TECHNICAL UNIVERSITY OF CLUJ-NAPOCA

ACTA TECHNICA NAPOCENSIS

Series: Applied Mathematics, Mechanics, and Engineering
Vol. 59, Issue II, June, 2016

DYNAMICS EQUATIONS FOR A CAR INSPECTION MOBILE STRUCTURE

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Abstract: *The paper is devoted to establishing the dynamics equations, by an analysis of kinematic and dynamic behavior, for a mobile robot, called RmITA. Based on geometric modeling of the structure, there will be determined the kinematic constraints that affect the structure. Also, the mathematical model used to determine the dynamics equations, will be based on new concepts in advanced mechanics, based on important scientific researches of the main author, concerning the acceleration energy. In keeping the fact that the mathematical models of the mobile platforms are different besides the other robots types, due to no holonomic constraints, the dynamic control functions, will be established according to restrictions for motion.*

Key words: *mobile robots, dynamics equations, control, acceleration energy, no holonomic constraints.*

1. INTRODUCTION

The development of robotic systems and their implementation in the various processes of the manufacturing or inspection, have undeniable advantages, highlighted by carrying goods, inspections of quality in less time, increasing labor productivity, accident prevention, all these issues having an important contribution to enhancing the quality of life and economic development of the users of these systems. In the paper, is considered a mobile structure, presented in the Fig. 1, able to help the human operator in cars inspection, by collecting data and send information to a computer after which, will be reviewed by an inspector, who will conclude about the state of the car.

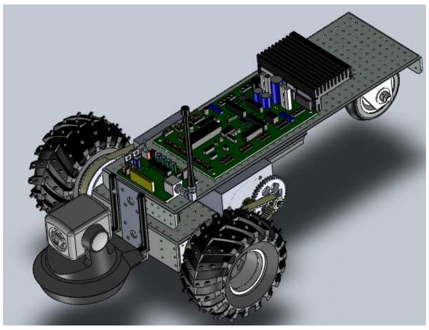


Fig 1 The RmITA Robot Structure

The robot is characterized by a differential shift commonly used in moving mobile robots. The structure is equipped with pan-and-tilt camera, characterized by two degrees of freedom, consisting in two rotations as can be seen from Fig.1.

The drive wheel is done in pairs, so that the two wheels on each side are driven by a motor. Thus, the proposed differential robotic system is characterized by four wheels, powered by two engines on each side of the structure. Based on these considerations, a straight line movement of the movable mechanical structure is obtained when the pair of wheels on one side rotates at the same speed and in the same direction, with the other pair of wheels on the opposite side (see Fig.2).

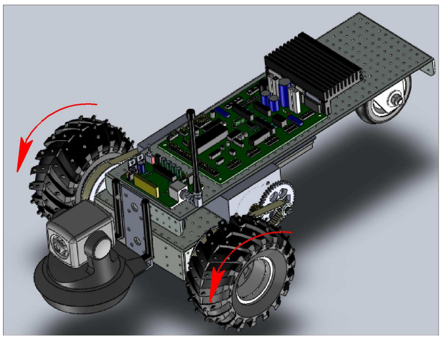
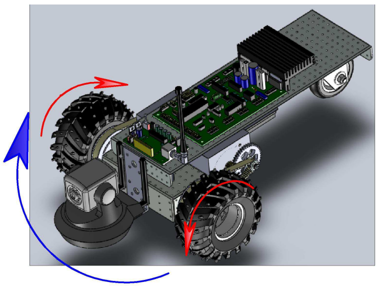
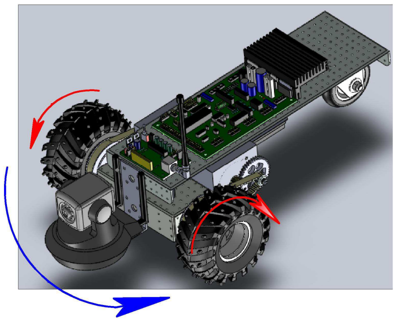


Fig.2 – Straight line motion of the inspection robot

The orientation of the robot is achieved by moving the robot wheels on one side in one direction, for example, the movement of the other two wheels in the opposite direction, as can be seen in Fig.3.



a. Turning right



b Turning left

Fig.3 – The orientation of the inspection robot

Thus the desired trajectories can be obtained by changing the angular speed and/or direction of the wheels on each side.

According to it's configuration the RmITA is considered a mobile robot that move over a plane, hence, it`s configuration space has two translational and one rotational degree of freedom; the rotation axis is perpendicular to the translations. The common characteristic of the robot is that cannot autonomously produce a velocity which is transversal to the axle of it's wheels, this constraint being a nonholonomic constraint. In other words, the vehicle cannot move transversally instantaneously, but it can reach any position and orientation by moving backward and forward while turning.

2. THE EXPRESSIONS FOR GEOMETRY AND KINEMATICS FOR RmITA ROBOT

Mobile robots are performing a plane-parallel motion, so that the law of motion, relative to the fixed reference system is:

$$\underset{(3\times1)}{{}^{0}\overline{X}} = \begin{bmatrix} \overline{r}_p \\ \theta \end{bmatrix} = \begin{bmatrix} x_p(t) = q_1(t) \\ y_p(t) = q_2(t) \\ \theta(t) = q_3(t) \end{bmatrix} \qquad (1)$$

The above relation contains three independent parameters, characterizing the position and orientation of the mobile robot, therefore the mobile robot in finite displacement has three degrees of freedom, as can be observed from Fig. 4.

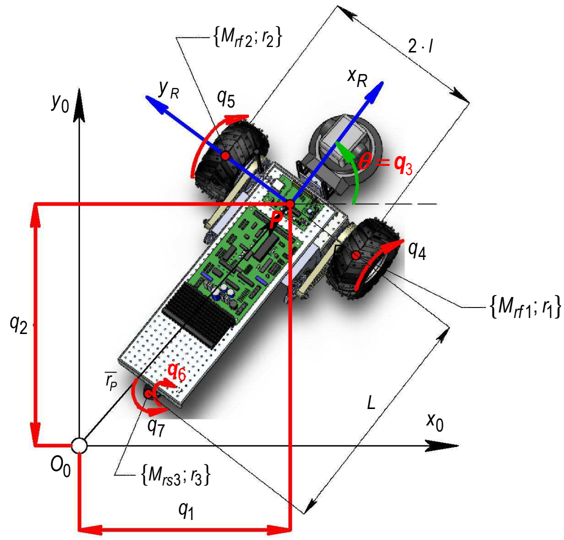


Fig.4 The independent parameters for RMITA robot

The inverse matrix representing the orienting of fixed reference frame with respect to mobile frame, is presented according to [1], [2] as:

$$R(\overline{z}, q_3) = \begin{bmatrix} cq_3 & -sq_3 & 0 \\ sq_3 & cq_3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad {}^{R}_{0}[R] = {}^{0}_{R}[R]^{-1} = \begin{bmatrix} cq_3 & sq_3 & 0 \\ -sq_3 & cq_3 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \qquad (2)$$

where $cq_i = \cos q_i$, and $sq_i = \sin q_i$.

According to Fig. 4 the independent parameters which characterizing the geometry of the robot in finite displacements are:

$$\overline{X}(t) = \left[q_i(t);\ i = 1 \to 5 \right]^T. \qquad (3)$$

The column vector of operational velocities, which expresses the absolute movement of the mobile robot is:

$${}^{0}\dot{\overline{X}} = \left[\dot{x}_p\ \dot{y}_p\ \dot{\theta} \right]^T = \left[\dot{q}_1\ \dot{q}_2\ \dot{q}_3 \right]^T; \qquad (4)$$

If the movement of mobile structure is achieved only after x_R axis, resulting that the sliding on the y_R axis is not possible in infinitesimal displacements, the velocity vector has the following form: ${}^{R}\dot{\overline{X}} = \left[{}^{R}\dot{x}_p\ 0\ {}^{R}\omega \right]^T$, and as a result, it appears the sliding constraint along y_R axis.

After a few transformations, there are resulting the velocities of characteristic point P projected onto the fixed reference system as:

$$\dot{X} = \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \omega = \dot{q}_3 \end{bmatrix} = \begin{bmatrix} cq_3 & 0 \\ sq_3 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \frac{r}{2} \cdot (\dot{q}_4 + \dot{q}_5) \\ \frac{r}{2 \cdot l} \cdot (\dot{q}_4 - \dot{q}_5) \end{bmatrix}. \quad (5)$$

In keeping with the constraints, according to Fig. 1 there can be written the kinematical constrains that are applied in the system, which can be expressed in the next differential relations:

Table 1

Nr.	***The kinematical restrictions for RmITA***
1	$-sq_3 \cdot dq_1 + cq_3 \cdot dq_2 + \sum_{i=3}^{7} 0 \cdot dq_i = 0$
2	$cq_3 \cdot dq_1 + sq_3 \cdot dq_2 +$ $+l \cdot dq_3 - r_{rf} \cdot dq_4 + \sum_{i=5}^{7} 0 \cdot dq_i = 0$
3	$cq_3 \cdot dq_1 + sq_3 \cdot dq_2 - l \cdot dq_3 +$ $+0 \cdot dq_4 - r_{rf} \cdot dq_5 + \sum_{i=6}^{7} 0 \cdot dq_i = 0$
4	$-s(q_3 + q_7) \cdot dq_1 + c(q_3 + q_7) \cdot dq_2 -$ $-L \cdot cq_7 \cdot dq_3 + \sum_{i=4}^{7} 0 \cdot dq_i = 0$
5	$c(q_3 + q_7) \cdot dq_1 + s(q_3 + q_7) \cdot dq_2 -$ $-L \cdot sq_7 \cdot dq_3 + \sum_{i=4}^{5} 0 \cdot dq_i - r_{rs} \cdot dq_6 + 0 \cdot dq_7 = 0$

In the above expressions, the first equation is provided to prevent the translation along the axis y_R of the platform, the equations two and three require a pure rolling without slipping of the driving wheels, four and five restrict the sliding along the y_3, y_4, axes respectively of the front wheels, and the last two imposing a roll without slipping of the driven wheel.

It results from the previous table that there are five link relations between the seven elementary displacements, so there are two independent parameters in elementary displacements, hence it results that the robot is subjected to no holonomic constraints.

3. ACCELERATION ENERGY FOR RmITA MOBILE ROBOT

In order to establish the dynamics equations there is used as starting point the Lagrange-Euler equations for nonholonomic links [2], [3]:

$$\frac{\partial E_A^{tot}}{\partial \ddot{q}_j} + Q_f^j + Q_g^j = Q_m^j + \sum_{i=1}^{5} \lambda_i \cdot a_{ij}. \quad (6)$$

where E_A^{tot} represents the total acceleration energy of the mobile robot, while Q_f^j, Q_g^i and Q_m^i are the generalized friction, gravitational and driving forces. In the same equations λ_i represent undetermined Lagrange parameters, while a_{ij} are considered the coefficients of the elementary displacements dq_j.

According to [2]-[4], the acceleration energy from (6), is expressed as:

$$\begin{aligned} E_A^i(q_k;\dot{q}_k;\ddot{q}_k) &= \frac{1}{2} \cdot M \cdot {}^i\dot{\bar{v}}_{C_i}^T \cdot {}^i\dot{\bar{v}}_{C_i} + \\ &+ \frac{1}{2} \cdot \left\{ {}^i\dot{\bar{\omega}}_i^T \cdot {}^iI_i^* \cdot {}^i\dot{\bar{\omega}}_i + \left[{}^i\bar{\omega}_i \times {}^iI_i^* \cdot {}^i\bar{\omega}_i \right] \right\} + \\ &+ \frac{1}{2} \cdot {}^i\dot{\bar{\omega}}_i^T \cdot \left[{}^i\bar{\omega}_i \times {}^iI_i^* \cdot {}^i\bar{\omega}_i \right] + \\ &+ \frac{1}{2} \cdot \bar{\omega}_i^T \cdot \left[{}^i\bar{\omega}_i^T \cdot Tr\left({}^iI_{pi}\right) \cdot {}^i\bar{\omega}_i - {}^i\bar{\omega}_i^T \cdot {}^iI_{pi} \cdot {}^i\bar{\omega}_i \right] \cdot {}^i\bar{\omega}_i \end{aligned} \quad (7)$$

In the previous expression, M is the mass corresponding to mechanical system, ${}^iI_i^*$ which is the axial centrifugal inertia tensor and ${}^iI_{pi}$ the inertia tensor planar centrifugal that characterizes the entire kinetic assembly (i), relative to the frame $\{i\}$, applied in the mass center of each link C_i. In the same expression, ${}^i\bar{v}_{C_i}$ and ${}^i\dot{\bar{v}}_{C_i}$ are the velocity and the acceleration of mass center, ${}^i\bar{\omega}_i$ and ${}^i\dot{\bar{\omega}}_i$ are the angular velocity and acceleration of the kinetic link (i) relative to the moving frame $\{i\}$.

For the considered mobile robot, the acceleration energy is rewritten as:

$$\begin{aligned} &E_A^{tot}\left(q_i;\dot{q}_i;\ddot{q}_i \;\; i=1\to 7\right) = E_A^{pl}\left(q_i;\dot{q}_i;\ddot{q}_i \;\; i=1\to 3\right) + \\ &+ \sum_{j=1}^{3} E_A^i\left(q_i;\dot{q}_i;\ddot{q}_i \;\; i=1\to 7\right). \end{aligned} \quad (8)$$

where E_A^{pl} represents the acceleration energy of the robot without wheels, and E_A^i is the acceleration energy of the robot]s wheels. In keeping with (7) and(8), the acceleration energy for RmITA, is:

$$
\begin{aligned}
E_A^{tot} = {} & \frac{M_{rf}}{2}\cdot\begin{bmatrix}\left(\ddot{q}_1+l\cdot\ddot{q}_3\cdot cq_3-l\cdot\dot{q}_3^2\cdot sq_3\right)^2+\\ +\left(\ddot{q}_2+l\cdot\ddot{q}_3\cdot sq_3+l\cdot\dot{q}_3^2\cdot cq_3\right)^2+\\ +\left(\ddot{q}_1-l\cdot\ddot{q}_3\cdot cq_3+l\cdot\dot{q}_3^2\cdot sq_3\right)^2+\\ +\left(\ddot{q}_2-l\cdot\ddot{q}_3\cdot sq_3-l\cdot\dot{q}_3^2\cdot cq_3\right)^2\end{bmatrix}+\\
& +\frac{I_{\Delta rf}}{2}\cdot\begin{pmatrix}\ddot{q}_4^2+\ddot{q}_3^2+\dot{q}_4^4+\dot{q}_3^4+\frac{3}{2}\cdot\dot{q}_3^2\cdot\dot{q}_4^2+\\ +\ddot{q}_5^2+\dot{q}_5^4+\frac{3}{2}\cdot\dot{q}_3^2\cdot\dot{q}_5{}^2\end{pmatrix}+\\
& +\frac{M_{rs}}{2}\cdot\left\{\left[\ddot{q}_1+L\cdot\left(\ddot{q}_3\cdot sq_3+\dot{q}_3^2\cdot cq_3\right)\right]^2+\right.\\
& \left.+\left[\ddot{q}_2-L\cdot\left(\ddot{q}_3\cdot cq_3-\dot{q}_3^2\cdot sq_3\right)\right]^2\right\}+ \qquad (9)\\
& +\frac{I_{\Delta rs}}{2}\cdot\left(\ddot{q}_6^2+\frac{1}{2}\cdot\ddot{q}_7^2+\dot{q}_6^4+\frac{1}{2}\cdot\dot{q}_7^4+\frac{3}{2}\cdot\dot{q}_6^2\cdot\dot{q}_7^2\right)+\\
& +\frac{1}{2}\cdot M_{pl}\cdot\left(\ddot{q}_1^2+\ddot{q}_2^2\right)-M_{pl}\cdot{}^{R}x_C\cdot\ddot{q}_3\cdot\left(\ddot{q}_1\cdot sq_3-\ddot{q}_2\cdot cq_3\right)-\\
& -M_{pl}\cdot{}^{R}x_C\cdot\dot{q}_3^2\cdot\left(\ddot{q}_1\cdot cq_3+\ddot{q}_2\cdot sq_3\right)+\frac{1}{2}\cdot I_{\Delta pl}\cdot\left(\ddot{q}_3^2+\dot{q}_3^4\right)
\end{aligned}
$$

where, M_{rf} and M_{rs} are the mass of the back respectively of the front wheels, M_{pl} the mass of the mobile platform, $I_{\Delta pl}$ is the inertia moment of the robot, $I_{\Delta rs}$ and $I_{\Delta rf}$ the inertia moment of the back and front wheels with respect to O_z axis.

4. THE STUDY OF GENERALIZED FORCES ACTING ON RmITA ROBOT

To determine generalized frictional forces, RmITA mobile system is considered as one body, as presented in Fig.5.

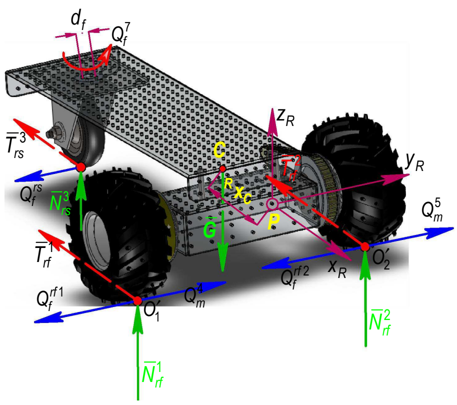


Fig.5 – The generalized forces

So, based on fundamental theorems of Newtonian dynamics (or the principle D`Alembert), according to the theorem of solidification, is written the fictional dynamic equilibrium equations in relation to the reference system located at the point P, expressed as follows:

$$
\begin{aligned}
Q_f^{rf} &= \mu\cdot r_{rf}\cdot\frac{M\cdot g\cdot\left({}^{R}x_C-\mu\cdot r_{rs}\right)+\Delta_T\cdot\left(Q_m^4+\Delta_0\cdot Q_m^5\right)}{2\cdot\left[\mu\cdot\left(r_{rf}-r_{rs}\right)+L\right]}\\
Q_f^{rs} &= \mu\cdot r_{rs}\cdot\frac{M\cdot g\cdot\left(\mu\cdot r_{rf}+L-{}^{R}x_C\right)+\Delta_T\cdot\left(Q_m^4+\Delta_0\cdot Q_m^5\right)}{\left[\mu\cdot\left(r_{rf}-r_{rs}\right)+L\right]} \qquad (10)\\
Q_f^{7} &= \frac{d_f}{2}\cdot\mu_7\cdot\mu\cdot\frac{M\cdot g\cdot\left({}^{R}x_C-\mu\cdot r_{rf}\right)+\Delta_T\cdot\left(Q_m^4+\Delta_0\cdot Q_m^5\right)}{\left[\mu\cdot\left(r_{rs}-r_{rf}\right)+L\right]}
\end{aligned}
$$

In (10), according to Fig.5, $Q_m^{4,5}$ are the driving moments of the; N_{rf} normal reaction of the front wheels; N_{rs} normal reaction on the rear wheels; $Q_f^{rf}=\mu\cdot r_{rf}\cdot N_{rf}$ moments of friction on the front wheels; $Q_f^{rs}=\mu\cdot r_{rs}\cdot N_{rs}$ moments of friction on the rear wheels; $Q_f^7=\left(d_f/2\right)\cdot\mu_7\cdot N_7$ moment of friction of the driven wheel, and d_f is the diameter of the shaft where is fixed the driven wheel, and:

$\Delta_T=\left\{\left(+1, Translation^{+}\right);\ \left(-1, Translation^{-}\right);\ \left(+1, Orientation\right)\right\};$

$\Delta_0=\left\{\left(+1, Translation\right);\left(-1, Orientation\right)\right\}$

According to [4], the gravitational driving forces are determined with:

$$
Q_g^j=\sum_{i=1}^{7} M_j\cdot{}^{0}\bar{g}^{T}\cdot\frac{\partial\bar{r}_j}{\partial q_i}=0 \qquad (11)
$$

The system of differential equations of motion characterizing the movement of RmITA is presented in tabular form, according to Table 2.

4. DYNAMICS EXPRESSIONS FOR THE MOBILE ROBOT

Based on the expressions of Table 2, representing the dynamic equations of the robot RMIT is noted that in the direct dynamic analysis, the unknowns are the generalized coordinates $\left\{q_j; j=1\rightarrow 7\right\}$ and Lagrange multipliers $\lambda_i; i=1\rightarrow 5$, and for the and inverse dynamic analysis (dynamic control), the unknowns are the generalized driving moments $\left\{Q_m^j; j=4\rightarrow 5\right\}$.

As follows from the above considerations, the dynamic model of a mechanical structure is represented analytically by a system of differential equations that define the linkages between the generalized coordinates $\{q_j; j=1\to 7\}$, or their derivatives and generalized forces acting on each element of the mechanical structure.

Table 2

No	The differential motion equations for *RmITA*
1	$\ddot{q}_1\cdot(M_{pl}+2\cdot M_{rf}+M_{rs})+(L\cdot M_{rs}-M_{pl}\cdot {}^{R}x_C)\cdot(\ddot{q}_3\cdot sq_3+\dot{q}_3^2\cdot cq_3)=$ $=-\lambda_1\cdot sq_3+cq_3\cdot(\lambda_2+\lambda_3)-\lambda_4\cdot s(q_3+q_7)+\lambda_5\cdot c(q_3+q_7)$
2	$\ddot{q}_2\cdot(M_{pl}+2\cdot M_{rf}+M_{rs})+(L\cdot M_{rs}-M_{pl}\cdot {}^{R}x_C)\cdot(\dot{q}_3^2\cdot sq_3-\ddot{q}_3\cdot cq_3)=$ $=\lambda_1\cdot cq_3+sq_3\cdot(\lambda_2+\lambda_3)+\lambda_4\cdot c(q_3+q_7)+\lambda_5\cdot s(q_3+q_7)$
3	$\ddot{q}_3\cdot(I_{\Delta f}+I_{\Delta p}+L^2\cdot M_{rs}+2\cdot M_{rf}\cdot l^2)+(L\cdot M_{rs}-M_{pl}\cdot {}^{R}x_C)\cdot(\ddot{q}_1\cdot sq_3-\ddot{q}_2\cdot cq_3)=$ $=l\cdot(\lambda_2-\lambda_3)-\lambda_4\cdot L\cdot cq_7-\lambda_5\cdot L\cdot sq_7$
4	$I_{\Delta f}\cdot\ddot{q}_4+Q_f^{rf}\cdot\mathrm{sgn}(\dot{q}_4)=Q_m^4-\lambda_2\cdot r_{rf}$
5	$I_{\Delta f}\cdot\ddot{q}_5+Q_f^{rf}\cdot\mathrm{sgn}(\dot{q}_5)=Q_m^5-\lambda_3\cdot r_{rf}$
6	$I_{\Delta s}\cdot\ddot{q}_6+Q_f^{rs}\cdot\mathrm{sgn}(\dot{q}_6)=-\lambda_5\cdot r_{rs}$
7	$\frac{I_{\Delta s}}{2}\cdot\ddot{q}_7+Q_f^{7}\cdot\mathrm{sgn}(\dot{q}_7)=0$

In the study of dynamic control functions of a structure, based on structure movement on a determined trajectory, to achieve some points, velocities and accelerations imposed by working task, the driving system must overcome the generalized external technological and gravitational forces, which are usually specific to mechanical transmissions or structure of the mechanical system. The dynamics equations of the robot RmITA presented in Table 2, are highlighting the complexity of dynamic control problem. The motion of the mobile structure has to be studied considering the fact that the structure, to achieve the target point, cannot simultaneously realize the positioning (translation) and orientation, so the two movements will be analyzed independently.

To perform a rectilinear translational motion, in accordance with its constructive structure, an essential condition derives from the fact that moments motors driving the wheels must be equal, $\{Q_m^4=Q_m^5\}$ deducted from the following equivalences:

$$\{(\ddot{q}_4=\ddot{q}_5),\ q_3=cst.,(\dot{q}_3,\ddot{q}_3)=0,\ q_7=0\} \quad (12)$$

Hence, taking into account the relationships $\{(4)\ and\ (5)\}$, it is noted that $(\lambda_2=\lambda_3)$. Multiplying the first expression in Table 2 with (cq_3), and the second with (sq_3) by summation, according to the conditions contained in the expression (12), follows:

$$(M_{pl}+2\cdot M_{rf}+M_{rs})\cdot(\ddot{q}_1\cdot cq_3+\ddot{q}_2\cdot sq_3)=2\cdot\lambda_2+\lambda_5 \quad (13)$$

There is introduced the notation:

$$\varepsilon_{rf}=\ddot{q}_{4,5}=\frac{\ddot{q}_1\cdot cq_3+\ddot{q}_2\cdot sq_3}{r_{rf}};\qquad \varepsilon_{rs}=\ddot{q}_6=\frac{\ddot{q}_1\cdot cq_3+\ddot{q}_2\cdot sq_3}{r_{rs}} \quad (14)$$

with the observation that $\{\varepsilon\}$ takes the first expression in the case of translation along q_1 or the value of the second term of (14) when the RmITA robot moves along q_2. Substituting $\{(4),(5)\}$, the driving moment's expressions for translation are:

$$Q_m^4=Q_m^5=\frac{r_{rf}}{2}\cdot\begin{bmatrix}(M_{pl}+2\cdot M_{rf}+M_{rs})\cdot(\ddot{q}_1\cdot cq_3+\ddot{q}_2\cdot sq_3)+\\ +\frac{1}{r_{rs}}(I_{\Delta s}\cdot\ddot{q}_6+Q_f^{rs}\cdot\mathrm{sgn}(\dot{q}_6))\end{bmatrix}+ \quad (15)$$
$$+I_{\Delta f}\cdot\ddot{q}_4+Q_f^{rf}\cdot\mathrm{sgn}(\dot{q}_4)$$

In order to realize a translation, it must $\{(q_1,q_2)=cst.\}$, resulting that:

$$\{(\dot{q}_1,\dot{q}_2)=0,\ (\ddot{q}_1,\ddot{q}_2)=0\} \quad (16)$$

Due to mechanical constitution as illustrated in Fig. 4, another condition for achieving orientation structure is $\{Q_m^4=-Q_m^5\}$ that in keeping with Table 2, leads to the following:

$$\{(\ddot{q}_5=-\ddot{q}_4)\} \quad (17)$$

This, involving the following equalities:

$$\{(-\lambda_2=\lambda_3)\} \quad (18)$$

According to restriction $\{(4)\}$ from Table 1, and taking account of the conditions(16), it can be deduced:

$$\{L\cdot\cos q_7=0\} \quad (19)$$

resulting that:

$$q_7=\pm\frac{\pi}{2} \quad (20)$$

Substituting the expressions (16)-(20) in Table 1, after dividing with (cq_7) results:

$$\ddot{q}_3 \cdot \left(I_{\Delta f} + I_{\Delta p} + L^2 \cdot M_{rs} + 2 \cdot M_{rf} \cdot l^2\right) = 2 \cdot l \cdot \lambda_2 - \lambda_5 \cdot L \quad (21)$$

From $\{(5)\}$ belonging to Table1 and taking into account (16)results:

$$\ddot{q}_4 = -\ddot{q}_5 = \frac{l}{r_{rf}} \cdot \ddot{q}_3 ; \qquad \ddot{q}_6 = \frac{L}{r_{rs}} \cdot \ddot{q}_3 \quad (22)$$

The expressions for the driving moments in the case of orientation for the RmITA mobile robot are:

$$Q_m^4 = -Q_m^5 = \frac{r_{rf}}{2 \cdot l} \begin{bmatrix} \ddot{q}_3 \cdot \left(I_{\Delta f} + I_{\Delta p} + L^2 \cdot M_{rs} + 2 \cdot M_{rf} \cdot l^2\right) + \\ + \frac{L}{r_{rs}} \left(I_{\Delta s} \cdot \ddot{q}_6 + Q_f^{rs} \cdot \mathrm{sgn}(\dot{q}_6)\right) \end{bmatrix} + \quad (23)$$
$$+ I_{\Delta f} \cdot \frac{l}{r_{rf}} \cdot \ddot{q}_3 + Q_f^{rf} \cdot \mathrm{sgn}(\dot{q}_4)$$

Also, in the two aforementioned expressions, it is observed that total driving moment, has a static component Q_{ms}^i, which is due to weight of the mechanical system, the resistance forces and a dynamic component Q_{md}^i. Thus, the total driving moment is expressed as:

$$\left\{ Q_m^i = Q_{ms}^i + Q_{md}^i \,,\ (i = 4{,}5) \right\} \quad (24)$$

The generalized variables found in the moving equations that express motion of the mobile system, will be replaced by polynomial function of time according to the working process, where is implemented RmITA structure.

6. CONCLUSIONS

In the paper there were determined the dynamic control functions for a mobile platform RmITA. For this, in achieving the differential equations of motion that characterizes the mobile structure, first there were used the geometry equations, leading to direct kinematic model. Based on this, by calling the specific dynamic equations of mechanical systems with holonomic links, are resulting the expressions which are governing the differential motion in configuration space for RmITA mobile platform.

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Ecuatiile dinamicii pentru un robot mobil utilizat in inspectii auto

Lucrarea este dedicată stabilirii ecuatiilor dinamicii, printr-o analiză a comportamentului cinematic si dinamic, pentru un robot mobil, numit RmITA. Pe baza modelării geometrice a structurii, exista constrângeri cinematice care afectează structura, care o integrează în clasa sistemelor mecanice neolonome. De asemenea, modelul matematic utilizat pentru a determina ecuatiile dinamicii, se bazează pe noi concepte in mecanica avansat, pe baza unor cercetări stiintifice importante ale autorului principal, în ceea ce priveste energia acceleratiilor. In conformitate cu faptul că modelele matematice ale platformelor mobile sunt diferite fata de celelalte tipuri de roboti, datorită constrângerilor cinematice, funcţiile de control dinamic, vor fi calculate în funcţie de restricţiile de mişcare.

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