computed using the projection of the wrist center in the base X-Y plane:

$$\theta^1 = atan2(y_{wc}, x_{wc}) \tag{3}$$

After θ_1, one can compute θ_2 and θ_3 using planar geometry.

It can be defined:

$$r = \sqrt{x_{wc}^2 + y_{wc}^2}$$

$$s = z_{wc} - d^1 \tag{4}$$

Now one can use the triangle formed by links a_2 and a_3 and the wrist center.

The law of cosines gives:

$$\cos(\theta^3) = \frac{(r^2 + s^2 - a^{22} - a^{32})}{(2 \cdot a^2 \cdot a^3)} \tag{5}$$

$$\theta^3 = atan2\left(\sqrt{1 - cos^2(\theta^3)}, \cos(\theta^3)\right) \tag{6}$$

$$\theta^2 = atan2(s, r) - atan2(a^3 \cdot \sin(\theta^3), a^2 + a^3 \cdot \cos(\theta^3)) \tag{7}$$

This solves the positioning part of the wrist in the inverse kinematics.

The last step is to compute the orientation angles θ_4, θ_5, and θ_6.

First, one must calculate the rotation matrix R_{03} based on θ_1, θ_2, and θ_3:

$$R^{03} = R_{z(\theta^1)} \cdot R_{y(\theta^2)} \cdot R_{y(\theta^3)} \tag{8}$$

Then one must compute:

$$R^{36} = R^{03\mathrm{T}} \cdot R^{06} \tag{9}$$

From R_{36}, which represents the orientation of the wrist, the Euler angles can be extracted using the eq. 8. The Euler angles θ_4, θ_5, and θ_6 will be extracted using a chosen convention (e.g., ZYX):

$$\theta^4 = atan2\big(R^{36}(2{,}3), R^{36}(1{,}3)\big) \tag{10}$$

$$\theta^5 = \arccos\big(R^{36}(3{,}3)\big) \tag{11}$$

$$\theta^6 = atan2\big(R^{36}(3{,}2), -R^{36}(3{,}1)\big) \tag{12}$$

These equations complete the inverse kinematics solution using a geometric method.

All the equations assume that the Denavit-Hartenberg parameters of the robot are known. This includes link lengths a_1, a_2, a_3 and offsets d_4 and d_5.

Multiple solutions may exist due to the nature of the trigonometric equations, and the physical joint limits must be considered to select the feasible configuration.

3. ROBOT DYNAMICS

The inverse dynamic model of a 6-DOF serial robot, such as ABB IRB 1200-7/0.7, is expressed as the relation below:

$$\tau = M(q) \cdot \ddot{q} + C(q,\dot{q}) \cdot \dot{q} + G(q) + J(q)^{\mathrm{T}} \cdot F_{ext} \quad (13)$$

Where:

- q expresses joint positions $[q_1, q_2, \ldots, q_6]$;
- $\dot{q}$ expresses joint velocities;
- $\ddot{q}$ expresses joint accelerations;
- τ expresses joint torques;
- M(q) is considered the joint-space inertia matrix;
- C(q, q̇) is the Coriolis and centrifugal matrix;
- G(q) is considered the gravity vector;
- J(q) is the Jacobian matrix;
- F_ext are the external forces at the end-effector.

The inertia matrix is computed from the formula of the total kinetic energy:

$$M_{ij(q)} = \Sigma_{\mathrm{k}} \left[m_k \cdot \left(\frac{\partial r_{ck}}{\partial q_{\mathrm{i}}}\right)^{\mathrm{T}} \left(\frac{\partial r_{ck}}{\partial q_{\mathrm{j}}}\right) + \left(\frac{\partial \omega_k}{\partial q_{\mathrm{i}}}\right)^{\mathrm{T}} \cdot I_k \cdot \left(\frac{\partial \omega_k}{\partial q_{\mathrm{j}}}\right) \right] \quad (14)$$

The gravity vector is derived from the potential energy:

$$G_{\mathrm{i}}(q) = \Sigma_{\mathrm{k}} \left[m_k \cdot g \cdot \frac{\partial h_{k(q)}}{\partial q_{\mathrm{i}}} \right] \quad (15)$$

The Coriolis and centrifugal terms use Christoffel symbols:

$$C_{ij(q,\dot{q})} = \Sigma_{\mathrm{k}} \left[c_{ijk} \cdot \dot{q}_k \right]$$
$$c_{ijk} = ½ \left(\frac{\partial M_{ij}}{\partial q_k} + \frac{\partial M_{ik}}{\partial q_j} - \frac{\partial M_{jk}}{\partial q_i} \right) \quad (16)$$

The external force applied at the end-effector is projected using the Jacobian:

$$\tau_{ext} = J(q)^{\mathrm{T}} \cdot F_{ext} \quad (17)$$

Final form of the inverse dynamic model is presented in eq. 18:

$$\tau = M(q) \cdot \ddot{q} + C(q,\dot{q}) \cdot \dot{q} + G(q) + J(q)^{\mathrm{T}} \cdot F_{ext} \quad (18)$$

4. ROBOT DYNAMICS SIMULATIONS

The formulas presented above were implemented in Simulink using an Embedded MATLAB Function block. The Simulink block diagram is presented in Figure 2.

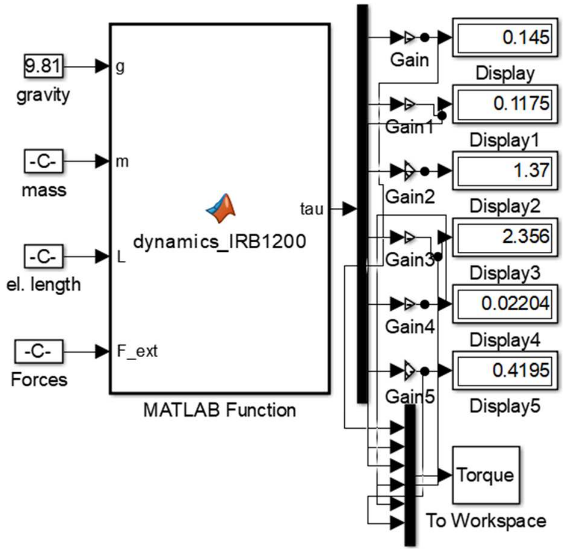

Fig. 2. Block diagram for the dynamic model of the ABB IRB 120 robot

This block diagram from Simulink characterizes the dynamic behavior of the ABB IRB 1200 industrial serial robot. The block titled, “dynamics_IRB1200” determines in real time the resistive moments in the kinematic couplings, necessary to compensate for: the gravity force, the masses of the moving elements, the external forces and the robot geometry.

The inputs to the block include the gravitational acceleration, the mass of each moving element, the lengths of the elements and the external wrench acting on the robot’s EE. The output data from the function block is a resistive torque vector, of six elements, corresponding to each joint. The resistive torque values are displayed and scaled using amplification blocks. The results are displayed for each kinematic coupling. The model, however, assumes the presence of harmonic reducers in the couplings. The transmission ratios are estimated at approximately 160:1 for joints 1 to 3 and 100:1 for joints 4 to 6 [15], [16], [17], [18], [19]. These harmonic drives significantly influence the required motor torque. They provide high positioning accuracy and torque multiplication. The torque output is also sent to the MATLAB workspace for further analysis. The current simulation setup is used to evaluate the forces in robot joints in typical industrial applications involving precision motion or interaction with an external load.

5. CONCLUSIONS

This paper attempts to demonstrate the importance of the Digital Twin concept in the context of Industry 4.0 and the more recent transition to the concept of Industry 5.0. The paper categorically highlights the ability of this model to clearly integrate the kinematic and dynamic models of industrial robots used in various manufacturing processes.

The approach presented in the paper facilitates simulation, optimization and functional validation without any intervention on the physical system of the robot. The development of a rigorous dynamic model of the ABB IRB1200 robot is presented. It starts from the Denavit-Hartenberg parameters and considers the implementation in simulation environments such as MATLAB/Simulink. The model takes into account the integration of imposed trajectories and external forces, which ensures a direct correlation between the working conditions and the real behavior of the robot.

Through simulation, the resistant moments in the kinematic couplings are acquired and subsequently in further research the energy consumption can be taken into account. The model allows testing of optimal working configurations through the presented dynamic model. A dynamic model of the robot is essential in modern manufacturing processes, where flexibility, energy efficiency and wear reduction are priorities.

Simulation based on the dynamic model even offers a viable and efficient alternative to physical testing and contributes to reducing costs, design time and at the same time increasing safety in robotic cells. The present work proposes a scalable solution, adaptable to different industrial scenarios.

As a result, the present research offers a valuable contribution to the field of robotics and digital twins. It opens clear directions for future research, such as real-time integration of physical data, the use of artificial intelligence for adaptation to dynamic conditions and definitely the extension of models to complex collaborative systems.

6. REFERENCES

[1] Liu, M., Fang, S., Dong, H., Xu, C., "Review of Digital Twin about Concepts, Technologies, and Industrial Applications", *Journal of Manufacturing Systems*, 2021. https://doi.org/10.1016/j.jmsy.2020.06.017

[2] Kousi, N., Michalos, G., Makris, S., "Digital Twin for Designing and Reconfiguring Human-Robot Collaborative Assembly Lines", *Applied Sciences*, 2021. https://doi.org/10.3390/app11104620

[3] Grieves, M., Vickers, J., "Digital Twin: Mitigating Unpredictable, Undesirable Emergent Behavior in Complex Systems", *Springer*, 2016. https://doi.org/10.1007/978-3-319-38756-7_4

[4] Boschetti, G., Sinico, T., "Designing Digital Twins of Robots Using Simscape Multibody", *Robotics*, 2024. https://doi.org/10.3390/robotics13040062

[5] Crenganiș, M. et al., "Digital Twin Virtual Approach of Robot-Based Incremental Sheet-Metal Forming Process", *Acta Technica Napocensis*, 2024.

[6] Jiang, Y., Yin, S., Li, K., Luo, H., Kaynak, O., "Industrial Applications of Digital Twins", *Philosophical Transactions of the Royal Society A*, 2021. https://doi.org/10.1098/rsta.2020.0360

[7] Colombo, A. et al., "Industrial Cyber-Physical Systems: A Backbone of the Fourth Industrial Revolution", *IEEE Industrial Electronics Magazine*, 2017. https://doi.org/10.1109/MIE.2017.2648857

[8] Li, M., Wang, H., "Enabling Improved Learning Capability of Industrial Robots with Knowledge Graph Towards Intelligent Digital Twins", *IEEE CSCWD*, 2022. https://doi.org/10.1109/CSCWD54268.2022.9776063

[9] Petru, C.D. et al., "Development of a Digital Twin for the ABB IRB 1200 Robot in Sheet

Metal Forming Processes", *Procedia Computer Science*, 2024. https://doi.org/10.1016/j.procs.2024.08.253

[10] Szybicki, D. et al., "Application of Digital Twins in Designing Safety Systems for Robotic Stations", *Electronics*, 2024. https://doi.org/10.3390/electronics13214179

[11] Burghardt, A. et al., "Programming of Industrial Robots Using Virtual Reality and Digital Twins", *Applied Sciences*, 2020. https://doi.org/10.3390/app10020486

[12] Li, J. et al., "Task Incremental Learning-Driven Digital-Twin Predictive Modeling for Customized Metal Forming", *Robotics and Computer-Integrated Manufacturing*, 2024. https://doi.org/10.1016/j.rcim.2023.102647

[13] Katyara, S. et al., "High-Fidelity Digital Twinning of Agile Manufacturing Systems", *arXiv preprint*, 2024. https://arxiv.org/abs/2409.10784

[14] Petru, C.-D., Morariu, F., Breaz, R.-E., Crenganiş, M., Racz, S.-G., Gîrjob, C.-E., Bârsan, A., & Biriş, C.-M. Development of a Small CNC Machining Center for Physical Implementation and a Digital Twin, Applied Sciences, vol. 15, no. 10, pp. 5549, MDPI, 2025. https://www.mdpi.com/2076-3417/15/10/5549

[15] ABB Robotics. (2016). IRB 1200 Product Manual – Spare Parts. 3HAC046984-001.

[16] Harmonic Drive AG. (2023). FB Series Component Set Catalog. harmonicdrive.net.

[17] Picea Motion. (2022). Harmonic Drive Reducer Comparison and Selection Guide. www.piceamotiondrive.com.

[18] Leobotics. (2021). ABB IRB 1200 Instruction Manual. www.leobotics.fr.

[19] Mahdi Yazdanpour “A Modular 6-DOF Collaborative Robot Arm with Worm Gear Transmission and Embedded Control”, International Conference on Control, Automation and Diagnosis (ICCAD) doi: 10.1109/ICCAD64771.2025.11099195.

Digital Twin pentru roboți industriali utilizați în sistemele de producție

Rezumat: Lucrarea își propune să abordeze într-un mod structurat etapele necesare în dezvoltarea unui (Digital Twin) pentru roboți industriali care poate fi utilizat în aplicații de fabricație. Scopul principal este de a facilita proiectarea, optimizarea și validarea digitală a celulelor robotizate, accentul fiind pus pe integrarea modelării cinematice și dinamice a roboților, simularea forțelor tehnologice și validarea funcțională. Lucrarea prezintă dezvoltarea modelului dinamic, începând cu construcția modelului 3D și importul acestuia în Simulink. În plus, integrează ecuațiile cinematice și dinamice ale robotului și include traiectoriile dorite a fi urmate de efectorul final. Aplicarea forțelor externe este corelată cu răspunsurile dinamice ale robotului (cupluri rezistente în articulații sau consum de energie). Modelul dinamic dezvoltat permite astfel testarea și optimizarea configurațiilor de lucru, fără a fi nevoie de intervenție fizică asupra sistemului real. Abordarea prezentată de lucrare facilitează o înțelegere aprofundată a interacțiunii dintre parametrii procesului și comportamentul dinamic al robotului. Lucrarea contribuie la dezvoltarea unei soluții robuste, eficiente din punct de vedere energetic, care poate fi adaptată cerințelor producției moderne.

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