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ON THE MATHEMATICAL ALGORITHM OF SOME ANCIENT DRACHMAS MINTING TIME

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Abstract: *It was selected three Dyrrachium drachmas issued between 229-30 BC of the "Treasure of Cehei" to establish minting time. There were measured dimensions, weights, and hardness. Finally they were subjected to X-ray diffraction determining the crystallographic parameter, grain size, and material relative deformation that existed at the measurements time. It was established the amount of silver as 96 – 98 weight %. Relative deformation was considered conclusive of all the results that led to the determination years of minting drachmas and those that were magister monetarius who supervised for, under whose authority coins were issued. They are, in order of age, Phereneikos in 164.5 BC, then Meniskos in 115 BC, and the last Xenon 103 BC.*

Key words: *ancient drachmas, XRD patterns, relative deformation*

1. INTRODUCTION

Coin was a symbol, since its appearance in Lydia [1], a universal medium of exchange and payment.

Coin was also the art and technology peak at its minting time. Latest technology was incorporated into the manufacture produces it. The issuer is identified by obverse and reverse signs usually, as a guarantee of coin quality and quantity of content material, often not consistent with the written content of the coin. The minting place results from signs. The place is necessary to identify the issuer.

Time of issue depends on the local dating system. Sometimes dating is often implicit, not explicit. Besides local dating, if any, equivalence accepts dating with some difficulties.

A coin named drachma was issued in the ancient Greek town of Dyrrachium, today Durres in Albania. Drachma had 96 – 98 and 78 - 92 weight % silver and copper as balance [2, 3]. A drachma weighted of 3.4 g. Drachmas minting technology may consist in alloy making, pouring it into blanks and hammering signs on obverse and reverse on blanks (on the large surface of an anvil the lower punch was placed (reverse and obverse), blank above, then upper punch was holding with pliers (obverse and reverse) and hammer blow was applied). Everything is done under the supervision of a magister monetarius.

Dating ancient drachmas is difficult. They were produced from 229 to 30 BC (Dyrrachium battle took place in 48 BC), from historical sources. Issuing date is not explicitly or even implicitly on them.

Three Dyrrachium drachmas, issued different times, were selected to establish hammering date, using historical sources.

Incidentally discovered on 26 April 1986 by two students, Dacian Cehei treasure (Şimleul Silvaniei city, Sălaj County) consists of three bracelets, a chain, a brooch and 552 Dyrrhachium drachmas (445 originals and 7 imitations), Figure 1. In less than 24 hours after discovery, after a brief rest in the Museum of History and Art premises of Zalău County, treasure takes the road to Bucharest, goes to Cabinet 2, and then enter the National Bank heritage, being returned 17 years later to the mentioned Museum. In very short period of time available - from the discovery of the treasure until the dispatch of the day in Bucharest - Eugen Chirilă reputed numismatist of Cluj-Napoca, along with the director of the County Museum of History and Art County at that time, Al. V. Matei, manage to photograph the pieces, to distribute the coins on 26 magistrates and weigh only one coin of each set. It was indicating only average resulting from their weighing together numerous other coins in each group [4].

Fig. 1. Overview photo of Dacian Cehei treasure.

Regarded as one of the great treasures of the first century BC of Transylvania in particular and the Dacia generally, we considered the appropied reevaluation of the scientific aspect in a way that would highlight the true historical meaning of documentation issues achieved at the moment of discovery, for reasons beyond of the authors will, as it was presented before.

2. EXPERIMENTAL METHODOLOGY AND RESULTS

Drachmas of Figure 2, obverse (a) and reverse (b) have dimensions, mass, and H_V hardness presented in Table 1. Magister monetarius is written on the drachmas obverse and was introduced in Table 1 to identify the time when they were hammered.

Dimensions were measured with a 0.01 mm precision micrometer and mass were determined with a 0.1 mg precision electronic balance.

Vickers hardness, H_V, was measured with a Vickers hardness tester. It is the strength to penetration of an indenter in the studied area. Vickers indenter is a diamond squire base pyramid, with the apex angle of 136°. The applied force is 100 N. Load maintenance is 15 seconds. Hardness was measured on the obverse at all coins; indenter application was made on the ox body. Hardness is the wear strength, the money circulation and handling.

The three coins were subjected to X-ray diffraction on the obverse and reverse, using a Shimadzu XRD-6000 diffractometer equipped with Cu anticathode X-ray tube. Used radiation had wavelength of $\lambda_{CuK\alpha1}$ = 154.0598 pm. XRD patterns are shown in Figure 3. Diffractograms results appear in Table 2. Silver and copper (as CuO) diffraction peaks have been identified.

Fig. 2. Analyzed drachmas: a - obverse, b - reverse; 1 - drachma 1, 2 - drachma 2, 3 - drachma 3.

Dimension, mass and H_V hardness of drachmas 1, 2, and 3 Table 1.

Drachma	Magister monetarius	When coins were hammered	Dimension, d_1xd_2, mm	Mass, g	Hardness, H_V, MPa
1	Obv. MENISKOS Rev. DIONISIOU	200 – 33 BC	17.35x18.01	2.9021	799
2	Obv. XENON Rev. PHILODAMOU	After 229 BC	17.78x17.01	3.1163	879
3	Obv. PHERENEIKOS	229 – 100 BC	18.39x18.22	3.2816	851

	Rev. DAMENOS				

Note: d_1 - vertical, d_2 - horizontal. Obv. – Obverse, Rev. – Reverse.

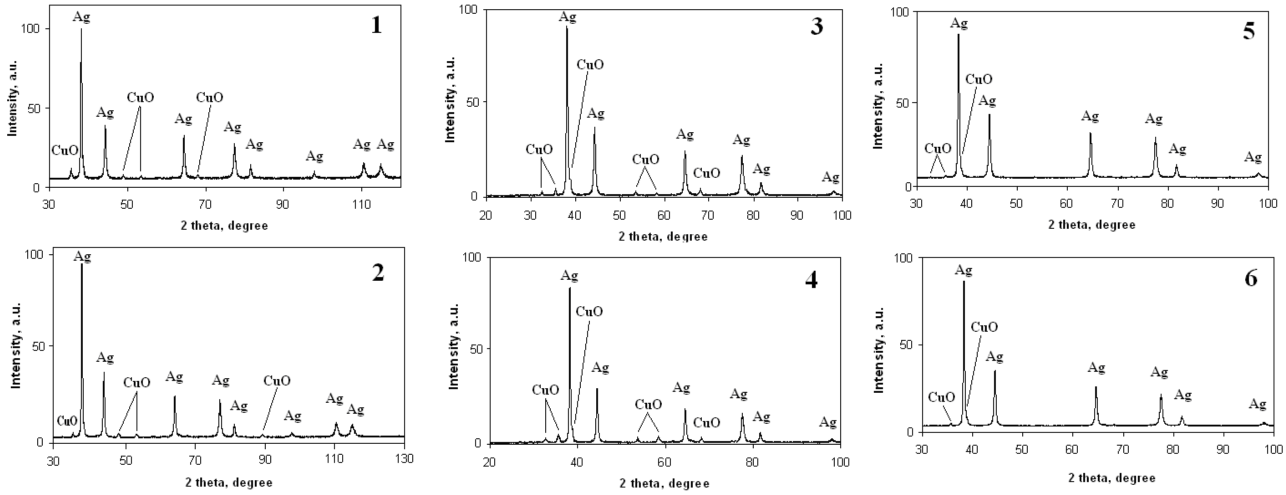

Fig. 3. Coins XRD patterns: 1 - drachma 1 obverse, 2 - drachma 2 reverse, 3 - drachma 2 obverse, 4 - drachma 2 reverse, 5 - drachma 3 obverse, 6 - drachma 3 reverse.

XRD results of Figure 3 Table 2.

	Drachma 1 obverse			Drachma 1 reverse		
Nr.	$2\Theta^o$	a, pm	$B_{obs}{}^o$	$2\Theta^o$	a, pm	$B_{obs}{}^o$
1	38.26	409.4612	0.351852	38.33	406.7450	0.318182
2	44.43	407.8134	0.474747	44.49	407.2913	0.435185
3	64.61	408.0150	0.496241	68.68	407.6213	0.505051
4	77.52	408.4130	0.636364	75.59	408.1025	0.656566
5	81.67	408.4223	0.545455	81.84	407.7229	0.549451
6	97.99	408.6316	0.811765	98.08	408.3728	0.751880
7	110.62	408.6912	0.912500	110.73	408.4200	0.913042
8	114.98	408.8396	1.200000	115.16	408.4312	1.133334
a_o	408.900			408.601		

	Drachma 2 obverse			Drachma 2 reverse		
Nr.	$2\Theta^o$	a, pm	$B_{obs}{}^o$	$2\Theta^o$	a, pm	$B_{obs}{}^o$
1	38.18	408.2831	0.387097	38.16	408.4891	0,263158
2	44.33	408.6868	0.457831	44.32	408.7744	0.517647
3	64.56	408.2968	0.533333	64.45	408.9184	0.492958
4	77.48	408.5906	0.684211	77.46	408.6795	0.580645
5	81.57	408.8353	0.629032	81.57	408.8353	0.512195
6	98.01	408.5696	0.957143	97.93	408.8177	0.913978
a_o	408.699			408.911		

	Drachma 3 obverse			Drachma 3 reverse		
Nr.	$2\Theta^o$	a, pm	$B_{obs}{}^o$	$2\Theta^o$	a, pm	$B_{obs}{}^o$
1	38,46	405.4219	0.340000	38.26	407.4612	0.270270
2	44,62	406.1650	0.388889	44.43	407.8134	0.350000
3	64,72	407.3967	0.420561	64.56	408.2968	0.410000
4	77,67	407.7485	0.514019	77.51	408.4574	0.540984

5	81,82	407.8050	0.458824	81.65	408.5048	0.500000
6	98,24	407.8589	0.614754	97.98	408.6616	0.682243
a_o	408.752			408.969		

Crystallographic parameter, a, was calculated on the base of 2Θ° of Table 2, using:

$$a = \lambda\ (h^2 + k^2 + l^2)^{1/2}/(2.\sin\Theta), \quad (1)$$

where:
λ - wavelength of used X-ray ($\lambda_{Cuk\alpha1}$ = 154.0598 pm);
h, k, l, - Miller indices of diffraction plane;
Θ - diffraction angle (half Bragg angle - 2Θ).
We used equation (1), silver crystallizes in face-cantered cubic system (CFC).

It was used the last squares method for the calculation of crystallographic parameter at zero error, a_o. As extrapolation function was used:

$$f = \cos^2\Theta/\sin\Theta + \cos^2\Theta/\Theta, \quad (2)$$

where: the last Θ is in radians.

Expression (2) provides linearization of **a** value over Bragg angles of 30°. Calculated a_o values are introduced in Table 1.

In order to calculate the crystallite size, d, and relative deformation, η, Williamson – Hall method [5] is used, starting with formula:

$$B_{size+strain}.\cos\Theta = K\lambda/d + \eta.\sin\Theta, \quad (3)$$

where:
$B_{size+strain}$ – width of the peak half-height due to the size and relative deformation (strain) of the crystallites, in radians;
K – constant (0,9).

From the relative deformation, η is calculated remnant stress, σ:

$$\sigma = \eta.E, \quad (4)$$

where:
E - modulus of elasticity of the material.

It is admitted the diffraction profile of the peak, Figure 3, as a Gaussian profile [6], so:

$$B_{size+strain} = (B_{obs}^2 - B_{inst}^2)^{1/2}, \quad (5)$$

where:
B_{obs} - width of observed peak half-height;
B_{inst} – instrument correction - the width of peak half-height of a standard substance with perfect crystal lattice, the large granules and without deformation. It was preferred Gauss distribution; the result is at Cauchy or Taylor distribution [5].

In determining the instrument correction, B_{inst}, a sample of crystalline silicon was subjected to X-ray diffraction on Shimadzu XRD 6000 difractometer ($\lambda_{Cuk\alpha1}$ = 154.0598 pm). The resulted XRD pattern is presented in Figure 4 and diffraction data in Table 3.

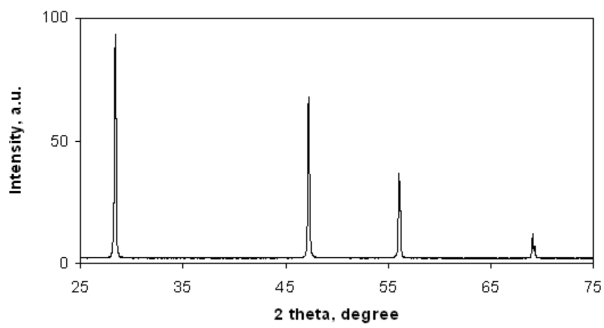

Fig. 4. XRD pattern of crystalline silicon.

Table 3. Diffraction results of the crystalline silicon

Nr.	2Θ°	I, ‰	B°
1	28.25	1000	0.158824
2	47.10	490	0.182692
3	55.92	260	0.240506
4	68.93	75	0.291866

XRD pattern, Figure 4, and diffraction data, Table 3, compared to known literature data [7] show the substance is silicon, giving FoM = 0.5060 (Figure of Merit - coincidence of Figure 4 pattern with [7] pattern). Applying the method of least squares, B values, considering a linear dependence 2Θ° apparent relationship gives instrument constant, B_{inst}^o:

$$B_{inst}^o = 0.051369 + 0.003339.(2\Theta^o). \quad (6)$$

By using relation (5) are determined $B_{size+strains}$, which is inserted into (3) to obtain the crystallite size, d, and the relative deformation, η, Table 4.

Calculation of crystallite size, d, and the relative deformation, η

Table 4.

	Drachma 1 obverse			Drachma 1 reverse		
Nr.	B_{obs} °	B_{inst} °	$B_{size+strain}$ °	B_{obs} °	B_{inst} °	$B_{size+strain}$ °
1	0.351852	0.179119	0.302847	0.318182	0.179353	0.262816
2	0.474747	0.199721	0.430693	0.435185	0.199921	0.386546
3	0.496241	0.267102	0.418224	0.505051	0.267336	0.428495
4	0.636364	0.310208	0.555635	0.656566	0.310442	0.578537
5	0.545455	0.324065	0.438752	0.549451	0.324633	0.443294
6	0.811765	0.378558	0.718092	0.751880	0.378858	0.649454
7	0.912500	0.420729	0.809718	0.913042	0.421096	0.810138
8	1.200000	0.435287	1.118269	1.133334	0.435888	0.921172
	d = 45.523 nm η = 6.851×10^{-3}			d = 45.925 nm η = 6.172×10^{-3}		

	Drachma 2 obverse			Drachma 2 reverse		
Nr.	B_{obs} °	B_{inst} °	$B_{size+strain}$ °	B_{obs} °	B_{inst} °	$B_{size+strain}$ °
1	0.387097	0.178852	0.344332	0.263158	0.178785	0.193101
2	0.457831	0.199387	0.412145	0.517647	0.199353	0.477720
3	0.533333	0.266935	0.461725	0.492958	0.266568	0.414668
4	0.684211	0.310075	0.609917	0.580645	0.310008	0.490962
5	0.629032	0.325731	0.539332	0.512195	0.323731	0.396915
6	0.957143	0.378624	0.879031	0.913978	0.378357	0.831987
	d = 45.534 nm η = 8.051×10^{-3}			d = 59.990 nm η = 7.535×10^{-3}		

	Drachma 3 obverse			Drachma 3 reverse		
Nr.	B_{obs} °	B_{inst} °	$B_{size+strain}$ °	B_{obs} °	B_{inst} °	$B_{size+strain}$ °
1	0.340000	0.179787	0.288577	0.270270	0.179119	0.202391
2	0.388889	0.200355	0.333305	0.350000	0.199721	0.299519
3	0.420561	0.267469	0.324549	0.410000	0.266935	0.311200
4	0.514019	0.310709	0.409482	0.540984	0.310175	0.443233
5	0.458824	0.324566	0.324309	0.500000	0.323998	0.380822
6	0.614754	0.379392	0.483719	0.682243	0.378524	0.567605
	d = 29.040 nm η = 0.495×10^{-3}			d = 72.502 nm η = 5.758×10^{-3}		

The measurements and calculations results (using equations 1-6) are shown in Table 5. Values are averaged on the obverse and reverse.

Table 5.

Measurements and calculations results of diffraction patterns (Tables 2 and 4)

Drachma	a_o, pm	d_{ave}, nm	$\eta_{medave} \times 10^3$
1	408.750	45.759	6.512
2	408.805	52.751	7.788
3	408.860	50.772	3.127

It is noted from Table 1 that the hardness H_V place coins in the following order: 1, 3, and 2, assuming that the relaxation occurs in time, the hardness decreases with time increasing, resulting coin 1 is oldest, coin 2 is nearest to us, and coin 3 is intermediate as a result of hammering time. But historical evidence does not confirm this, with the result that the oldest is the coin 3, coin 1 is hammered after coin 2, facts considered in coin 2 timing.

It is expected that the relative deformation, η, decreasing exponentially with time, t, after the relationship:

$$\eta_t = \eta_o.e^{-kt}, \quad (7)$$

where:
η_t - relative deformation at time t;
η_o - Relative deformation at initial time (t = 0);
k – relaxation constant (1/year) - the relative deformation a year reducing fraction;
t - time since the initial deformation (in years).

Considering that the coins contain silver, copper as alloying element, and same impurity and made in the same shop, with the same technology, there are carried out three equations, for each corresponding coin, following the model of equation (7):

$$\eta_1 = \eta_o.e^{-kt1}; \quad (8)$$

$$\eta_2 = \eta_o.e^{-kt2}; \quad (9)$$

$$\eta_3 = \eta_o.e^{-k.t3}. \quad (10)$$

Logarithm equations (8) and (10), then subtracting them results:

$$\ln\eta_1 - \ln\eta_3 = k(t_3 - t_1), \quad (11)$$

where:

$$k = (\ln\eta_1 - \ln\eta_3)/(t_3 - t_1). \quad (12)$$

Substituting numerical data: years of hammering (mid range of issue), Table 1, reference year (when measurements were made, 2014) and η, from Table 6, results:
k = 0.014820 1/year (13)
Subtracting equation (9) logarithm of equation (8) and considering logarithmic t_1 = 115 + 2014 = 2129, results:
t2 = 2117 years, from which subtracting 2014 results the year 103 BC. The results appear in Table 6.

Dating drachmas Table 6.

Drachma	Considered average hammering year	Current age, years	Calculated seniority, years	Hammering year
1	115 BC	2129		
2			2117	103 BC
3	164.5 BC	2178.5		

Interpretation and calculation results of 1, 2, and 3 drachmas dating appear in Table 7.

Dating drachmas hammering Table 7.

Drachma	Hammering year
1	115 BC
2	103 BC
3	164.5 BC

3. DISCUSSION OF RESULTS

There were issued drachmas in Greek city Dyrrachium, by magister monetarius: Meniskos (200-33 BC), Xenon (after 229 BC) and Phereneikos (229-100 BC) cannot be set on hammering year and order of hammering. We have analyzed the three coins, which contain 96-98 and 78-92 wt % silver, the remainder copper. The coins were obtained from "Dacian Treasure of Cehei".

Drachmas should have a weight of 3.4 g, but due to wear in circulation have different weights. Weight of coins was the numbering criterion in Table 1. If the wear in circulation should be constant (per year) and identical (due to a particular alloy used) then the weight of the oldest coin (coin 1) would be the smallest, hardest (coin 2) would be the latest and intermediate should be (coin 3), but we expect that wear is not commensurate with the time due to different intensity circulation, and different alloy content. So we cannot establish dating by weight loss.

Vickers hardness was measured on the obverse of each coin. Relaxation and decrease hardness of the material creates the impression that coin 1 is the oldest one coin 2 closest to us, and coin 3 is intermediate as hammered, which do not correspond to historical dating

Drachmas were subjected to X-ray diffraction on the obverse and reverse. Silver peaks are numerous and very intense, while those of copper (as CuO) are few and extremely weak, which indicates and confirms the high title of silver compared to copper. No traces of other metals or elements in patterns.

Treasure was formed after 30 BC and until the discovery (1986 AD) was in lying place for 2000 years, during which no circulation wear material was removed from coins, the designs are in very good condition. No copper oxidised (because no air contact) and oxide was not removed in circulation, keeping the state treasury forming.

Crystallographic parameters, a_o, obverse - reverse mean are in the order of drachmas 1 (408.750 pm), 2 (408.805 pm), and 3 (408.860 pm), close values to those of silver (408.55 pm) [8] but little larger, very different from the copper (361.505 pm) [9] indicating that the addition of copper is small, its influence is negligible, possibly the silver content is 96-98 wt %.

There were determined the crystallite size, d, in the order of 45.759 nm (drachma 1) 50.772 nm (drachma 3), and 52.751 nm (drachma 2), and relative deformation, η, in the order of $3.127x10^{-3}$ (drachma 3), $6.512x10^{-3}$ (drachma 1), and 7.788 $x10^{-3}$ (drachma 2) also by X-ray diffraction. To consider reducing the relative deformation with time sounds credible when setting the coin hammering data and not crystallite size variation. Considering the time dependence of relative deformation by an exponential ($\eta_t = \eta_o e^{-kt}$) and the oldest is drachma 3, issued in 164.5 BC (average hammering known time) and drachma 1 issued in 115 BC (average hammering known time), it results for drachma 3 that was hammered in 103 BC. The relative deformation decrease in time is known as relaxation in technique.

4. CONCLUSIONS

After considering reducing the relative deformation appear as years of issuance for the three drachmas follows: first drachma 3 hammered under the authority and supervision of the magister monetarius Phereneikos (164.5 BC), the second drachma 1 issued under the authority and supervision of Meniskos as magister monetarius (115 BC) and the third is drachma 3 issued under the authority and supervision of Xenon as magister monetarius (103 BC).

This may determine and order on the position of magister monetarius of involved persons: the first being Phereneikos, then Meniskos, the last of which is Xenon.

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ASUPRA ALGORITMULUI MATEMATIC AL DATĂRII BATERII UNOR DRAHME ANTICE

Rezumat: *S-au ales trei drahme Dyrrachium emise între 229 – 30 î.Hr. din „Tezaurul de la Cehei“ pentru stabilirea datării baterii. Li s-au măsurat dimensiunile, greutăţile şi durităţile. În final au fost supuse difracţiei cu raze X determinându-se parametrul cristalografic, dimensiunea granulelor şi deformarea remanentă existent la data determinărilor. S-a determinat cantitatea de argint ca fiind de 96 – 98 % greutate. Dintre toate rezultatele au fost considerate concludente deformarea relativă, care a dus la determinarea anilor baterii drahmelor şi a celor ce au fost magister monetarius care au supraveghiat, sub autoritatea căora s-au emis monedele. Aceştia sunt, în ordinea vechimii, Phereneikos în 164,5 î.Hr., apoi Meniskos în 115 î.Hr. şi ultimul Xenon în 103 î.Hr.*

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