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STUDY OF HUMAN BODY MOBILITY THROUGH MECHANICAL MODELING

Simona Gabriela JURCO, Eugen JURCO, Gheorghe TOMOAIA, Mariana ARGHIR

Abstract: *The work presents a model with five degrees of freedom of the human body subjected to external vibration. It analyzes the mobility of material system assimilated to the human body. It shows the corresponding simulation of a model note 5-SPP, which is composed of two feet, which is on a vibrating platform, which transmit the movement to the pelvis. In this paper is analyzing the stability of mechanical system human body equated for each segment separately. Each foot has two segments, and the fifth is the pelvis. From the study, shows that the movement of each segment is stable, so it does not cause malfunctions of the body over which it has conducted this study.*
Key words: *human body mobility, external mechanical vibrations, stability diagrams*

1. INTRODUCTION

Vibration exposure causes a general complex distribution action of forces and oscillatory movements in the human body.

The location and nature of the sensations can widely vary depending on the vibration frequency, the vibration direction and other factors. Whole-body vibration results in the workplace or experiments using a vibrating platform.

This can causes unpleasant sensations giving rise to discomfort, capacity reduction (eg.: decreased visual acuity) or is even a health risk (eg.: tissue destruction or physiological changes) [2].

2. MECHANICAL MODEL WITH FIVE DEGREES OF FREEDOM

Mechanical body model represents a human body in standing position, sitting on a rigid support from which the oscillation takes movement. The body is divided in five masses, respecting the anatomical position of the elements. It takes into account only the translational motion along the axis Oz; the rotational motion does not occur. So the model has five degrees of mobility. To simplify the model we have renamed 5-EPP (bring five equations corresponding to the two legs with to segments each of them and pelvis).

Table 1.
Model characteristics coefficients [1]

Characteristics		Measurement Unit	Value
Name	Symbol		
elasticity coefficient	$k_1=k_4$	N/m	25500
elasticity coefficient	$k_2=k_5$	N/m	53640
elasticity coefficient	$k_3=k_6$	N/m	8941
damping coefficient	$c_1=c_4$	Ns/m	378
damping coefficient	$c_2=c_5$	Ns/m	3651
damping coefficient	$c_3=c_6$	Ns/m	298
tibia mass	$m_1=m_4$	Kg	3,57
femur mass	$m_2=m_5$	Kg	4,17
pelvis mass	m_3	Kg	16,17

Input signal is based on two harmonics and has the form for one leg:

$$u(t)=c_1\dot{u}+k_1u \qquad (1)$$

This is the same and for the second leg. In this situation:

k_1u and k_4u - spring forces transmitted from the tibia due to the excitation foot;

$c_1\dot{u}=c_4\dot{u}$ – forces of damping for the excitation foot.

$$k_1u=k_4u=k_1u_0\sin\omega t=31000\cdot 6\cdot 10^{-5}\sin\omega t=1.86\sin\omega t \qquad (2)$$

$$c_1\dot{u}=c_4\dot{u}=c_1u_0\omega\cos\omega t = \\ =3970\cdot 6\cdot 10^{-5}\cdot 147.18\cos\omega t=35\cos\omega t \qquad (3)$$

where:

$\omega=2\pi f=147.18$ rad/s;

f = 23.437 Hz - frequency vibration;
$u_0 = 6x10^{-5}$ m - excitation amplitude.

2.1 The system of differential equations

Mass balance equations system (4) for each mechanical mass components (m_1, m_2, m_3, m_4, m_5) are write to obtain the mathematical model for the mechanical model from figure 1. The body is considered in standing position on a vibrating platform.

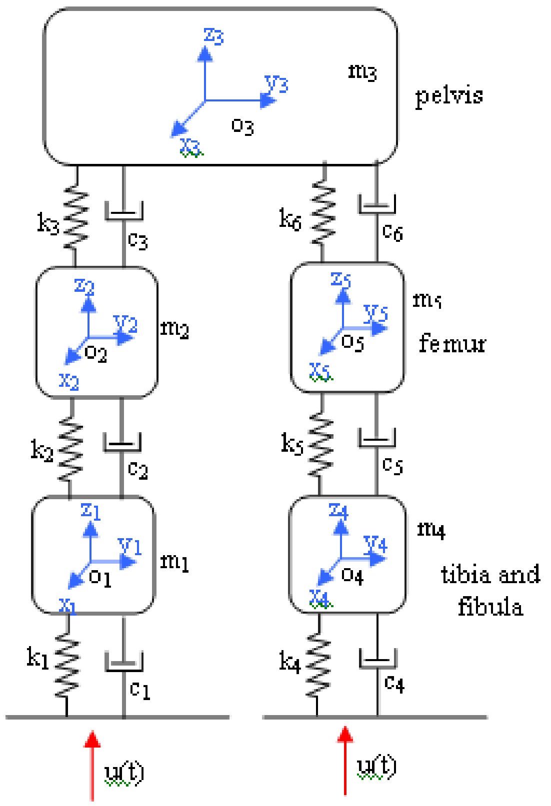

Fig. 1 Simplified mechanical models of human pelvis and legs with five degrees of mobility

The equations system is:

$$\begin{cases} m_1\ddot{z}_1 = -c_1(\dot{z}_1-\dot{u})-k_1(z_1-u)+c_2(\dot{z}_2-\dot{z}_1)+ \\ k_2(z_2-z_1) \\ m_2\ddot{z}_2 = -c_2(\dot{z}_2-\dot{z}_1)-k_2(z_2-z_1)+c_3(\dot{z}_3-\dot{z}_2)+ \\ +k_3(z_3-z_2) \\ m_4\ddot{z}_4 = -c_4(\dot{z}_4-u)-k_4(z_4-u)+c_5(\dot{z}_5-\dot{z}_4)+ \\ +k_5(z_5-z_4) \\ m_5\ddot{z}_5 = -c_5(\dot{z}_5-\dot{z}_4)-k_5(z_5-z_4)+c_6(\dot{z}_3-\dot{z}_5)+ \\ +k_6(z_3-z_5) \\ m_3\ddot{z}_3 = -c_3(\dot{z}_3-\dot{z}_2)-k_3(z_3-z_2)+c_6(\dot{z}_3-\dot{z}_5)- \\ -k_6(z_3-z_5) \end{cases} \quad (4)$$

- pass all member two unknown derivatives in the left side:

$$\begin{cases} m_1\ddot{z}_1 + c_1(\dot{z}_1-\dot{u}) + k_1(z_1-u) - c_2(\dot{z}_2-\dot{z}_1) - \\ -k_2(z_2-z_1) = 0 \\ m_2\ddot{z}_2 + c_2(\dot{z}_2-\dot{z}_1) + k_2(z_2-z_1) - c_3(\dot{z}_3-\dot{z}_2) - \\ -k_3(z_3-z_2) = 0 \\ m_4\ddot{z}_4 + c_4(\dot{z}_4-\dot{u}) + k_4(z_4-u) - c_5(\dot{z}_5-\dot{z}_4) - \\ -k_5(z_5-z_4) = 0 \\ m_5\ddot{z}_5 + c_5(\dot{z}_5-\dot{z}_4) + k_5(z_5-z_4) - c_6(\dot{z}_3-\dot{z}_5) - \\ -k_6(z_3-z_5) = 0 \\ m_3\ddot{z}_3 + c_3(\dot{z}_3-\dot{z}_2) + k_3(z_3-z_2) - c_6(\dot{z}_3-\dot{z}_5) + \\ +k_6(z_3-z_5) = 0 \end{cases} \quad (5)$$

- unknown system by adding the following differential equation becomes:

$$\begin{cases} m_1\ddot{z}_1 + \dot{z}_1(c_1+c_2) + z_1(k_1+k_2) - c_2\dot{z}_2 - k_2z_2 = \\ = c_1\dot{u} + k_1u \\ m_2\ddot{z}_2 + \dot{z}_2(c_2+c_3) - c_2\dot{z}_1 + z_2(k_2+k_3) - k_2z_1 - \\ -c_3\dot{z}_3 - k_3z_3 = 0 \\ m_4\ddot{z}_4 + \dot{z}_4(c_4+c_5) + z_4(k_4+k_5) - c_5\dot{z}_5 - \\ -k_5z_5 = c_4\dot{u} + k_4u \\ m_5\ddot{z}_5 + \dot{z}_5(c_5+c_6) - c_5z_4 + z_5(k_5+k_6) - \\ -k_5z_4 - c_6\dot{z}_3 - k_6z_3 = 0 \\ m_3\ddot{z}_3 + \dot{z}_3(c_3+c_6) - c_3\dot{z}_2 + z_3(k_3+k_6) - \\ -k_3z_2 - c_6\dot{z}_5 - k_6z_5 = 0 \end{cases} \quad (6)$$

The (4) system can be written in matrix form.

$$[M]\cdot\{\ddot{z}\}+[C]\cdot\{\dot{z}\}+[K]\cdot\{z\}=\{u\} \quad (7)$$

where:
[M] – matrix of inertia coefficients;
[C] – damping coefficients matrix;
[K] – stiffness matrix coefficients;
$\{z\}$ – displacements vector;
$\{\dot{z}\}$ – velocities vector;
$\{\ddot{z}\}$ – accelerations vector;
$\{u\}$ – forced excitation vector (external)
Those expressions are:

$$[M] = \begin{bmatrix} m_1 & 0 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 & 0 \\ 0 & 0 & m_4 & 0 & 0 \\ 0 & 0 & 0 & m_5 & 0 \\ 0 & 0 & 0 & 0 & m_3 \end{bmatrix} \quad (8)$$

$$\{\ddot{z}\}=\begin{Bmatrix}\ddot{z}_1\\ \ddot{z}_2\\ \ddot{z}_4\\ \ddot{z}_5\\ \ddot{z}_3\end{Bmatrix} \tag{9}$$

$$[C]=\begin{bmatrix} c_1+c_2 & -c_2 & 0 & 0 & 0\\ -c_2 & c_2+c_3 & -c_3 & 0 & 0\\ 0 & 0 & 0 & c_4+c_5 & -c_5\\ 0 & 0 & -c_6 & -c_5 & c_5+c_6\\ 0 & -c_3 & c_3+c_6 & 0 & -c_6 \end{bmatrix} \tag{10}$$

$$\{\dot{z}\}=\begin{Bmatrix}\dot{z}_1\\ \dot{z}_2\\ \dot{z}_4\\ \dot{z}_5\\ \dot{z}_3\end{Bmatrix} \tag{11}$$

$$[K]=\begin{bmatrix} k_1+k_2 & -k_2 & 0 & 0 & 0\\ -k_2 & k_2+k_3 & -k_3 & 0 & 0\\ 0 & 0 & 0 & k_4+k_5 & -k_5\\ 0 & 0 & -k_6 & -k_5 & k_5+k_6\\ 0 & -k_3 & k_3+k_6 & 0 & -k_6 \end{bmatrix} \tag{12}$$

$$\{z\}=\begin{Bmatrix}z_1\\ z_2\\ z_4\\ z_5\\ z_3\end{Bmatrix} \tag{13}$$

$$\{u\}=\begin{Bmatrix}u_s(t)\\ 0\\ u_d(t)\\ 0\\ 0\end{Bmatrix} \tag{14}$$

but

$$\begin{aligned} u_s(t)&=c_1\dot{u}+k_1u\\ u_s(t)&=c_4\dot{u}+k_4u\\ u_d(t)&=c_1\dot{u}+k_1u\\ u_d(t)&=c_4\dot{u}+k_4u \end{aligned} \tag{15}$$

• is given as (isolated), higher order derivatives with respect to each unknown quantity, and results the system of scalar differential equations;

$$\begin{aligned}
\ddot{z}_1&=\frac{1}{m_1}\begin{bmatrix}-c_1\dot{z}_1+c_1\dot{u}-k_1z_1+k_1u+c_2\dot{z}_2-c_2\dot{z}\\ +k_2z_2-k_2z_1\end{bmatrix}\\
\ddot{z}_2&=\frac{1}{m_2}\begin{bmatrix}-c_2\dot{z}_2+c_2\dot{z}_1-k_2z_2+k_2z_1+c_3\dot{z}_3-\\ -c_3\dot{z}_2+k_3z_3-k_3z_2\end{bmatrix}\\
\ddot{z}_4&=\frac{1}{m_4}\begin{bmatrix}-c_4\dot{z}_4+c_4\dot{u}-k_4z_4+k_4u+c_5\dot{z}_5-\\ -c_5\dot{z}_4+k_5z_5-k_5z_4\end{bmatrix}\\
\ddot{z}_5&=\frac{1}{m_5}\begin{bmatrix}-c_5\dot{z}_5+c_5z_4-k_5z_5+k_5z_4+c_6\dot{z}_3-\\ -c_6\dot{z}_5+k_6z_3-k_6z_5\end{bmatrix}\\
\ddot{z}_3&=\frac{1}{m_3}\begin{bmatrix}-c_3\dot{z}_3+c_3\dot{z}_2-k_3z_3+k_3z_2+c_6\dot{z}_3+\\ +c_6\dot{z}_5-k_6z_3+k_6z_5\end{bmatrix}
\end{aligned} \tag{16}$$

• state two are grouped as unknown second derivatives to achieve the same connections

$$\begin{aligned}
\ddot{z}_1&=\frac{1}{m_1}\begin{bmatrix}-\dot{z}_1(c_1+c_2)+c_1\dot{u}-z_1(k_1+k_2)+k_1u+\\ +c_2\dot{z}_2+k_2z_2\end{bmatrix}\\
\ddot{z}_2&=\frac{1}{m_2}\begin{bmatrix}-\dot{z}_2(c_2+c_3)+c_2\dot{z}_1-z_2(k_2+k_3)+k_2z_1+\\ +c_3\dot{z}_3+k_3z_3\end{bmatrix}\\
\ddot{z}_4&=\frac{1}{m_4}\begin{bmatrix}-\dot{z}_4(c_4+c_5)+c_4\dot{u}-z_4(k_4+k_5)+k_4u+\\ +c_5\dot{z}_5+k_5z_5\end{bmatrix}\\
\ddot{z}_5&=\frac{1}{m_5}\begin{bmatrix}-\dot{z}_5(c_5+c_6)+c_5z_4-z_5(k_5+k_6)+k_5z_4+\\ +c_6\dot{z}_3+k_6z_3\end{bmatrix}\\
\ddot{z}_3&=\frac{1}{m_3}\begin{bmatrix}-\dot{z}_3(c_3+c_6)+c_3\dot{z}_2-z_3(k_3+k_6)+k_3z_2+\\ +c_6\dot{z}_5+k_6z_5\end{bmatrix}
\end{aligned} \tag{17}$$

• the introduction of excitation (requesting) system of differential equations:

$$\begin{aligned}
\ddot{z}_1&=\frac{1}{m_1}\begin{bmatrix}-\dot{z}_1(c_1+c_2)+c_1u_0\omega\cos\omega t-\\ z_1(k_1+k_2)++k_1u_0\sin\omega t+c_2\dot{z}_2+k_2z_2\end{bmatrix}\\
\ddot{z}_2&=\frac{1}{m_2}\begin{bmatrix}-\dot{z}_2(c_2+c_3)+c_2\dot{z}_1-z_2(k_2+k_3)+\\ k_2z_1++c_3\dot{z}_3+k_3z_3\end{bmatrix}\\
\ddot{z}_4&=\frac{1}{m_4}\begin{bmatrix}-\dot{z}_4(c_4+c_5)+c_4u_0\omega\cos\omega t-z_4\\ (k_4+k_5)+k_4u_0\sin\omega t+c_5\dot{z}_5+k_5z_5\end{bmatrix}\\
\ddot{z}_5&=\frac{1}{m_5}\begin{bmatrix}-\dot{z}_5(c_5+c_6)+c_5z_4-z_5(k_5+k_6)+\\ k_5z_4++c_6\dot{z}_3+k_6z_3\end{bmatrix}\\
\ddot{z}_3&=\frac{1}{m_3}\begin{bmatrix}-\dot{z}_3(c_3+c_6)+c_3\dot{z}_2-z_3(k_3+k_6)+\\ k_3z_2++c_6\dot{z}_5+k_6z_5\end{bmatrix}
\end{aligned} \tag{18}$$

3. THE 5-EPP CORRESPONDING PROGRAMME MECHANICAL MODEL OF THE HUMAN OPERATOR

Program calculates velocity and movement system elements shown in the figures below. Integration time of 10 seconds was considered. Integration was performed with Runge-Kutta method of fourth order using Simulink software package. Each equation is modelled separately. Thus each subsystem corresponds to an entire model. For displacement equation z_1, was obtained from the modelling system diagram shown in Figure 2, and follows as in Figure 3 for z_2, in Figure 4 for z_3, in Figure 5 for z_4, and in Figure 6 for z_5.

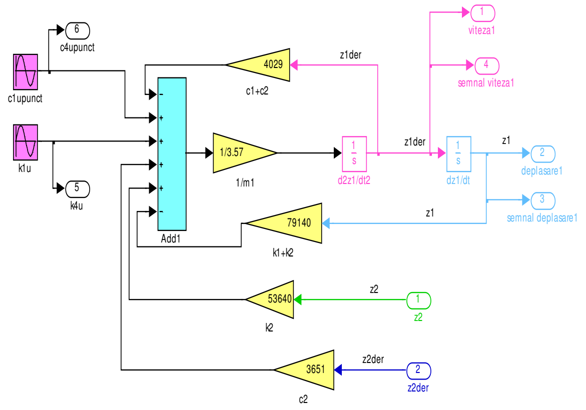

Fig. 2 Subsystem z_1 5-EPP program properly

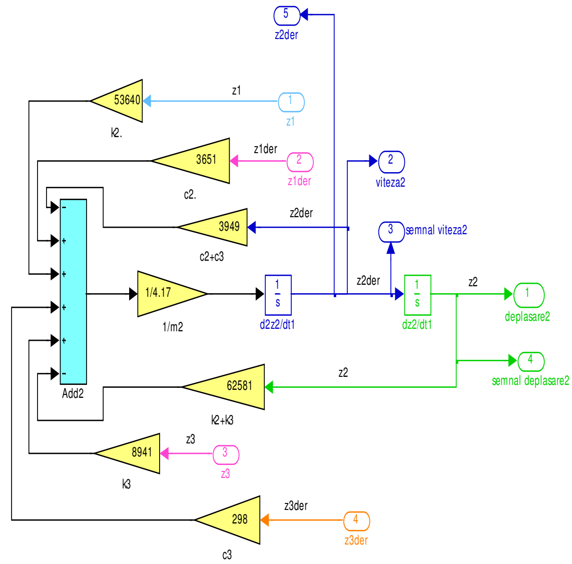

Fig. 3 Subsystem z_2 5-EPP program properly

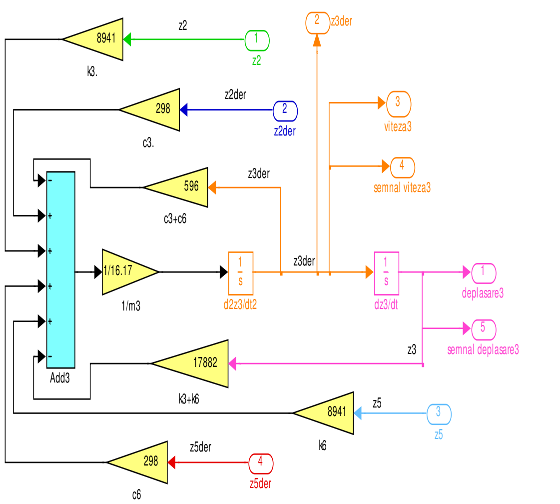

Fig 4. Subsystem z_3 5-EPP program properly

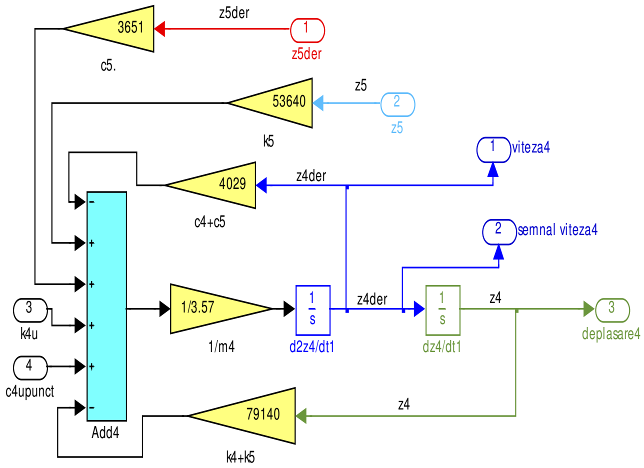

Fig 5 Subsystem z_4 5-EPP program properly

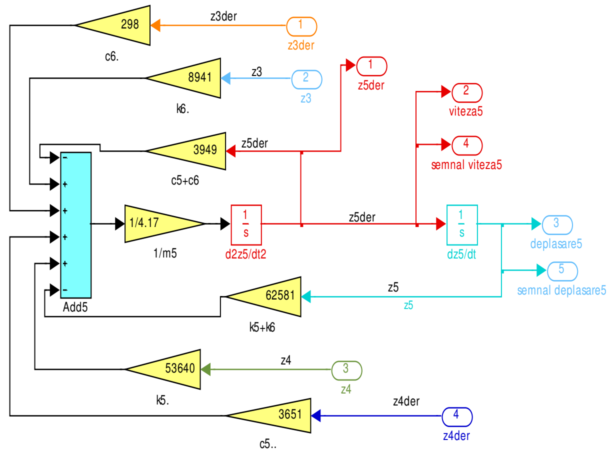

Fig. 6. Subsystem z_5 5-SPP program property

3.1 Graphical representations

Figure 7 and 8 corresponds to the variation of travel time for z_1 and z_4 generalized coordinate, Figure 9 and 10 correspond to the z_2 and z_5 generalized coordinate, and Figure 11 is for the z_3 generalized coordinate.

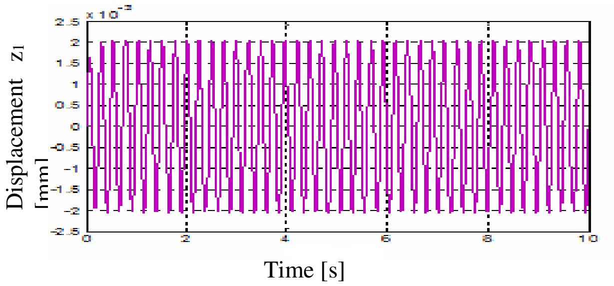

Fig.7 Changes in travel time for generalized coordinate z_1

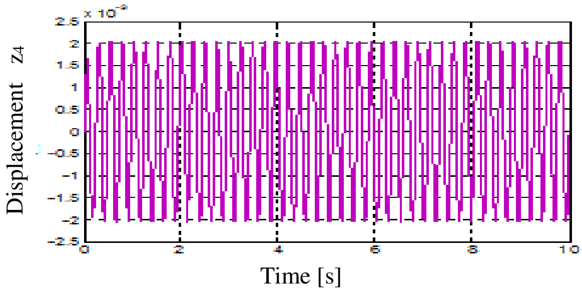

Fig.8 Changes in travel time for z_4 generalized coordinate

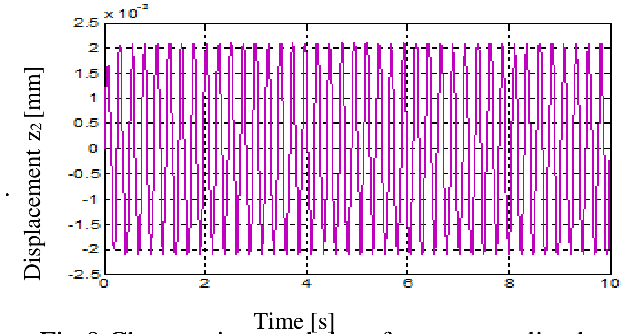

Fig.9 Changes in travel time for z_2 generalized coordinate

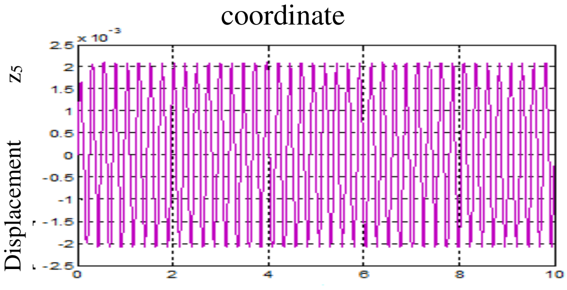

Fig.10 Changes in travel time for z_5 generalized coordinate

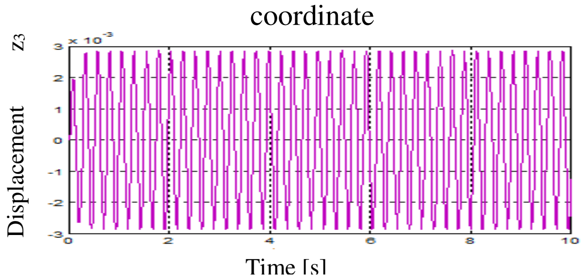

Fig.11 Changes in travel time for z_3 generalized coordinate

3.2. The stability of the mechanical system assimilated to the human body

Knowing the time variation law for each generalized coordinate and its velocity, as integration results of the differential equations system (18), the combination between them (coordinate and velocity) shows us the motion stability for the given part of the human body under the action of the external vibration.

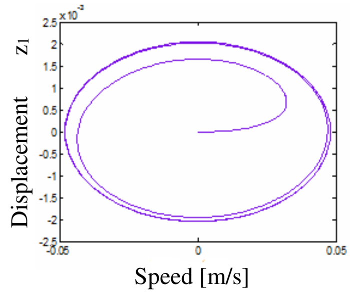

Fig.12 System stability for z_1 coordinate

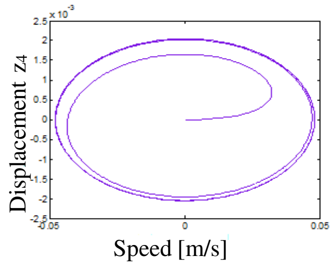

Fig.13 System stability for z_4 coordinate

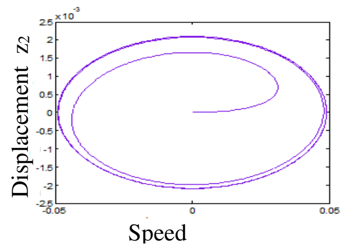

Fig.14 System stability for z_2 coordinate

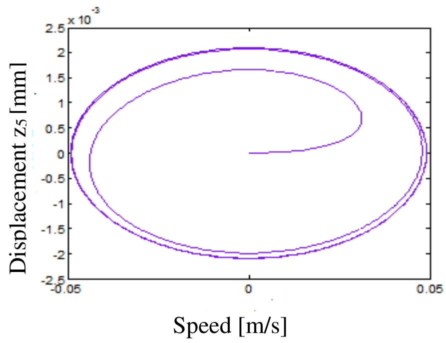

Fig.15 System stability for z_5 coordinate

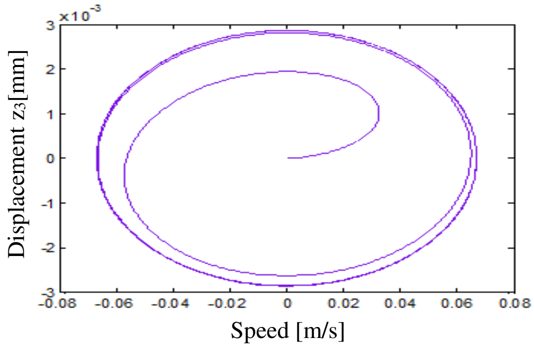

Fig.16 System stability for z_3 coordinate

4. CONCLUSION

1. The human body is a natural system, can be considered as a material system having three mechanical characteristics: mass, elasticity and dumping.
2. The human body can be approximate all or divided in different parts, function of the direction of study, or function of the necessary results.
3. In this paper the idea was to consider the human body formed with five different parts: two legs (with two segments each of them) and pelvis. The name was established as 5-SPP. This study was necessary for the comparison with the experiment that follows.

In terms of approximating the human system with five degrees of mobility, as expected is seen in table element corresponding m_3 pelvis area, the elements still receives signals from both legs of the human operator. The result obtained by integrating the system of differential equations is $3x10^{-3}$m if the pelvis (m_3), 1,2 $x10^{-4}$ (m_1) and $2x10^{-3}$m (m_4) for tibia, 2,1 $x10^{-3}$m (m_2) and $2,1x10^{-3}$m (m_5) to femur. Symmetrical system should equal out, which turned out.

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Studiul mobilităţii corpului uman prin modelare mecanică

Rezumat: *Lucrarea prezintă un model mecanic cu cinci grade de libertate al corpului uman supus la vibraţii exterioare. Se analizează mobilitatea sistemului material asimilat corpului uman. Se prezintă simularea corespunzătoare unui model notat 5-SPP, care este compus din cele doua picioare, care se spijină pe o platformă vibratoare, care transmit mişcarea la pelvis. In lucrare se analizează stabilitatea sistemului mecanic asimilat corpului uman, pentu fiecare segment în parte, fiecare picior are două segmente, iar cel de al cincilea segment este pelvisul. Din studiul efectuat, rezultă că mişcarea fiecărui segment este stabilă, deci nu produce disfuncţionalităţi corpului asupra căruia s-a efectuat acest studiu.*

Simona Gabriela JURCO, Dr. Eng., researcher, University of Agriculture Science and Veterinary Medicine Cluj-Napoca, E-mail: urs_simona@yahoo.com, Telephone (+) 40744966005;

Eugen JURCO, Dr. Eng., Assistant Professor, University of Agriculture Science and Veterinary Medicine Cluj-Napoca, E-mail: jurco_eugen@yahoo.com;

Gheorghe TOMOAIA, Dr. Prof. , University of Medicine and Pharmacy Cluj - Napoca, Romania, E-mail: tomoaia2000@yahoo.com;

Mariana ARGHIR, Prof. Dr. Ing., Technical University of Cluj-Napoca, B-dul Muncii, Nr. 105-103, Romania E-mail: marianaaghir@yahoo.com, Tel: 0729. 108.327.