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ANALYTICAL CALCULATION AND DETERMINATION OF VIBRATION MODES OF THE ULTRASONIC SYSTEM USED TO IMPROVE THE WIRE PRODUCTION PROCESS

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Abstract: *Ultrasounds represent mechanical vibrations with frequencies generally between 18... and 100000 Hz. Ultrasonic applications are found in destructive testing, ultrasonic cleaning, ultrasonic welding, ultrasonic machining, ultrasonic motors, sonochemistry, ultrasonic flow meters, material characterization, ultrasonography, therapeutic ultrasound, ultrasonic surgery, liposuction, water treatment, air pollution control. A great novelty at the moment is the destruction of cancer cells using the phenomenon of ultrasonic cavitation. In industry, one of the possible, but relatively little studied applications refers to reducing the friction coefficient that occurs in the wire drawing process. This article presents the analytical calculation of the main characteristics of the ultrasonic transducer and the determination using FEM of the vibration eigenmodes, useful in the ultrasonic activation of the drawing die.*
Key words: *ultrasonic, vibrations, wiredrawing, friction, reduction*

1. INTRODUCTION

Wire drawing and drawing are technological processes for obtaining semi-finished products by plastic deformation, in which the metal material under the action of a pulling force is forced to pass through a calibrated hole of a tool, which is smaller than the initial section of the material. The tools for wire drawing are called dies, and for drawing — dies. If the tensile force is provided by a drum, drum or roller, on which the deformed material is wound, the process is called wire drawing; if the tensile force is exerted by a machine element with rectilinear movement, and the deformed products (bars or pipes) are obtained straight, the technological process is called drawing. In wire drawing, the state of mechanical stress is complex and consists of a compressive stress to which is added a tensile stress, which deforms the crystalline structure, through sliding and elongation, which allow the preservation of the cohesion of the structure of the material to be processed. The wire drawing machine is presented in figure 1 where by this method wires with diameters Φ = 9.5, 12, 15, 19 and 25 mm can generally be obtained. Figure 1a presents the view of a wire drawing machine and figure 1b the image of wire drawing die.


a)


b)

Fig. 1. Wire drawing technology; a. – wire drawing machine; b.- wire drawing die.

One of the most important problems that arise with this technology is the very high energy consumption necessary to carry out the process since the deformation of the material is done cold and the friction forces are very high.

To optimize the wire drawing process, one of the most important methods, if not the only method, is to apply the ultrasonic vibration field to the drawing die. Using this method, it is known that the reduction of the friction coefficient [1] and therefore of the power required for processing is approximately 25%. The phenomenon has been studied by several authors both theoretically and practically [2,3]. Thomas Sednaoui &all studied [3] using an experimental stand the variation of the friction coefficient depending on the vibration amplitude and found a substantial reduction from the value of 1.6 to the value of 0.4 μm if the vibration amplitude increases from 0.5 μm to 3.5 μm. at a frequency f = 25 KHz. The experiments were carried out in the case of palpating with the finger a plate that oscillates in the ultrasonic field. Pham, T.M. Twiefel, J. studied in [4] the reduction of the friction coefficient at the contact between an elastomer and a metal. In their research [5], Diana Angelica Torres & all also proposed to measure the reduction of the coefficient of friction when palpating with a finger a vibrating surface in the ultrasonic range by means of longitudinal waves as a function of speed.

In this regard, a method for analytical calculation of the main characteristics of an ultrasonic system as well as the frequencies and vibration modes of the system thus determined will be presented. The presented research continues a previous activity [6] in which the final shape of the ultrasonic concentrator was designed in the form of a 3rd. degree polynomial equation as presented in Fig. 2.

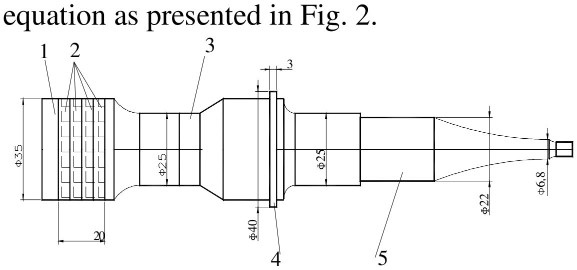


Fig. 2. Ultrasonic system; 1.- reflector; 2. - piezoceramic elements; 3. – ultrasonic amplifier; 4. – nodal flange; 5. – ultrasonic concentrator

2. ANALITYCAL CALCULATION AND FEM OF THE ULTRASONIC SYSTEM

The calculation of ultrasonic system consists in two parts. First part is represented by the analytical calculation of the system and the second part by the FEM of the vivration modes.

2.1 Elements of the analitical calculation for the ultrasonic system

To calculate and dimension the ultrasonic system, according to [7,8], in the first step, the initial data necessary to solve the equations describing the vibratory behavior of piezoceramic materials are introduced, namely [9]:

- piezoelectric material prmittivity:

$$\varepsilon_0 = 9.69 \text{ [F/m]} \quad (1)$$

– resonance frequency:

$$f_0 = 20000 \text{ Hz} \quad (2)$$

– resonance pulsation:

$$\omega_0 = 2\pi f_0 = 12.5 \cdot 10^4 \quad (3)$$

- particle displacement:

$$\xi = 0.5 \cdot 10^{-7} \text{ m} \quad (4)$$

- electrical input power:

$$P_{in} = 1500\ W \quad (5)$$

- acoustical intensity [13,16]:

$$J = \frac{Z_c \cdot (2 \cdot \pi \cdot f_0)^2 \cdot \xi^2}{2} = 1246\ \cdot 10^3 \quad (6)$$

where Z_c – acoustic impedance; f_0 – resonance frequency; ξ – particle displacement; Z_c = 80 kRayl

- acoustical and mechanical efficiency:

$$\eta = \ 0.75 \quad (7)$$

- electromechanical coupling factor - defined as the ratio between the energy transformed into oscillations and the energy introduced into the ultrasonic system, or as the efficiency of transforming electrical energy into mechanical energy [10,11]:

$$\zeta = 0.74 \quad (8)$$

- electroacoustic efficiency [4,5]:

$$\eta_{ea} = 0.94 \quad (9)$$

- piezoceramic material density [6,7]:
-

$$\rho = 7.8 \cdot 10^3 \ \text{[Kg/m}^3\text{]} \quad (10)$$

- piezoceramic material Young modulus:

$$Y = 5.1 \cdot 10^{10} \ \text{[N/m}^2\text{]} \quad (11)$$

- Poisson coefficent:

$$\nu = 0.25 \quad (12)$$

- relative permitivity at 1 Hz:

$$\varepsilon_{rp} = 2600 \quad (13)$$

- loss angle:

$$\delta_p = 0.72 \text{ deg} \ ; \ \text{tg}(\delta_p) = 0.0125 \quad (14)$$

- piezoelectric constant:

$$d_{33} = 370 \cdot 10^{-12} \ \text{[m/V]} \quad (15)$$

The mechanical properties of aluminum for the ultrasonic amplifier and steel for ultrasonic concentrator like density, Poisson coefficient and Young modulus are well known and will not be presented again. Using the input data presented above, the dimensions of the transducer composed of a passive element (the reflector) and an active element were calculated, this assuming the determination of the following parameters:

- speed of ultrasound propagation through components [11]:

$$v_p = \sqrt{\frac{E_p(1-\nu_p)}{\rho_p(1+\nu_p)(1-2\nu_p)}} = 2801 \ \text{m/s} \quad (16)$$

$$v_{Ol} = \sqrt{\frac{E_{Ol}(1-\nu_{Ol})}{\rho_{Ol}(1+\nu_{Ol})(1-2\nu_{Ol})}} = 5952 \ \text{m/s} \quad (17)$$

$$v_{Al} = \sqrt{\frac{E(1-\nu)}{\rho(1+\nu)(1-2\nu)}} = 6148 \ \text{m/s} \quad (18)$$

where: v_p is the speed of ultrasound propagation in the piezoceramic element; v_{Ti} - is the speed of ultrasound propagation in the reflector, v_{OL} - speed of ultrasound propagation in the concentrator.

Ultrasound wavelength:

$$\lambda_p = \frac{v_p}{f_0} = 140 \text{ mm} \quad (19)$$

$$\lambda_{Al} = \frac{v_{Al}}{f_0} = 297 \text{ mm} \quad (20)$$

$$\lambda_{Ol} = \frac{v_{Ol}}{f_0} = 307 \text{ mm} \quad (21)$$

where: λ_p is the wave length of the piezoceramic material; λ_{TI} - wave length of the reflector; λ_{Ol} - wave length of the in concentrator.

Dimensions of components according to the longitudinal direction of propagation of ultrasonic vibrations in $\lambda/4$ are:

$$d_{Ol} = \frac{\lambda_{Ol}}{4} = 41.75 \text{ mm} \quad (22)$$

$$d_p = \frac{\lambda_p}{4} = 20 \text{ mm} \quad (23)$$

Since this dimension is related to the possibilities of construction and acquisition of piezoceramic discs, in the construction of the piezoceramic transducer four piezoceramic discs with a thickness of 6 mm were used.

The total thickness of the piezoceramic block is 24 mm.

The radiation area of the active element must be correlated with the input power and the required acoustic intensity and is:

$$A_p = \frac{P_{in}}{I_a \cdot \eta_{ea}} = 0.015 \text{ m}^2 \quad (24)$$

where A_p is the radiation area of the piezoceramic disc package.

- radius of the active element, r_p:

$$r_p = \sqrt{\frac{A_p}{\pi}} = 21.5 \text{ mm} \quad (25)$$

Taking into account the constructive possibilities regarding the choice of piezoceramic elements, the radius of the piezoceramic element that was available is $r_p = 16\, mm$.

- electromagnetic transformation coefficient:

$$n_p = k_p \cdot Y_p \cdot \frac{A_p}{d} = 2.75 \quad (26)$$

- acoustic impedances of the transducer elements:

$$Z_p = \rho_p \cdot v_p \cdot A_p = 2.56E4 \text{ Kg/s} \quad (27)$$

$$Z_{OL} = \rho_{pOl} \cdot v_{Ol} \cdot A_{Ol} = 4.3E4 \text{ Kg/s} \quad (28)$$

$$Z_{Ti} = \rho_{Ti} \cdot v_{Ti} \cdot A_{Ti} = 1.98E4 \text{Kg/s} \quad (29)$$

where ρ_{pOl}, ρ_{Ti} are the densities of the piezoceramic material, the concentrator and the reflector, respectively; $v_p, v_{Ol}, v_{Ti} -$ volumes of the piezoceramic transducer, the concentrator and the reflector; $A_p, A_{Ol}, A_{Ti} -$ areas of the presented volumes.

The effective electrical characteristics required to produce acoustic power under mechanical resonance conditions are:

- the electrical stress, up given by the relationship:

$$U_p = (\alpha_0 \cdot Z_p \cdot P_{in} \cdot \eta_{ea})^{1/2} / n_p\, \eta_{am} = 1.45 \cdot 10^2 \text{ V} \quad (30)$$

where $\alpha_0 = 0.85$

- the electrical capacitance of the active element, Cp given by the relationship:

$$C^p = \frac{\varepsilon_0 \varepsilon^{rp} A^p}{d^p} = 1.24\, nF \quad (31)$$

To write the relations compactly, the following notations are used:

$$Z_1 = Z^p = 2.56 \cdot 10^4 \text{ Kg/s} \quad (32)$$

$$Z_2 = Z^p + Z^{Ti} = 4.54 \cdot 10^4 \text{ Kg/s} \quad (33)$$

$$\tau = 2\rho_p \cdot v_p \cdot A_{Ti} = .19 \cdot 10^{-4} \text{ Kg/s} \quad (34)$$

$$Z_3 = jZ_2 + \tau + \tau_p = 10^4 (j \cdot 4.54 + 5.19 + 1.38) \text{ Kg/s} \quad (35)$$

$$Z_4 = Z_p + Z_{Ol} = 6.86 \cdot 10^4 \text{ Kg/s} \quad (36)$$

$$Z_5 = jZ_4 + \tau_p = 10^4 (j \cdot 6.86 + 1.38) \quad (37)$$

$$Z_6 = Z_p + Z_{Ol} = 2.986 \cdot 10^4 \text{ Kg/s} \quad (38)$$

- ideal resonant acoustic power,

$$P_{ai0} = \frac{(2\alpha_0 \cdot \rho_0 \cdot v_p \cdot A_{Ti} \cdot \eta_p^2 \cdot U_p^2) \frac{Z_4}{Z_4 - Z_1} \eta_{am}^2}{\tau^2} = 1.35 \cdot 10^3\, W \quad (39)$$

- acoustic power at resonance with consideration of radiation losses:

$$P_{ai0} = \frac{(2\alpha_0 \cdot \rho_0 \cdot v_p \cdot A_{Ti} \cdot \eta_p^2 \cdot U_p^2) \frac{Z_4}{Z_4 - Z_1} \eta_{am}^2}{\tau + \tau_p{}^2} = 0.68 \cdot 10^3\, W \quad (40)$$

2.2 Finite element modeling of the ultrasonic system vibrations

The results provided by the ANSYS software mainly refer to the analysis of the deformation states and the stress state that appear inside the structure as a result of the application of the potential difference on the surfaces of the piezoceramic elements.

The analysis was carried out using two types of discretization elements, namely SOLID98 for the discretization of piezoelectric elements and Solid187. The analysis performed is of the modal type, i.e. the free vibration frequencies and the vibration modes corresponding to them are calculated. In the ultrasonic frequency range (f= 18000……100000 Hz) the software calculates four free vibration frequencies, namely f_1 = 19753 Hz, f_2 = 19778, f_3 = 20067 Hz and f_4 = 20106 Hz.

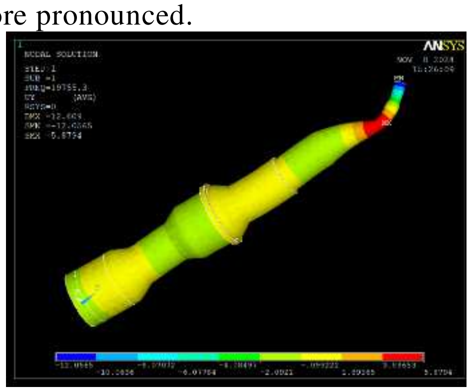
The analysis of the vibration modes presents in the figure 3 the displacement along OY axis. Their maximum values are presented in Table 2. In this case, the maximum amplitude occurs at frequency f_2 = 19778 Hz.

Table 2

The amplitude of oscillations relative to the OY axis

Frequency [Hz]	f1 =19753	f2 = 19778	f3 = 20067	f4 = 20106
Displacement [µm]	12.05	12.39	8.32	1.44

In this case, the maximum amplitude occurs at frequency f_2 = 19778 Hz. The first presented mechanical stress analysis refers to the shear stresses for the YOZ plane where it can be observed that unlike the stresses calculated by reference to the three axes, in this situation the variation of stresses is much more pronounced.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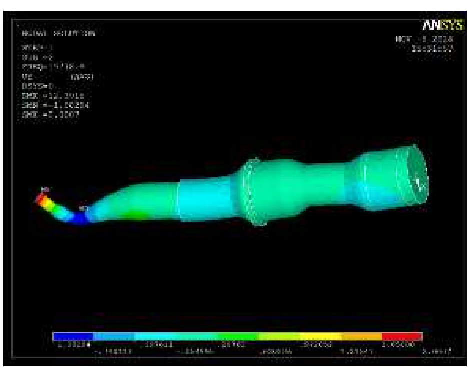
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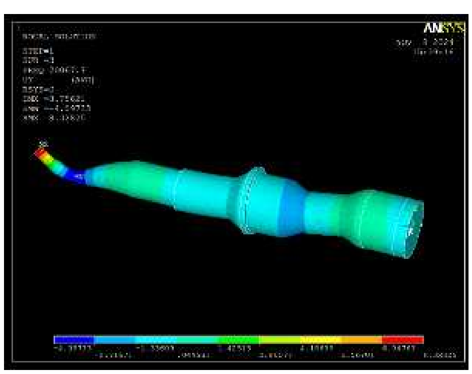
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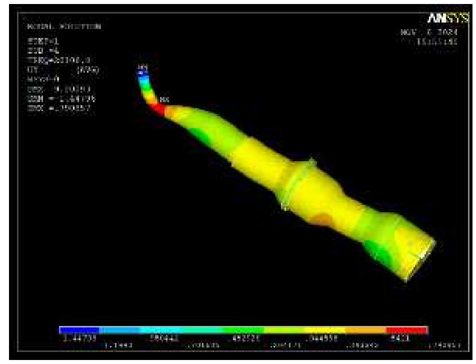
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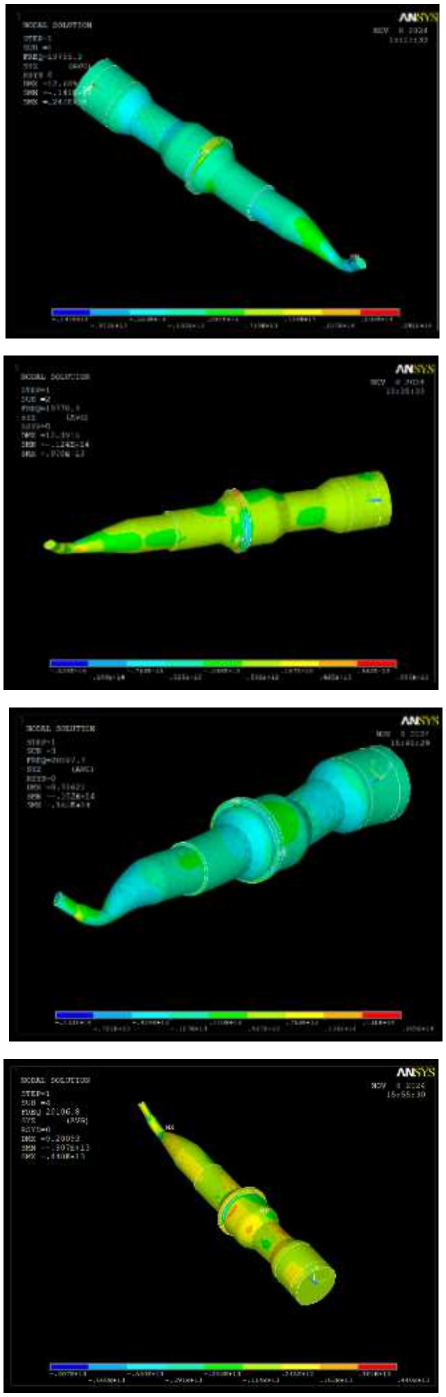
Fig. 3. Displacement valus along the OY axis; a.- f = 19753 Hz; b. – 19778 Hz; c. – 20067 Hz; d. – 20106 Hz.

For the frequency f_1 = 19755 Hz at the diameter jumps along the transducer, intervals of stresses located in the yellow and blue color area are recorded, the highest values of which are calculated in the nodal point of the vibrations, where, towards the free end of the concentrator, it bends. Also, high values are found in the embedding area of the ultrasonic system, that is, in the nodal flange. In figure 4a, the stresses calculated in this case are presented. For the second resonant frequency f_2 = 19778 Hz in figure 4b the variation of shear stresses is presented, which is also quite non-uniform. Maximum values are recorded in the nodal flange as well as in the oscillation area of the free tip of the ultrasonic concentrator. A somewhat dangerous situation occurs at the interface between the piezoceramic elements and the ultrasonic reflector, i.e. on the opposite side of the ultrasound concentrator. However, the jumps in the stress values are not very large in this situation.

For the third vibration frequency f_3 = 20067 Hz, the stresss in the area of the piezoceramic elements are minimal, being located in the range (-0.12·E13.....0.17·E13) N/m^2. The maximum stresses values are calculated in the area of the nodal flange as is natural but not presenting any danger regarding the structural integrity of the piezoceramic transducer.

For the third vibration frequency f_3 = 20106 Hz, in figure 4d, stresses are calculated whose variation is quite important along the length of the ultrasonic transducer. As can be seen, in the area located between the piezoceramic elements and the nodal flange, important stresss (red and ocher colors) are calculated along with the stress with minimum value.



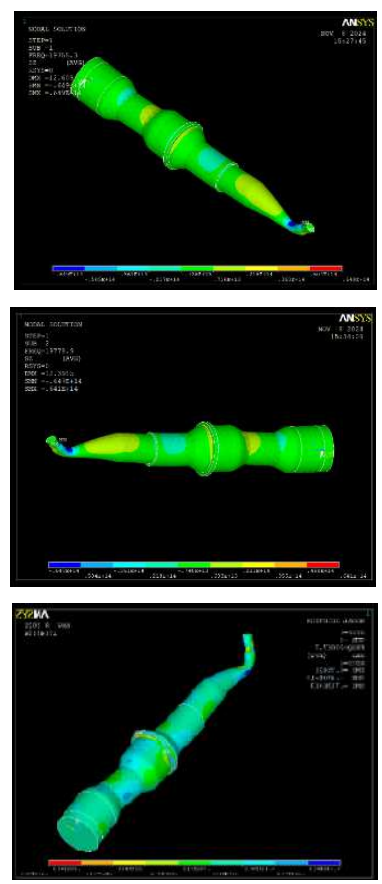
a) b) c) d)

Fig. 4. Presentation of the calculation of shear stresses reported to the YOZ plane; a.- f = 19753 Hz; b. – 19778 Hz; c. – 20067 Hz; d. – 20106 Hz.

Also, in the area located towards the ultrasonic concentrator, that is, in the area of the amplifier, important variations of the shear stresses appear, especially in the area of the oscillation mode. From this point of view of the stresses in this plane, the last free vibration frequency is not very recommended to be used in practice.

The second plane of calculation of shear stresses is XOZ, a plane in which the behaviors at the different vibration frequencies are also different. Figure 5a presents the stresses occurring at frequency f_1 = 19753 Hz, a frequency that does not produce stresses with very large jumps. Their variation is relatively small, the maximum values also being calculated in the nodal flange area. Even in the inflection area of the vibration node, the stresses are not very important. Figure 5b presents the stresses for frequency f_2 = 19778 Hz, stresses that also do not register important variations.

In the area of the piezoceramic elements, their values are the lowest with a zero value in the interval (-0.29 E13....0.1 E13) N/m^2. For the vibration frequency f_3 = 20067 Hz, a greater variation of the stresses along the concentrator and relatively important deformations of it are observed also in the area located between the nodal flange and the piezoceramic elements.



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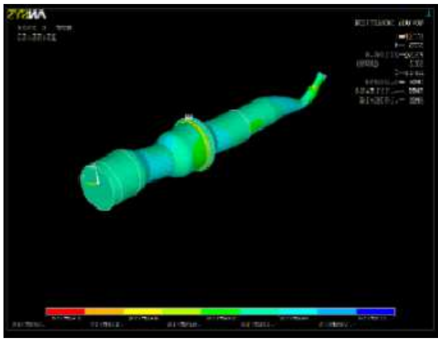

d)

Fig. 5. Presentation of the calculation of shear stresses reported to the XOZ plane; a.- f = 19753 Hz; b. – 19778 Hz; c. – 20067 Hz; d. – 20106 Hz..

For the fourth free vibration frequency f_4 = 20106 Hz, the variation of the stresses is relatively smaller, the areas with maximum stresses both in tension and in compression being also found in the nodal flange area.

3. CONCLUSION

The paper presents the analytical and finite element calculation of an ultrasonic system used to introduce a vibratory field for ultrasonic activation of the dies used in wire drawing. Very important is the correlation between the analytical design performed for the 20 KHz vibration frequency. for which the system was dimensioned and the results of the FEM analysis which found vibration modes very close to this value.

Using these vibration frequencies the vibratory system will operate with optimal efficiency, maximum vibration amplitudes and minimum energy consumption. This system will reduce the friction coefficient between the wire and the die through the ultrasonic lubrication phenomenon.

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Calculul analitic și determinarea modurilor de vibrație ale sistemului ultrasonic utilizat pentru îmbunătățirea procesului de trefilare a sârmei

Ultrasunetele reprezintă vibrații mecanice cu frecvențe în general între 18... și 100000 Hz. Aplicațiile ultrasonice se găsesc în controlul nedistructiv, curățarea cu ultrasunete, sudarea cu ultrasunete, prelucrarea cu ultrasunete, motoarele cu ultrasunete, sonochimia, debitmetrele cu ultrasunete, caracterizarea materialelor, ultrasonografie, ultrasunete terapeutice, chirurgie cu ultrasunete, liposucție, tratarea apei, controlul poluării aerului. O mare noutate în prezent este distrugerea celulelor canceroase folosind fenomenul de cavitație ultrasonică. În industrie, una dintre aplicațiile posibile, dar relativ puțin studiate, se referă la reducerea coeficientului de frecare care apare în procesul de tragere a sârmei. Acest articol prezintă calculul analitic al principalelor caracteristici ale traductorului ultrasonic și determinarea folosind FEM a modurilor proprii de vibrație, utile în activarea cu ultrasunete a matriței de tragere.

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