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DYNAMIC MODEL FOR THE MOBILE ROBOT RmITA

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Abstract: *In the paper, there will be determined the dynamic control functions for a mobile platform, denoted RmITA. To achieve this goal, the determination of the differential equations of motion that characterizing the mobile structure, are used the equations of the direct kinematic model. Based on this, by appealing specific equations of mechanical systems with nonholonomous links, there are resulting the expressions which are governing the differential motion in configuration space of the mobile platform.*
Key words: *advanced mechanics, robotics, dynamics, motion trajectory, control.*

1. INTRODUCTION

The mobile robots are greatly expanded in recent years, currently they being applied in all areas of human activity. The robot is a natural result of the evolution of automated machines tools, as a result of technical developments - scientific, the robot being defined as a technology able to replace or assist the human in the exercise of various actions on machines or production lines. In this context, it is obvious the complexity of the issues regarding the construction and operation of robots and especially their control.

In keeping with the fact that the robots during operations are performing moving trajectories situated in the configuration space, or in the Cartesian space, it's imposed a continuous control of kinematic parameters from every driving motor, in order to achieve a proper control of the dynamic parameters. Robots with directional motion trajectory planning are requiring complicated maneuvers to achieve the desired orientation and position whether human operator intervenes, or autonomous robot performs these operations. The robot shown in Figure 1 called RmITA is characterized by a differential driving, commonly used in moving of mobile robots. The drive wheel is made by two wheels on each side, both being driven by stepper motors. [1]

Fig.1 The mobile robot RmITA

The RmITA structure besides the two independent wheels has a driven wheel, making a plane-parallel motion, so that in the configuration space performs two translations and a rotation, each of the three movements are parallel and independent of each other. From this feature, the independence of movements, it appears that the mobile robot cannot move laterally at a velocity that is transverse to the axis of the wheels; this constraint is called nonholonomous constraint. In other words, the robot cannot move transversely, but may be in any position and orientation, moving forward - backward, followed by a rotation around its central axis. On these considerations, the straight movement of the mobile mechanical structure is obtained when the wheels are rotating at the same speed and in the same direction (see Figure 2).

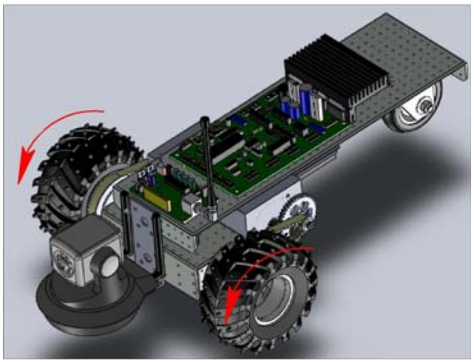

Fig.2 Linear displacement of the mobile structure

The orientation is accomplished by moving one driving wheel of the robot in one direction, and the other wheel in the opposite direction. Thus, in accordance with Figure 3, the desired trajectory is obtained by changing the angular speed and direction of drive wheels on each side.

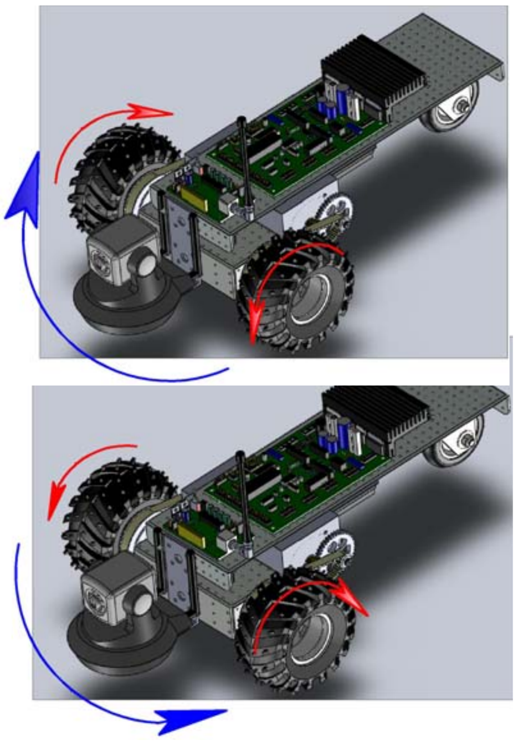

Fig.3 Orientation of the mobile structure

Taking into account the fact that the mobile structure have been presented and modeled in other scientific papers, [1], [2], this paper will be focused on determining the differential equations of motion which are the basis in obtaining the dynamic control functions.

2. THE ACCELERATION ENERGY

To analyze the dynamic behavior of the mobile robot, are used considerations based on advanced mechanics. Hence, applying Lagrange multipliers λ_i and performing transformations specific to nonholonomous mechanical systems, the determining of the dynamics equation is in accordance with the following expression [3]:

$$\frac{\partial E_A^{tot}}{\partial \ddot{q}_j} + Q_f^j + Q_g^j = Q_m^j + \sum_{i=1}^{5} \lambda_i \cdot a_{ij}, \qquad (1)$$

where E_A^{tot} represents the total accelerations energy of the mobile robot, a_{ij} are the coefficients of the elementary displacements $\left(dq_j,\ j=1\rightarrow 7\right)$ and Q_m^j, Q_f^j are representing the generalized driving force, respectively the generalized friction forces. The acceleration energy for the mobile robot RmITA is determined according to:

$$E_A^{tot}\left(q_i;\dot{q}_i;\ddot{q}_i, i=1\rightarrow 7\right) = E_A^{pl}\left(q_i;\dot{q}_i;\ddot{q}_i, i=1\rightarrow 3\right) + \sum_{j=1}^{4} E_A^i\left(q_i;\dot{q}_i;\ddot{q}_i, i=1\rightarrow 7\right), \qquad (2)$$

where E_A^{pl} is the acceleration energy corresponding to the robot without wheels, and E_A^i [4], is the acceleration energy of the component wheels of platform RmITA taken into study. In keeping with [4] and knowing that $\{cq_i = \cos q_i;\ sq_i = \sin q_i, i=1\rightarrow n\}$ the acceleration energy for RmITA structure is:

$$E_A^1 = \frac{M_{rf}}{2} \cdot \left[\left(\ddot{q}_1 + l \cdot \ddot{q}_3 \cdot \mathrm{c}\, q_3 - l \cdot \dot{q}_3^2 \cdot \mathrm{s}\, q_3 \right)^2 + \left(\ddot{q}_2 + l \cdot \ddot{q}_3 \cdot \mathrm{s}\, q_3 + l \cdot \dot{q}_3^2 \cdot \mathrm{c}\, q_3 \right)^2 \right] + \frac{I_{\Delta rf}}{2} \cdot \left(\ddot{q}_4^2 + \frac{1}{2} \cdot \ddot{q}_3^2 + \dot{q}_4^4 + \frac{1}{2} \cdot \dot{q}_3^4 + \frac{3}{2} \cdot \dot{q}_3^2 \cdot \dot{q}_4^2 \right); \qquad (3)$$

$$E_A^2 = \frac{M_{rf}}{2} \cdot \left[\left(\ddot{q}_1 - l \cdot \ddot{q}_3 \cdot \mathrm{c}\, q_3 + l \cdot \dot{q}_3^2 \cdot \mathrm{s}\, q_3 \right)^2 + \left(\ddot{q}_2 - l \cdot \ddot{q}_3 \cdot \mathrm{s}\, q_3 - l \cdot \dot{q}_3^2 \cdot \mathrm{c}\, q_3 \right)^2 \right] + \frac{I_{\Delta rf}}{2} \cdot \left(\ddot{q}_5^2 + \frac{1}{2} \cdot \ddot{q}_3^2 + \dot{q}_5^4 + \frac{1}{2} \cdot \dot{q}_3^4 + \frac{3}{2} \cdot \dot{q}_3^2 \cdot \dot{q}_5^2 \right); \qquad (4)$$

for the two driving wheels. For the driven wheel is obtained the expression:

$$E_A^3 = \frac{M_{rs}}{2} \cdot \left\{ \left[\ddot{q}_1 + L \cdot \left(\ddot{q}_3 \cdot \mathrm{s}\, q_3 + \dot{q}_3^2 \cdot \mathrm{c}\, q_3 \right) \right]^2 + \right.$$
$$\left. + \left[\ddot{q}_2 - L \cdot \left(\ddot{q}_3 \cdot \mathrm{c}\, q_3 - \dot{q}_3^2 \cdot \mathrm{s}\, q_3 \right) \right]^2 \right\} +$$
$$+ \frac{I_{\Delta rs}}{2} \cdot \left(\ddot{q}_6^2 + \frac{1}{2} \cdot \ddot{q}_7^2 + \dot{q}_6^4 + \frac{1}{2} \cdot \dot{q}_7^4 + \frac{3}{2} \cdot \dot{q}_6^2 \cdot \dot{q}_7^2 \right), \quad (5)$$

where $I_{\Delta rf}$ and $I_{\Delta rs}$ are the inertia moments of the forward and respectively backward wheels.

The acceleration energy for the platform is:

$$E_A^{pl} = \frac{1}{2} \cdot M_{pl} \cdot \left(\ddot{q}_1^2 + \ddot{q}_2^2 \right) -$$
$$- M_{pl} \cdot {}^R x_C \cdot \ddot{q}_3 \cdot \left(\ddot{q}_1 \cdot \mathrm{s}\, q_3 - \ddot{q}_2 \cdot \mathrm{c}\, q_3 \right) -$$
$$- M_{pl} \cdot {}^R x_C \cdot \dot{q}_3^2 \cdot \left(\ddot{q}_1 \cdot \mathrm{c}\, q_3 + \ddot{q}_2 \cdot \mathrm{s}\, q_3 \right) + \frac{1}{2} \cdot I_{\Delta pl} \cdot \left(\ddot{q}_3^2 + \dot{q}_3^4 \right) \quad (6)$$

In the previous expressions, there are used the notations: M_{pl} the mass of platform, M_{rs} the mass of backward wheel, M_{rf} the mass of one of the front wheel, ${}^R x_C$ is the mass centre of the platform and $I_{\Delta_{Pl}}$ is the mechanical inertia momentum of the RmITA structure. Knowing that the total acceleration energy of the mobile structure is the sum of the elements in the composition of the robot, according to the relation (2), results relation (8).

Based on fundamental theorems of Newtonian dynamics (or the principle of D'Alembert), [4] according to solidification theorem, there are presented frictional dynamic equilibrium equations expressed as in the relation (9), where Q_f^{rf}, Q_f^{rs} represents the friction moments on forward respectively on backward wheels, r_{rs}, r_{rf}, are the radius of the back, respectively front wheel, Q_f^7 is the friction moment from the joint of the driven wheel, and d_f is the diameter of the shaft on which is situated the driven wheel.

$$E_A^{tot} = \frac{M_{rf}}{2} \cdot \left[\left(\ddot{q}_1 + l \cdot \ddot{q}_3 \cdot \mathrm{c}q_3 - l \cdot \dot{q}_3^2 \cdot \mathrm{s}q_3 \right)^2 + \right.$$
$$+ \left(\ddot{q}_2 + l \cdot \ddot{q}_3 \cdot \mathrm{s}q_3 + l \cdot \dot{q}_3^2 \cdot \mathrm{c}q_3 \right)^2 +$$
$$+ \left(\ddot{q}_1 - l \cdot \ddot{q}_3 \cdot \mathrm{c}q_3 + l \cdot \dot{q}_3^2 \cdot \mathrm{s}q_3 \right)^2 +$$
$$\left. + \left(\ddot{q}_2 - l \cdot \ddot{q}_3 \cdot \mathrm{s}q_3 - l \cdot \dot{q}_3^2 \cdot \mathrm{c}q_3 \right)^2 \right] +$$
$$+ \frac{I_{\Delta rf}}{2} \cdot \left[\ddot{q}_4^2 + \ddot{q}_3^2 + \dot{q}_4^4 + \dot{q}_3^4 + \frac{3}{2} \cdot \dot{q}_3^2 \cdot \dot{q}_4^2 + \right.$$
$$\left. + \ddot{q}_5^2 + \dot{q}_5^4 + \frac{3}{2} \cdot \dot{q}_3^2 \cdot \dot{q}_5{}^2 \right] +$$
$$+ \frac{M_{rs}}{2} \cdot \left\{ \left[\ddot{q}_1 + L \cdot \left(\ddot{q}_3 \cdot \mathrm{s}q_3 + \dot{q}_3^2 \cdot \mathrm{c}q_3 \right) \right]^2 + \right.$$
$$\left. + \left[\ddot{q}_2 - L \cdot \left(\ddot{q}_3 \cdot \mathrm{c}q_3 - \dot{q}_3^2 \cdot \mathrm{s}q_3 \right) \right]^2 \right\} +$$
$$+ \frac{I_{\Delta rs}}{2} \cdot \left(\ddot{q}_6^2 + \frac{1}{2} \cdot \ddot{q}_7^2 + \dot{q}_6^4 + \frac{1}{2} \cdot \dot{q}_7^4 + \frac{3}{2} \cdot \dot{q}_6^2 \cdot \dot{q}_7^2 \right) + \quad (7)$$
$$+ \frac{1}{2} \cdot M_{pl} \cdot \left(\ddot{q}_1^2 + \ddot{q}_2^2 \right) - M_{pl} \cdot {}^R x_C \cdot \ddot{q}_3 \cdot \left(\ddot{q}_1 \cdot \mathrm{s}q_3 - \ddot{q}_2 \cdot \mathrm{c}q_3 \right) -$$
$$- M_{pl} \cdot {}^R x_C \cdot \dot{q}_3^2 \cdot \left(\ddot{q}_1 \cdot \mathrm{c}q_3 + \ddot{q}_2 \cdot \mathrm{s}q_3 \right) + \frac{1}{2} \cdot I_{\Delta pl} \cdot \left(\ddot{q}_3^2 + \dot{q}_3^4 \right).$$

$$Q_f^{rf} = \mu \cdot r_{rf} \cdot \frac{M_{pl} \cdot g \cdot \left({}^R x_C - \mu \cdot r_{rs} \right) + \Delta_T \cdot \left(Q_m^4 + \Delta_0 \cdot Q_m^5 \right)}{2 \cdot \left[\mu \cdot \left(r_{rf} - r_{rs} \right) + L \right]}$$

$$Q_f^{rs} = \mu \cdot r_{rs} \cdot \frac{M_{pl} \cdot g \cdot \left(\mu \cdot r_{rf} + L - {}^R x_C \right) + \Delta_T \cdot \left(Q_m^4 + \Delta_0 \cdot Q_m^5 \right)}{\left[\mu \cdot \left(r_{rf} - r_{rs} \right) + L \right]}; \quad (8)$$

$$Q_f^7 = \frac{d_f}{2} \cdot \mu_7 \cdot \mu \cdot \frac{M_{pl} \cdot g \cdot \left({}^R x_C - \mu \cdot r_{rf} \right) + \Delta_T \cdot \left(Q_m^4 + \Delta_0 \cdot Q_m^5 \right)}{\left[\mu \cdot \left(r_{rs} - r_{rf} \right) + L \right]},$$

In the same expression(8), there are introduced the following parameters:

$$\Delta_T = \left\{ \begin{array}{c} \left(+1,\ Translation^{+} \right) \\ \left(-1,\ Translation^{-} \right) \\ \left(+1,\ Oriention \right) \end{array} \right\};$$

$$\Delta_0 = \left\{ \left(+1,\ Translation \right);\ \left(-1,\ Oriention \right) \right\}$$

The gravitational forces, are determined in concordance with [4], as being: $Q_g^i = 0$.

The differential equations of motion which are defining the dynamic behavior of the mobile platform RmITA, are obtained by calculating the partial derivatives of total acceleration energy (7), an replacing them in the specific expression of nonholonomic mechanical systems (1), as follows:

$$\begin{aligned}&\ddot{q}_1\cdot\left(M_{pl}+2\cdot M_{rf}+M_{rs}\right)+\\&+\left(L\cdot M_{rs}-M_{pl}\cdot{}^{R}x_C\right)\cdot\left(\ddot{q}_3\cdot \mathrm{s}\,q_3+\dot{q}_3^2\cdot \mathrm{c}\,q_3\right)=\\&=-\lambda_1\cdot \mathrm{s}\,q_3+\mathrm{c}\,q_3\cdot\left(\lambda_2+\lambda_3\right)-\\&-\lambda_4\cdot \mathrm{s}\left(q_3+q_7\right)+\lambda_5\cdot \mathrm{c}\left(q_3+q_7\right);\end{aligned} \quad (9)$$

$$\begin{aligned}&\ddot{q}_2\cdot\left(M_{pl}+2\cdot M_{rf}+M_{rs}\right)+\\&+\left(L\cdot M_{rs}-M_{pl}\cdot{}^{R}x_C\right)\cdot\left(\dot{q}_3^2\cdot \mathrm{s}\,q_3-\ddot{q}_3\cdot \mathrm{c}\,q_3\right)=\\&=\lambda_1\cdot \mathrm{c}\,q_3+\mathrm{s}\,q_3\cdot\left(\lambda_2+\lambda_3\right)+\\&+\lambda_4\cdot \mathrm{c}\left(q_3+q_7\right)+\lambda_5\cdot \mathrm{s}\left(q_3+q_7\right);\end{aligned} \quad (10)$$

$$\begin{aligned}&\ddot{q}_3\cdot\left(I_{\Delta f}+I_{\Delta p}+L^2\cdot M_{rs}+2\cdot M_{rf}\cdot l^2\right)+\\&+\left(L\cdot M_{rs}-M_{pl}\cdot{}^{R}x_C\right)\cdot\left(\ddot{q}_1\cdot \mathrm{s}\,q_3-\ddot{q}_2\cdot \mathrm{c}\,q_3\right)=\\&=l\cdot\left(\lambda_2-\lambda_3\right)-\lambda_4\cdot L\cdot \mathrm{c}\,q_7-\lambda_5\cdot L\cdot \mathrm{s}\,q_7;\end{aligned} \quad (11)$$

$$I_{\Delta f}\cdot\ddot{q}_4+Q_f^{rf}\cdot \mathrm{sgn}\left(\dot{q}_4\right)=Q_m^4-\lambda_2\cdot r_{rf}; \quad (12)$$

$$I_{\Delta f}\cdot\ddot{q}_5+Q_f^{rf}\cdot \mathrm{sgn}\left(\dot{q}_5\right)=Q_m^5-\lambda_3\cdot r_{rf}; \quad (13)$$

$$I_{\Delta s}\cdot\ddot{q}_6+Q_f^{rs}\cdot \mathrm{sgn}\left(\dot{q}_6\right)=-\lambda_5\cdot r_{rs}; \quad (14)$$

$$\frac{I_{\Delta rs}}{2}\cdot\ddot{q}_7+Q_f^{7}\cdot \mathrm{sgn}\left(\dot{q}_7\right)=0, \quad (15)$$

where $\mathrm{sgn}\left(\dot{q}_i\right),\ i=4\rightarrow 7$ represents the signs of the velocities.

Based on (9)-(15), it can be noted that for direct dynamic analysis the unknowns are the generalized coordinates $\left\{q_j;\ j=1\rightarrow 7\right\}$ and the Lagrange multipliers $\lambda_i;\ i=1\rightarrow 5$. For the inverse dynamic analysis the unknowns are the generalized driving moments:

$$\left\{Q_m^j;\ j=4\rightarrow 5\right\}. \quad (16)$$

As shown in previously considerations, established for a mechanical structure the dynamic model is represented analytically by means of a system of differential equations which are defining the generalized coordinates, or their derivatives generalized forces acting on each element of the mechanical configuration.

3. DYNAMIC CONTROL FUNCTIONS FOR THE MOBILE STRUCTURE RMITA

In this study the dynamic control functions of a structure, regardless of its type, based on displacement structure a well-defined path, to achieve some points [5], speeds and accelerations imposed by the task, the motors must overcome the technological external generalized forces and gravity acting on the system which are generally well defined and generalized inertial forces due to motors, transmissions, mechanical, or structural strength of the mechanical system analyzed. The dynamic equations of the robot RmITA determined by (9)-(15) are highlighting the complexity of the dynamic control problem. The moving of mobile structure must be studied considering the fact that the structure, to achieve the target point is not performing simultaneously positioning (translation) and orientation, so the two movements will be analyzed independently, taking into account the differential kinematical restrictions [6], presented in the following expressions, in accordance with Figure 4:

$$-\mathrm{s}\,q_3\cdot dq_1+\mathrm{c}\,q_3\cdot dq_2=0\ ; \quad (17)$$

$$\mathrm{c}\,q_3\cdot dq_1+\mathrm{s}\,q_3\cdot dq_2+l\cdot dq_3-r_{rf}\cdot dq_4=0; \quad (18)$$

$$\mathrm{c}\,q_3\cdot dq_1+\mathrm{s}\,q_3\cdot dq_2-l\cdot dq_3-r_{rf}\cdot dq_5=0; \quad (19)$$

$$-\mathrm{s}\left(q_3+q_7\right)\cdot dq_1+\mathrm{c}\left(q_3+q_7\right)\cdot dq_2-L\cdot \mathrm{c}\,q_7\cdot dq_3=0; \quad (20)$$

$$\begin{aligned}&\mathrm{c}\left(q_3+q_7\right)\cdot dq_1+\mathrm{s}\left(q_3+q_7\right)\cdot dq_2-\\&-L\cdot \mathrm{s}\,q_7\cdot dq_3-r_{rs}\cdot dq_6=0.\end{aligned} \quad (21)$$

The expression (17) is providing the preventing condition of platform translation along the axis y_R, the equations (18) and (19) are the conditions for pure rolling without slipping of the driven wheels, (20) is restricting the sliding of driven wheel along the transversal axis, and (21) is the required condition for pure rolling without slipping of the driven wheel.

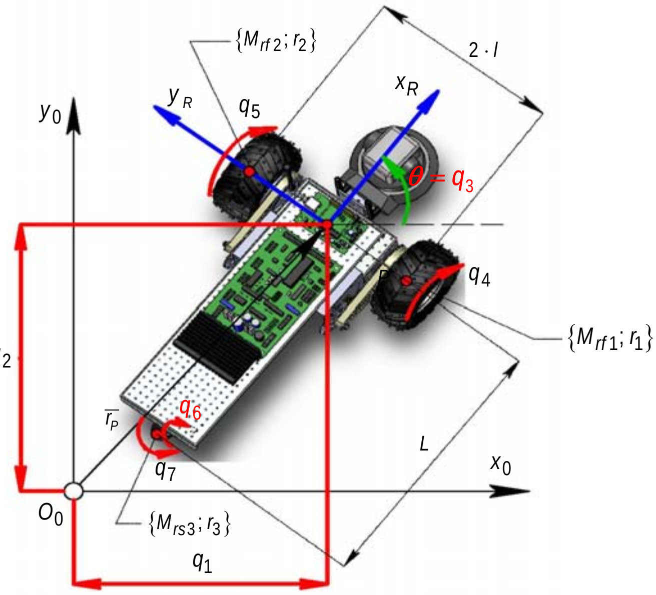

Figure 4 Independent parameters in finite displacements of RmITA mobile robot

According to previous considerations, for the robot to perform a linear translation motion in accordance with its design structure, an essential condition derives from the fact that the driving wheel moments must be equal:

$$\left\{Q_m^4 = Q_m^5\right\}, \tag{22}$$

which is conducting to equivalences:

$$\left\{(\ddot{q}_4 = \ddot{q}_5),\ q_3 = cst.,\ (\dot{q}_3, \ddot{q}_3) = 0,\ \ q_7 = 0\right\}.$$

In these circumstances, taking into account the relationships, (12) and (13) the driving moments for the RmITA structure, in order to execute a linear displacement, according to (22) are determined as follows:

$$Q_m^4 = Q_m^5 = \frac{r_{rf}}{2}\cdot\Big[\left(M_{pl} + 2\cdot M_{rf} + M_{rs}\right)\cdot\left(\ddot{q}_1\cdot cq_3 + \ddot{q}_2\cdot sq_3\right) + \frac{1}{r_{rs}}\left(I_{\Delta rs}\cdot\ddot{q}_6 + Q_f^{rs}\cdot \mathrm{sgn}(\dot{q}_6)\right)\Big] + I_{\Delta rf}\cdot\ddot{q}_4 + Q_f^{rf}\cdot\mathrm{sgn}(\dot{q}_4). \tag{23}$$

Due to mechanical constitution [1] according to independent parameters in finite displacements, presented in Figure 4, for the orientation of the structure is necessary the condition:

$$\left\{Q_m^4 = -Q_m^5\right\}, \tag{24}$$

which is leading to:

$\left\{(\ddot{q}_5 = -\ddot{q}_4)\right\}$ respectively $\left\{(-\lambda_2 = \lambda_3)\right\}$. (25)

In keeping with the previous considerations, the expression for the moments of the two driving wheels of the mobile robot RmITA, in order to execute the orientation is:

$$Q_m^4 = \frac{r_{rf}}{2\cdot l}\cdot\Big[\ddot{q}_3\cdot\left(I_{\Delta rf} + I_{\Delta pl} + L^2\cdot M_{rs} + 2\cdot M_{rf}\cdot l^2\right) + \frac{L}{r_{rs}}\left(I_{\Delta rs}\cdot\ddot{q}_6 + Q_f^{rs}\cdot\mathrm{sgn}(\dot{q}_6)\right)\Big] + I_{\Delta rf}\cdot\frac{l}{r_{rf}}\cdot\ddot{q}_3 + Q_f^{rf}\cdot\mathrm{sgn}(\dot{q}_4) = -Q_m^5; \tag{26}$$

The expressions (23) and (26) are expressing the driving moments of the two driving wheels of the mobile robot RmITA. The same expressions also are representing the dynamic control functions. Also, the two aforementioned expressions, are containing a static component Q_{ms}^i, which is due to weight of the mechanical system, the resistance forces and a dynamic component Q_{md}^i. Thus, the total torque can express, in accordance with:

$$\left\{Q_m^i = Q_{ms}^i + Q_{md}^i\ ,\ \ (i = 4,5)\right\}. \tag{27}$$

4. CONCLUSIONS

The contents of this paper is linked to dynamic modeling, of a mobile robot, denoted RmITA, for which are established the expressions which are governing the dynamic behavior. Based on geometric modeling of the structure there have been presented the kinematical constraints which influence the structure and also are integrating the mobile platform in the class of nonholonomous mechanical systems.

Based on new concepts in the advanced mechanics of the mechanical systems, it was established the total acceleration energy for the mobile robot. In its final formula it can be seen that are appearing parameters such as the position of the rotation axis (central axis), the mechanical moment of inertia.

The dynamic model of the structure is based on acceleration energy, which is substituted in the specific expression of nonholonomous mechanical systems which is leading to the determination of the differential equations of motion of the mobile robot RmITA. Also in this part of the paper were determined the expressions of the two driving moments of the wheels that moving the mechanical system.

Their determination was made on the conditions required for a straight displacement, orientation and kinematic restrictions of robot.

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Modelul dinamic pentru structura mobilă RmITA

Rezumat: In lucrare, au fost stabilite funcțiile de control dinamic pentru o platformă mobilă, denumită RmITA. Pentru a atingerea acestui deziderat, adică determinarea ecuațiilor diferențiale de mișcare ce caracterizează structura mobilă, au fost utilizate ecuațiile modelului cinematic direct. Astfel, apelând ecuațiile specifice sistemelor mecanice cu legături neolonome, au rezultat expresiile dinamice, care guvernează mișcarea diferențială în spațiul configurațiilor platformei mobile RmITA.

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