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THE GEOMETRIC MODEL OF THE TRTTRR1 MODULAR SERIAL ROBOT

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Abstract: The structure of the paper has three parts: a theoretical approach, the geometric model of the TRTTRR1 modular serial robot and conclusions, followed by a reference list. After a general presentation of the geometrical modeling of robots, the rotation matrices method is mentioned. A brief description of the mechanical structure of TRTTRR1 modular serial robot is then performed and the algorithm for direct modeling is presented in detail, after that, for the above mentioned robot. The column vector of the operational coordinates, defining the position and the orientation of the robot's gripper is determined, with respect to the fixed frame.

Key words: modular serial robot, geometric model, rotation matrices method.

1. THEORETICAL APPROACH

The fundamental equations for the direct and inverse geometric model, according to [1] and [2] can be written as:

$$\overline{X}^0 = f(\overline{q}) \tag{1}$$

$$\overline{q} = f^{-1} \left(\overline{X}^0 \right), \tag{2}$$

where f represents the linear operator of direct transformation of the column vector \overline{q} into \overline{X}^0 and f^{-1} is a non-linear inverse transformation operator of the vector \overline{X}^0 into the vector \overline{q} .

The problem of direct geometric modeling (DGM) can be solved in one of the following methods: the vector method, the method of rotation matrices, the method of PG-type composite operators, the method of Denavit-Hartenberg composite operators, the method of exponential matrices.

The inverse geometrical model (IGM) is mathematically expressed by the relation (2). In the case of inverse modeling, the position of the gripper with respect to the fixed frame (T_0) is assumed to be known and the column vector \bar{q} of the generalized coordinates is determined, containing the geometric control function from each driving joint k ($k = 1 \div n$) of the robot. For a given robot, the number (k) of the degrees of freedom (DOF) is constant, but the number (m) of the operational coordinates can be a variable, but it does not exceed the number of the generalized coordinates. From this reason, in order to achieve the geometric control, the condition $m \le k$ must be ensured.

The system of equations (2) from the inverse geometric model is generally a system of nonlinear and transcendental equations, which in some cases can be solved by algebraic or geometric methods.

1.1 The Rotation Matrices Method

A convenient method, with the view of the matrix calculus, is the rotation matrices method [1], [3], which uses 3×3 rotation matrices and 3×1 position vectors in order to obtain the geometric model of the transformations from an element sequence (k-1, k).

The figure 1 depicts the mechanical structure of a serial robot with (*n*) DOF, having an open kinematic chain. The mechanical structure of the robot consists in (*n*+1) rigid elements linked together through (*n*) kinematic joints of rotation (R) or translation (T). A reference mobile frame (T_k) is attached to the origin of each element *k* ($k = 1 \div n$), and at the robot base, at a point O_0 belonging to its frame, the fixed frame (T_0) is attached.



Fig. 1. The kinematic structure of a robot with *n* DOF

The DGM, according to [2], assumes that the robot mechanical structure is situated into a known configuration, represented by the vector \overline{q} of the generalized coordinates.

The position of the frame (T_n) , rigidly linked to the gripper, with respect to the frame (T_0) , can be determined as:

- the position O_n of the frame (T_n) is determined by the vector $\overline{p}_n = [p]^0$;

- the orientation of each axis of the frame (T_n) with respect to the frame (T_0) is determined using the rotation matrix $[R]_n^0$.

Successive iterations are used to solve the DGM problem. The column vector of the operational coordinates is determined as:

$$\begin{bmatrix} \overline{X} \end{bmatrix}^0 = \begin{bmatrix} p_x & p_y & p_z & \alpha_z & \beta_x & \gamma_y \end{bmatrix}^T = \\ = \begin{bmatrix} f_j & (q_k, k = 1 \div n), & j = 1 \div 6 \end{bmatrix}^T.$$
(3)

The equation (3) defines the position of the gripper with respect to the base frame (T_0) through the coordinates p_x , p_y , p_z of a point belonging to the gripper and by the elements α_z , β_x , γ_y of the rotation matrix R, defining the orientation.

2. THE GEOMETRIC MODEL OF THE TRTTRR1 MODULAR SERIAL ROBOT

2.1 The Mechanical Structure of the TRTTRR1 Modular Serial Robot

The 6-DOF robot presented in fig. 2 has a kinematic structure consisting in six modules, a fixed base and a gripper.



Fig. 2. The kinematic structure of the TRTTRR1 modular serial industrial robot

The module 1 represents the horizontal translation module of the entire robot; the module 2 is the rotation module of the robot arm; the module 3 is the vertical translation module. The module 4 represents the horizontal translation module of the robot arm; the modules 5 and 6 make together the orientation module of the gripper 7.

The figure 2 contains the following notations: l_i - the constructive parameters of the robot ($i = 1 \div 7$) and q_k - the generalized coordinates ($k = 1 \div 6$).

2.2 The Direct Geometric Model of the TRTTRR1 Modular Serial Robot Using the Rotation Matrices Method

The orientation matrices expressing the relative orientation of each frame with respect to the previous frame are the following:

$$[R]_{1}^{0} = I_{1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
(4)
$$[R]_{2}^{1} = R(\bar{z}_{2}; q_{2}) = \begin{bmatrix} cq_{2} & -sq_{2} & 0 \\ sq_{2} & cq_{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
(5)
$$[R]_{3}^{2} = I_{3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
(6)
$$[R]_{4}^{3} = I_{4} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
(7)

$$[R]_{5}^{4} = R(\bar{x}_{5}; q_{5}) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cq_{5} & -sq_{5} \\ 0 & sq_{5} & cq_{5} \end{bmatrix}$$
(8)

$$[R]_{6}^{5} = [\overline{y}_{6}; q_{6}] = \begin{bmatrix} cq_{6} & 0 & sq_{6} \\ 0 & 1 & 0 \\ -sq_{6} & 0 & cq_{6} \end{bmatrix}$$
(9)

$$[R]_{7}^{6} = I_{7} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
(10)

The relative position vectors of the origins O_i of the frames $O_i x_i y_i z_i$ $(i = 0 \div 7)$ with respect to the previous frame have the following matrix expressions:

$$\begin{bmatrix} \bar{r} \end{bmatrix}_{1}^{0} = \begin{bmatrix} 0 \\ l_{0} + q_{1} \\ 0 \end{bmatrix}; \quad \begin{bmatrix} \bar{r} \end{bmatrix}_{2}^{1} = \begin{bmatrix} 0 \\ 0 \\ l_{1} \end{bmatrix}; \quad (11)$$
$$\begin{bmatrix} \bar{r} \end{bmatrix}_{3}^{2} = \begin{bmatrix} 0 \\ 0 \\ l_{2} + q_{3} \end{bmatrix}; \quad \begin{bmatrix} \bar{r} \end{bmatrix}_{4}^{3} = \begin{bmatrix} l_{4} + q_{4} \\ 0 \\ l_{3} \end{bmatrix}; \quad (12)$$
$$\begin{bmatrix} \bar{r} \end{bmatrix}_{5}^{4} = \begin{bmatrix} l_{5} \\ 0 \\ 0 \end{bmatrix}; \quad \begin{bmatrix} \bar{r} \end{bmatrix}_{5}^{6} = \begin{bmatrix} l_{6} \\ 0 \\ 0 \end{bmatrix}; \quad \begin{bmatrix} \bar{r} \end{bmatrix}_{7}^{6} = \begin{bmatrix} l_{7} \\ 0 \\ 0 \end{bmatrix}. \quad (13)$$

According to [4] and [5], the absolute rotation matrices, expressing the orientation of each mobile frame with respect to the fixed frame are obtained with the following relations:

$$[R]_{2}^{0} = [R]_{1}^{0} \cdot [R]_{2}^{1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} cq_{2} & -sq_{2} & 0 \\ sq_{2} & cq_{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} cq_{2} & -sq_{2} & 0 \\ sq_{2} & cq_{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
(14)

$$[R]_{3}^{0} = [R]_{2}^{0} \cdot [R]_{3}^{2} = \begin{bmatrix} cq_{2} & -sq_{2} & 0\\ sq_{2} & cq_{2} & 0\\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} cq_{2} & -sq_{2} & 0\\ sq_{2} & cq_{2} & 0\\ 0 & 0 & 1 \end{bmatrix}$$
(15)

$$[R]_{4}^{0} = [R]_{3}^{0} \cdot [R]_{4}^{3} = \begin{bmatrix} cq_{2} & -sq_{2} & 0\\ sq_{2} & cq_{2} & 0\\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} cq_{2} & -sq_{2} & 0\\ sq_{2} & cq_{2} & 0\\ 0 & 0 & 1 \end{bmatrix}$$
(16)

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$$\begin{bmatrix} R \end{bmatrix}_{5}^{0} = \begin{bmatrix} R \end{bmatrix}_{4}^{0} \cdot \begin{bmatrix} R \end{bmatrix}_{5}^{1} = \begin{bmatrix} cq_{2} & -sq_{2} & 0 \\ sq_{2} & cq_{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & cq_{5} & -sq_{5} \\ 0 & sq_{5} & cq_{5} \end{bmatrix} = \begin{bmatrix} cq_{2} & -sq_{2}cq_{5} & sq_{2}sq_{5} \\ sq_{2} & cq_{2}cq_{5} & -cq_{2}sq_{5} \\ 0 & sq_{5} & cq_{5} \end{bmatrix} = \begin{bmatrix} cq_{2} & -sq_{2}cq_{5} & sq_{2}sq_{5} \\ 0 & sq_{5} & cq_{5} \end{bmatrix} = \begin{bmatrix} cq_{2} & -sq_{2}cq_{5} & sq_{2}sq_{5} \\ 0 & 1 & 0 \\ -sq_{6} & 0 & cq_{6} \end{bmatrix} =$$

$$= \begin{bmatrix} cq_{2}cq_{6} - sq_{2}sq_{5}sq_{6} & -sq_{2}cq_{5} & cq_{2}sq_{6} + sq_{2}sq_{5}cq_{6} \\ 0 & 1 & 0 \\ -sq_{6} & 0 & cq_{6} \end{bmatrix} =$$

$$= \begin{bmatrix} cq_{2}cq_{6} - sq_{2}sq_{5}sq_{6} & -sq_{2}cq_{5} & cq_{2}sq_{6} + sq_{2}sq_{5}cq_{6} \\ -cq_{5}sq_{6} & sq_{5} & cq_{2}cq_{5} & sq_{2}sq_{6} - cq_{2}sq_{5}cq_{6} \\ -cq_{5}sq_{6} & sq_{5} & cq_{2}cq_{5} & sq_{2}sq_{5}cq_{6} \end{bmatrix};$$

$$= \begin{bmatrix} cq_{2}cq_{6} - sq_{2}sq_{5}sq_{6} & -sq_{2}cq_{5} & cq_{2}sq_{6} + sq_{2}sq_{5}cq_{6} \\ -cq_{5}sq_{6} & sq_{5} & cq_{2}cq_{5} & sq_{2}sq_{6} - cq_{2}sq_{5}cq_{6} \\ -cq_{5}sq_{6} & sq_{5} & cq_{2}cq_{6} + cq_{2}sq_{5}sq_{6} \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} =$$

$$= \begin{bmatrix} cq_{2}cq_{6} - sq_{2}sq_{5}sq_{6} & cq_{2}cq_{5} & sq_{2}sq_{5}cq_{6} \\ -cq_{5}sq_{6} & sq_{5} & cq_{5}cq_{6} \end{bmatrix} \cdot$$

$$= \begin{bmatrix} cq_{2}cq_{6} - sq_{2}sq_{5}sq_{6} & cq_{2}cq_{5} & sq_{2}sq_{5}cq_{6} \\ -cq_{5}sq_{6} & sq_{5} & cq_{2}cq_{5} & sq_{2}sq_{5}cq_{6} \end{bmatrix} \cdot$$

$$= \begin{bmatrix} cq_{2}cq_{6} - sq_{2}sq_{5}sq_{6} & cq_{2}cq_{5} & sq_{2}sq_{6} - cq_{2}sq_{5}cq_{6} \\ -cq_{5}sq_{6} & sq_{5} & cq_{2}cq_{6} + cq_{2}sq_{5}cq_{6} \end{bmatrix} \cdot$$

$$= \begin{bmatrix} cq_{2}cq_{6} - sq_{2}sq_{5}sq_{6} & cq_{2}cq_{5} & sq_{2}sq_{6} - cq_{2}sq_{5}cq_{6} \\ -cq_{5}sq_{6} & sq_{5} & cq_{2}sq_{5}cq_{6} \end{bmatrix} \cdot$$

According to [6], the set of orientation independent parameters $(\alpha_z, \beta_x, \gamma_y)$ are determined by identifying the elements (2, 2), (3, 2) and (3, 3) from the matrix relation (19) with the homologous ones from the relation (20):

$$R(\alpha_z - \beta_x - \gamma_y) =$$
(3)

 $\begin{bmatrix} -s\alpha_{z}s\beta_{x}s\gamma_{y} + c\alpha_{z}c\gamma_{y} & -s\alpha_{z}c\beta_{x} & s\alpha_{z}s\beta_{x}c\gamma_{y} + c\alpha_{z}s\gamma_{y} \\ c\alpha_{z}s\beta_{x}s\gamma_{y} + s\alpha_{z}c\gamma_{y} & c\alpha_{z}c\beta_{x} & -c\alpha_{z}s\beta_{x}c\gamma_{y} + s\alpha_{z}s\gamma_{y} \\ -c\beta_{x}s\gamma_{y} & s\beta_{x} & c\beta_{x}c\gamma_{y} \end{bmatrix}$

The set of orientation independent parameters $(\alpha_z, \beta_x, \gamma_y)$ are described in the relation:

$$\begin{bmatrix} \alpha_z & \beta_x & \gamma_y \end{bmatrix}^T = \begin{bmatrix} q_2 & q_5 & q_6 \end{bmatrix}^T \quad (21)$$

The matrix equations below, known as the relative translation vectors, are used to express the position of each frame origin with respect to the previous frame:

$$\begin{bmatrix} \overline{p} \end{bmatrix}_{10} = \begin{bmatrix} \overline{r} \end{bmatrix}_{1}^{0} = \begin{bmatrix} 0 \\ l_{0} + q_{1} \\ 0 \end{bmatrix}; \quad \begin{bmatrix} \overline{p} \end{bmatrix}_{21} = \begin{bmatrix} R \end{bmatrix}_{1}^{0} \cdot \begin{bmatrix} \overline{r} \end{bmatrix}_{2}^{1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ l_{1} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ l_{1} \end{bmatrix}; \quad (22)$$

$$\begin{bmatrix} \overline{p} \end{bmatrix}_{32} = \begin{bmatrix} R \end{bmatrix}_2^0 \cdot \begin{bmatrix} \overline{r} \end{bmatrix}_3^2 = \begin{bmatrix} cq_2 & -sq_2 & 0 \\ sq_2 & cq_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ l_2 + q_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ l_2 + q_3 \end{bmatrix};$$
(23)

$$\begin{bmatrix} \overline{p} \end{bmatrix}_{43} = \begin{bmatrix} R \end{bmatrix}_{3}^{0} \cdot \begin{bmatrix} \overline{r} \end{bmatrix}_{4}^{3} = \begin{bmatrix} cq_{2} & -sq_{2} & 0 \\ sq_{2} & cq_{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} l_{4} + q_{4} \\ 0 \\ l_{3} \end{bmatrix} = \begin{bmatrix} (l_{4} + q_{4})cq_{2} \\ (l_{4} + q_{4})sq_{2} \\ l_{3} \end{bmatrix};$$
(24)

$$[\overline{p}]_{54} = [R]_4^0 \cdot [\overline{r}]_5^4 = \begin{bmatrix} cq_2 & -sq_2 & 0\\ sq_2 & cq_2 & 0\\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} l_5\\ 0\\ 0 \end{bmatrix} = \begin{bmatrix} l_5cq_2\\ l_5sq_2\\ 0 \end{bmatrix};$$
(25)

$$\begin{bmatrix} \overline{p} \end{bmatrix}_{65} = \begin{bmatrix} R \end{bmatrix}_{5}^{0} \cdot \begin{bmatrix} \overline{r} \end{bmatrix}_{6}^{5} = \begin{bmatrix} cq_{2} & -sq_{2}cq_{5} & sq_{2}sq_{5} \\ sq_{2} & cq_{2}cq_{5} & -cq_{2}sq_{5} \\ 0 & sq_{5} & cq_{5} \end{bmatrix} \cdot \begin{bmatrix} l_{6} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} l_{6}cq_{2} \\ l_{6}sq_{2} \\ 0 \end{bmatrix};$$
(26)

$$\begin{bmatrix} \overline{p} \end{bmatrix}_{76} = \begin{bmatrix} R \end{bmatrix}_{6}^{0} \cdot \begin{bmatrix} \overline{r} \end{bmatrix}_{7}^{6} = \begin{bmatrix} cq_{2}cq_{6} - sq_{2}sq_{5}sq_{6} & -sq_{2}cq_{5} & cq_{2}sq_{6} + sq_{2}sq_{5}cq_{6} \\ sq_{2}cq_{6} + cq_{2}sq_{5}sq_{6} & cq_{2}cq_{5} & sq_{2}sq_{6} - cq_{2}sq_{5}cq_{6} \\ -cq_{5}sq_{6} & sq_{5} & cq_{5}cq_{6} \end{bmatrix} \cdot \begin{bmatrix} l_{7} \\ 0 \\ 0 \end{bmatrix} = \\ = \begin{bmatrix} (cq_{2}cq_{6} - sq_{2}sq_{5}sq_{6})l_{7} \\ (sq_{2}cq_{6} + cq_{2}sq_{5}sq_{6})l_{7} \\ -cq_{5}sq_{6}l_{7} \end{bmatrix}.$$

$$(27)$$

The relations (22) – (27) are used to obtain the origin position of each frame with respect to the fixed frame $O_0 x_0 y_0 z_0$ from the robot base. It can be written that:

$$\begin{bmatrix} \overline{p} \end{bmatrix}_{1} = \begin{bmatrix} \overline{p} \end{bmatrix}_{10} = \begin{bmatrix} 0 \\ l_0 + q_1 \\ 0 \end{bmatrix}; \quad \begin{bmatrix} \overline{p} \end{bmatrix}_{2} = \begin{bmatrix} \overline{p} \end{bmatrix}_{1} + \begin{bmatrix} \overline{p} \end{bmatrix}_{21} = \begin{bmatrix} 0 \\ l_0 + q_1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ l_1 \end{bmatrix} = \begin{bmatrix} 0 \\ l_0 + q_1 \\ l_1 \end{bmatrix}; \quad (28)$$

$$[\overline{p}]_{3} = [\overline{p}]_{2} + [\overline{p}]_{32} = \begin{bmatrix} 0 \\ l_{0} + q_{1} \\ l_{1} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ l_{2} + q_{3} \end{bmatrix} = \begin{bmatrix} 0 \\ l_{0} + q_{1} \\ l_{1} + l_{2} + q_{3} \end{bmatrix};$$
(29)

$$\begin{bmatrix} \overline{p} \end{bmatrix}_{4} = \begin{bmatrix} \overline{p} \end{bmatrix}_{3} + \begin{bmatrix} \overline{p} \end{bmatrix}_{43} = \begin{bmatrix} 0 \\ l_{0} + q_{1} \\ l_{1} + l_{2} + q_{3} \end{bmatrix} + \begin{bmatrix} (l_{4} + q_{4})cq_{2} \\ (l_{4} + q_{4})sq_{2} \\ l_{3} \end{bmatrix} = \begin{bmatrix} (l_{4} + q_{4})cq_{2} \\ l_{0} + q_{1} + (l_{4} + q_{4})sq_{2} \\ l_{1} + l_{2} + l_{3} + q_{3} \end{bmatrix};$$
(30)

$$\begin{bmatrix} \overline{p} \end{bmatrix}_{5} = \begin{bmatrix} \overline{p} \end{bmatrix}_{4} + \begin{bmatrix} \overline{p} \end{bmatrix}_{54} = \begin{bmatrix} (l_{4} + q_{4})cq_{2} \\ l_{0} + q_{1} + (l_{4} + q_{4})sq_{2} \\ l_{1} + l_{2} + l_{3} + q_{3} \end{bmatrix} + \begin{bmatrix} l_{5}cq_{2} \\ l_{5}sq_{2} \\ 0 \end{bmatrix} = \begin{bmatrix} (l_{4} + l_{5} + q_{4})cq_{2} \\ l_{0} + q_{1} + (l_{4} + l_{5} + q_{4})sq_{2} \\ l_{1} + l_{2} + l_{3} + q_{3} \end{bmatrix}; \quad (31)$$

$$\begin{split} [\overline{p}]_{6} &= [\overline{p}]_{5} + [\overline{p}]_{65} = \begin{bmatrix} (l_{4} + l_{5} + q_{4})cq_{2} \\ l_{0} + q_{1} + (l_{4} + l_{5} + q_{4})sq_{2} \\ l_{1} + l_{2} + l_{3} + q_{3} \end{bmatrix} + \begin{bmatrix} l_{6}cq_{2} \\ l_{6}sq_{2} \\ 0 \end{bmatrix} = \begin{bmatrix} (l_{4} + l_{5} + l_{6} + q_{4})cq_{2} \\ l_{0} + q_{1} + (l_{4} + l_{5} + l_{6} + q_{4})sq_{2} \\ l_{1} + l_{2} + l_{3} + q_{3} \end{bmatrix}; (32) \\ [\overline{p}]_{7} &= [\overline{p}]_{6} + [\overline{p}]_{76} = \begin{bmatrix} (l_{4} + l_{5} + l_{6} + q_{4})cq_{2} \\ l_{0} + q_{1} + (l_{4} + l_{5} + l_{6} + q_{4})cq_{2} \\ l_{1} + l_{2} + l_{3} + q_{3} \end{bmatrix} + \begin{bmatrix} (cq_{2}cq_{6} - sq_{2}sq_{5}sq_{6})l_{7} \\ (sq_{2}cq_{6} + cq_{2}sq_{5}sq_{6})l_{7} \\ -cq_{5}sq_{6}l_{7} \end{bmatrix} = \\ = \begin{bmatrix} (l_{4} + l_{5} + l_{6} + q_{4})cq_{2} + (cq_{2}cq_{6} - sq_{2}sq_{5}sq_{6})l_{7} \\ l_{0} + q_{1} + (l_{4} + l_{5} + l_{6} + q_{4})sq_{2} + (sq_{2}cq_{6} + cq_{2}sq_{5}sq_{6})l_{7} \\ l_{0} + q_{1} + (l_{4} + l_{5} + l_{6} + q_{4})sq_{2} + (sq_{2}cq_{6} + cq_{2}sq_{5}sq_{6})l_{7} \\ l_{1} + l_{2} + l_{2} + q_{2} - cq_{3}sq_{4}l_{7} \end{bmatrix}. \end{split}$$

 $\begin{bmatrix} l_1 + l_2 + l_3 + q_3 - cq_5 sq_6 l_7 \end{bmatrix}$ The column vector of the operational coordinates defining the robot gripper position with respect to the fixed frame, by the coordinates p_{x_7} , p_{y_7} , p_{z_7} of a point of it and its orientation by the elements α_z , β_x , γ_y of the matrix *R*, are expressed as a matrix, by the relation:

$$\left[\overline{X}\right]^{0} = \begin{bmatrix} p_{x_{7}} \\ p_{y_{7}} \\ p_{z_{7}} \\ \alpha_{z} \\ \beta_{x} \\ \gamma_{y} \end{bmatrix} = \begin{bmatrix} (l_{4} + l_{5} + l_{6} + q_{4})cq_{2} + (cq_{2}cq_{6} - sq_{2}sq_{5}sq_{6})l_{7} \\ l_{0} + q_{1} + (l_{4} + l_{5} + l_{6} + q_{4})sq_{2} + (sq_{2}cq_{6} + cq_{2}sq_{5}sq_{6})l_{7} \\ l_{1} + l_{2} + l_{3} + q_{3} - cq_{5}sq_{6}l_{7} \\ q_{2} \\ q_{5} \\ q_{6} \end{bmatrix}.$$
(34)

3. CONCLUSIONS

The robot constructive parameters and the generalized coordinates are necessary to determine the geometric model of a robot.

Having the orientation matrices expressing the relative orientation of each frame with respect to the previous frame as well as the relative position vectors, after taking some steps, the position and orientation of the robot gripper with respect to the fixed frame can be determined.

The DGM defines the position and orientation of the gripper, expressed by the coordinates of the characteristic points and by the orientation of the gripper about this point. The results obtained in this paper, according to [2], [4], [5], [7], are further used in the kinematic and dynamic modeling of the TRTTRR1 modular serial robot. They can also be used in the determination of the gripper characteristic point trajectory.

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Modelul geometric al robotului serial modular TRTTRR1

Rezumat: Lucrarea este structurată pe trei părți: considerații teoretice, modelarea geometrică a robotului serial modular TRTTRR1 și concluzii, succedate de o listă bibliografică. După o prezentare generală a modelării geometrice a roboților se fac referiri la metoda matricelor de rotație. Se efectuează apoi o descriere a structurii mecanice a robotului serial modular TRTTRR1, după care este prezentat în detaliu algoritmul modelului geometric direct pentru robotul menționat. Se determină astfel vectorul coloană al coordonatelor operaționale care definesc poziția dispozitivului de prehensiune al robotului în raport cu sistemul de referință fix și orientarea acestuia.

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