

ACCURACY VERIFICATION OF 3D SHAPE MEASUREMENT BY TRANSPORT OF INTENSITY EQUATION USING SINGLE PIXEL IMAGING

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Abstract: Recently, demand for 3D shape measurement of transparent objects has been increasing. Phase measurement is used to measure such objects, but has drawbacks in terms of robustness and complexity of the optical system. In this work, we propose a highly robust and simple optics-based phase measurement method by combining Transport of Intensity Equation (TIE) and Single Pixel Imaging (SPI). In this report, we show that phase measurement can be performed using SPI images by numerical analysis, and then perform TIE using images obtained by SPI to reconstruct the phase. As a result, we confirmed that the quantitative nature of the phase was lost, but we were able to identify the difference between the height and the height of the object.

Key words: 3D Shape Measurement, Phase Measurement, Transport of Intensity Equation, Single Pixel Imaging, Ghost Imaging.

1. INTRODUCTION

In recent years, demand for 3D shape measurement of transparent objects has been increasing. The weak intensity of the reflected light from such an object makes it difficult to observe with ordinary optical instruments. In general, phase information is used to determine the 3D shape of such objects [1]. However, many existing phase measurement methods, such as interferometry, suffer from sensitivity to external disturbances, complex optical setups, and long acquisition times. Therefore, a fast phase measurement method with highly robust and simple optics is required.

The Transport of Intensity Equation (TIE) is an equation from which the phase distribution can be derived from the optical intensity distribution and its axial variation [2]. In TIE, the only values obtained from the measurement are the light intensity distribution and its axial variation. Therefore, in order to improve the robustness of the phase distribution obtained by solving TIE, it is necessary to perform light intensity distribution measurements with higher robustness. In other words, a highly robust optical intensity distribution measurement method is required. Single Pixel Imaging (SPI) is a method of reconstructing a target object based on the scattered light intensity obtained by illuminating the object with known patterns [3]. Within SPI, Ghost Imaging (GI) is known for its high noise tolerance by correlating illumination

patterns and scattered light intensity. However, GI provides only a two-dimensional image and lacks phase information.

In this work, we propose Single Pixel Phase Imaging (SPPI) by combining TIE and SPI. Highly robust phase measurement is realized by introducing highly robust images obtained by GI into TIE. The GI optical setup consists of a projector for pattern illumination, a sample, and a single-pixel detector, allowing for a simple experimental setup. In addition, most of the TIE work has been measuring the phase of transmitted light from transparent objects [4,5,6], but this can only be applied to the measurement of the shape of transparent objects, making it difficult to expand the scope of measurement. Therefore, in this work, the phase of reflected light is measured to make the phase measurement method applicable to more general objects. In this report, we verify the feasibility of the proposed method through numerical simulations and demonstrate its performance using an experimental optical setup.

2. PRINCIPLES OF SINGLE PIXEL PHASE IMAGING

The principle of the proposed method is shown in fig.1. In this method, a known pattern is first illuminated to the sample from a projector as in conventional GI, and the GI image is reconstructed by correlation calculation. GI is an

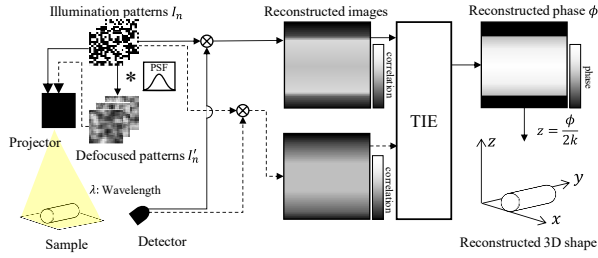


Fig. 1. Principle of SPPI.

imaging method that reconstructs an image based on the correlation operation between the illumination and detection light fluctuations. Illumination light $I_n(x, y)$ ($n = 1, 2, \dots, N$) with a known pattern is illuminated on the sample and N light response intensities B_n are detected. Both the illumination and the light response intensity have fluctuations $\Delta I_n, \Delta B_n$, and their correlation functions $G(x, y)$ is given by

$$G(x, y) = \langle \Delta B_n \Delta I_n(x, y) \rangle, \quad (1)$$

where $\langle \rangle$ denotes the ensemble mean. Since the correlation function is proportional to the light intensity distribution of the sample, the correlation function can be regarded as the light intensity distribution.

In solving TIE, we need light intensity distributions at different focal planes to derive $\partial I / \partial z$. TIE is an equation connecting the light intensity distribution and the phase distribution. The general equation for TIE is given by

$$-k \frac{\partial I(x, y)}{\partial z} = \nabla_{\perp} \cdot \{I(x, y) \nabla_{\perp} \phi(x, y)\}, \quad (2)$$

where k is wavenumber of illumination, $I(x, y)$ is intensity distribution, $\phi(x, y)$ is phase distribution, z is the direction of light travel, and $\nabla_{\perp}$ is the gradient operator in the x - y plane. Since the light intensity distribution $I(x, y)$ appears on both sides of eq. (2), and their units cancel out, the use of a correlation function in the TIE does not introduce any dimensional inconsistency. Of the unknowns other than ϕ , the axial derivative $\partial I / \partial z$ cannot be obtained directly. Therefore, in conventional TIE implementations, multiple intensity distributions are acquired by shifting the imaging plane along the z -axis, and the axial derivative is approximated using a finite difference method. This requirement for z -axis scanning increases measurement time.

In this work, instead of scanning the sample in the z -direction, a defocus pattern is generated by convolving a Point Spread Function (PSF) with a known pattern and illuminating the sample. In GI optics, it is possible to reconstruct the light intensity distribution by moving the sample from the focal plane of the pattern, but the resulting distribution is defocused. In this case, the pattern actually illuminated to the sample is defocused. If this defocused pattern is used as the illuminated pattern and the sample is placed at the focal plane of the pattern, the obtained distribution is the same as that obtained by moving the sample. Therefore, this can be regarded as a pseudo-movement in the z -direction. This eliminates the need for scanning of the optical system, which has been a bottleneck of TIE, thereby increasing the speed and eliminating error factors. Finally, the phase distribution is obtained by solving TIE. In this work, since the phase information of the reflected light is obtained, the light intensity distribution is not uniform. Therefore, the Universal Solution proposed by Zhang et al. is used to solve TIE [7]. The Universal Solution has been reported as a stable and accurate solution that can cope with large changes in the light intensity distribution. Since the phase measurement is performed using the illumination light of wave number k , the surface profile z can be estimated as:

$$z(x, y) = \frac{\phi(x, y)}{2k}. \quad (3)$$

3. NUMERICAL ANALYSIS OF TIE USING GI IMAGES

Numerical analysis was performed to validate the feasibility phase measurement is possible by TIE using GI images. fig. 2 shows the surface height distribution of the sample used in the numerical analysis. The illumination wavelength was set to 520 nm. According to Eq. (3), the maximum phase difference obtained from the sample is π radian. The pixel size was 100 μm , and the image resolution was 64×64 pixels. The illumination pattern used for GI was the Hadamard pattern, and 8,192 images were projected. To simulate the defocus of each pattern due to optical path differences, PSF

modeled as a Gaussian function was applied. This Gaussian function was designed so that its standard deviation increases linearly by 0.03 per μm of defocus. The axial travel distance was set to 10 μm .

The light intensity distribution reconstructed by GI is shown in Fig. 3. The phase distribution reconstructed by the TIE calculation is shown in fig. 4(a), and a comparison with the original phase distribution is shown in fig. 4(b). From fig. 4(a), the periodic structure, which is the original shape, is confirmed even in the derived phase. In fig. 4(b), the periodic structure is indeed consistent with the designed value. Thus, the effectiveness of SPPI in shape measurement is demonstrated.

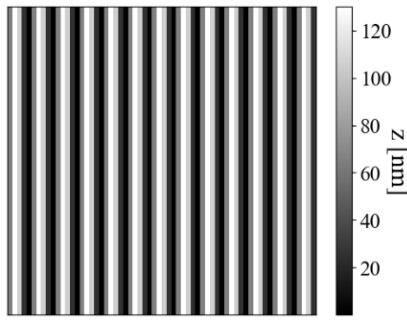


Fig. 2. Height distribution of the samples used in the numerical analysis.

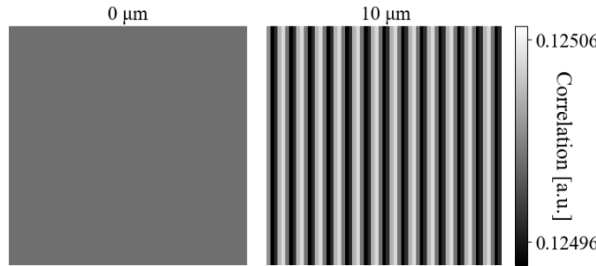


Fig. 3. Images at defocus 0 μm and 10 μm reconstructed by GI.

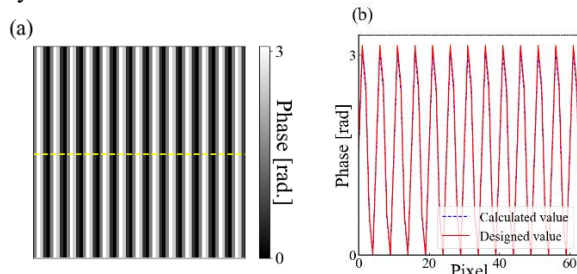


Fig. 4. Evaluation of the derived phase distribution. (a) Derived phase distribution. (b) Comparison of the derived phase distribution with the designed phase distribution.

4. EXPERIMENTAL RESULTS OF PHASE MEASUREMENT BY TIE USING GI IMAGES

To confirm the effectiveness of the proposed method, we conducted phase measurement experiments using TIE with GI. The sample is a block of Martensitic stainless steel with electroless nickel plating, on which a periodic sinusoidal pattern exists in an 8 mm $\times$ 6 mm area. The shape of the pattern has an amplitude of 33 nm with a spatial period of 1,000 μm . Since the phase resulting from the 33 nm height difference is about 0.8 radian, the reconstruction should be possible based on the results of the numerical analysis. In order to obtain a GI image to achieve more accurate defocusing conditions for GI imaging, the sample was physically translated instead of simulating defocus with patterns. The GI optical system is shown in fig. 5. The light source of the projector is a green LED, which illuminates Hadamard patterns 8,192 by 64 $\times$ 64 pixels. Pixel size is 200 μm per side. Δz indicates the distance traveled in the z direction, and GI images were obtained at positions at Δz positions from 0 to 100 μm in 10 μm increments.

The light intensity distribution reconstructed by GI is shown in fig. 6. To improve the accuracy of the GI image used in this work, 10 measurements were taken at each position and averaged. The area circled by the dotted line in the upper left corner is the area with the sinusoidal structure. In addition, since the upper part of the GI image is strongly affected by reflected light due to the position of the detector, the central part enclosed by the white solid line is used for phase measurement. TIE calculations were performed using the light intensity distribution obtained in fig. 6 with $\Delta z = 0$ and 100 μm . The reconstructed phase is

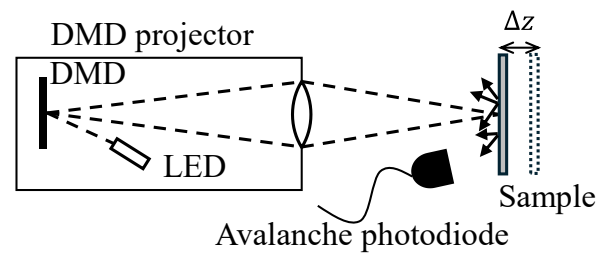


Fig. 5. Optical setup of TIE experiments using GI images.

shown in fig. 7 and the cross-sectional profile along the dotted line in fig. 7 is shown in fig. 8. In the TIE calculation, Fourier transforms are employed, which introduce boundary artifacts due to the periodicity assumption. To avoid introducing boundary artifacts, all pixel values in the outermost 3 pixels of the GI image are set to zero. From fig. 7, the width of the reconstructed phase is about 700 radian. This value is far from the approximately 0.8 radian that is expected to be obtained from the periodic structure. In other words, the quantitative accuracy of the TIE using the GI image is greatly compromised. One possible reason is the difference in the progression of intensity variations caused by the defocus of the projected pattern itself versus those arising from phase differences in the reflected light as it propagates. On the other hand, the cross-sectional profile in fig. 8 reveals a periodic concave-convex phase structure. In particular, the distance between the

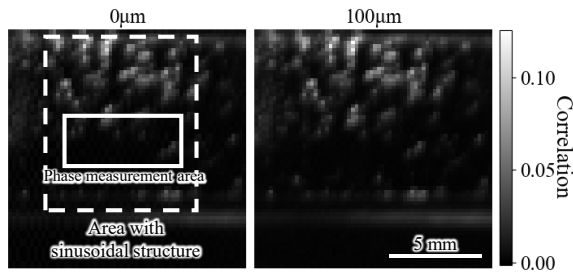


Fig. 6. Images at defocus 0 μm and 100 μm reconstructed by GI.

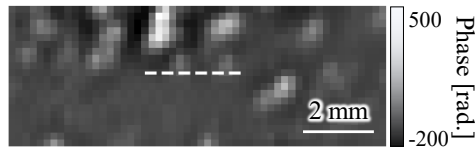


Fig. 7. Phase distributions derived by TIE from GI images at 0 μm and 100 μm .

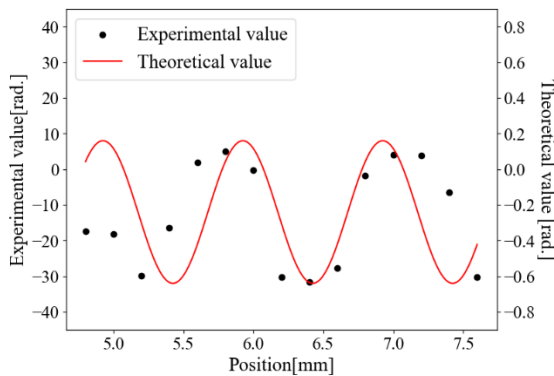


Fig. 8. Cross-sectional profile at the dotted line in fig. 7.

peaks of the central convexity are approximately 1 mm, which is consistent with the spatial frequency of the periodic structure of the sample. Therefore, it is considered that the height difference information is preserved by TIE using the GI image.

5. CONCLUSION

In this report, we proposed SPPI, which enables highly robust phase measurement by combining GI and TIE. Numerical results show that phase measurement is possible by solving TIE using GI images. In the TIE experiment using the GI image in actual optics, the 33 nm uneven shape of the sample could be recognized, although the quantitative accuracy was lost.

As a future prospect, we aim to develop a quantitative phase measurement by designing an appropriate defocus pattern, and by deriving numerically or experimentally the relationship between the GI image obtained when the pattern is defocused and the change in light intensity during defocusing due to phase difference.

6. REFERENCES

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