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DYNAMICS OF HUMAN BODY SUBJECTED TO VIBRATIONS ON A VIBRATING PLATE

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Abstract: *The paper presents a study of dynamics of human body subjected to vibrations on a vibrating platform, which the human body is placed in the position of sitting, inclined towards the direction of drive train vibrations. Designed system is an original development of the earlier studies. Dynamics of human body system assimilated to the usual material system corresponds of the human body, located in the vibrating field. In the paper are established the scalar differential equations, that govern the human body dynamics.*
Key words*: dynamics of human body segments, general vibrations, system of differential equations.*

1. INTRODUCTION

It is considered a human subject placed on a vibrating platform, which produces vertical vibrations. The body is placed at 45 ° inclined towards horizontal plate. A suggestive image can be date as in figure 1 [3].

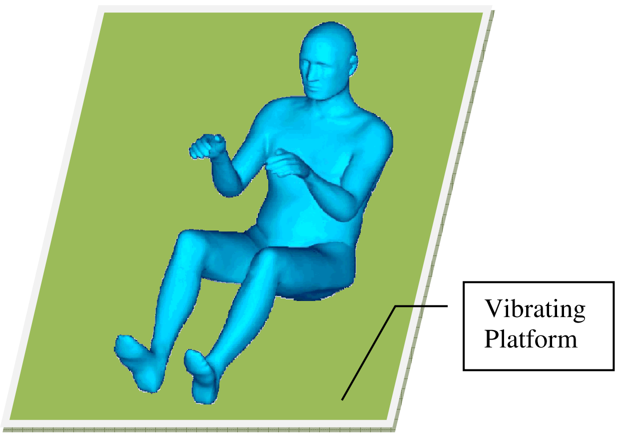

Fig. 1. Human body posture sitting on the vibrating platform [1]

It is considered that in this position, the area most exposed to vibration is the range of the legs up to his chest. Therefore, they are not taken into account the arms, neck and head. This part of the body is divided into six segments (Fig. 2) [4].

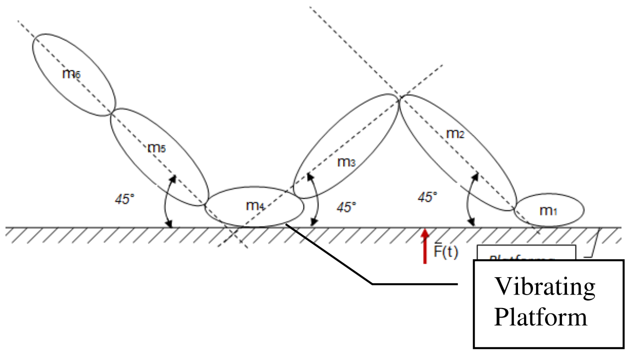

Fig. 2. Segmented biodynamic model of the human body [4] subjected of vibrations

The segments masses are: m_1 – mass of the dorsal region leg; m_2 – mass of the crurale region of the previous leg (the calf of the leg); m_3 – mass of the femoral region leg; m_4 – mass of the leg buttocks region; m_5 – mass of the abdomen; m_6 – mass of the thorax.

2. HUMAN BODY ON THE VIBRATING PLATFORM

This initial idea about the human body under the action of vibration on the vibrating platform, was modified, and the new situation are made for the action of the active force in the body base on the m_4 mass (Fig. 3).

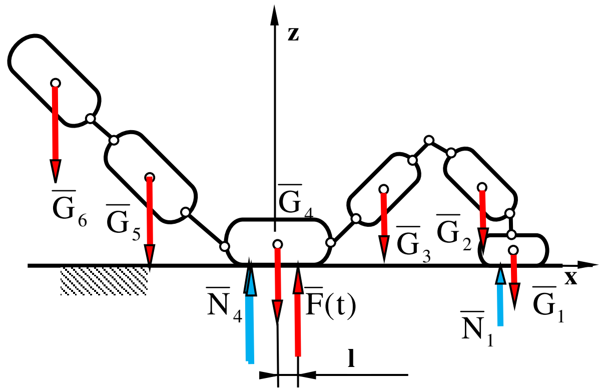

Fig. 3. Biomechanical scheme of the human body on the vibrating platform

In the Figure 3 are presented the gravity forces for the body segments (G_i , i=1, 2, ..., 6), the action force (F(t)), that acts over the body situated on the vibrating platform, and the linkage forces (N_1 and N_2) the normal reactions on the segments who are posted on the platform.

An important fact is that the action force is situated at the distance „l" about the application gravity force for the „4" segment, because here there is vertical direction of the mass centre position of the mechanical system. But the action force is divided into two different components, because they are direct applied on the subjected components of the human body.

3. MECHANICAL SCHEMES

The start figure of this part of the paper is the Figure 3, and the segment in each figure will be presents as the original position.

3.1. Mechanical Characteristics

The segment is considered formed with: mass, spring and, damper. The notations and the values are given in the Table no.1.

Table 1.

Mechanical Characteristic of the Human Segments

Crt. No.	Denomination	Symbol	U.M.
1.	Mass of the dorsal region leg	m_1	kg
2.	The active force over the dorsal region leg	$F_1(t)$	N
3.	Vertical elasticity constant of the dorsal region leg	k_{1V}	N/m
4.	Vertical damping constant of foot	c_{1V}	Ns/m
5.	Elasticity constant of the calf of the leg along a second axis of it	k_{2V}	N/m
6.	Damping constant of the calf of the leg along a second axis of it	c_{2V}	Ns/m
7.	Mass of the calf of the leg (shank)	m_2	kg
8.	Elasticity constant along a second axis of femur	k_{3V}	N/m
9.	Elasticity constant along the symmetry axis of femur	k_{3H}	N/m
10.	Damping constant along a second axis of femur	c_{3V}	Ns/m
11.	Damping constant along the symmetry axis of femur	c_{3H}	Ns/m
12.	Mass of the femur	m_3	kg
13.	Elasticity constant along a second axis of buttocks	k_{4V}	N/m
14.	Elasticity constant along the symmetry axis of the buttocks	k_{4H}	N/m
15.	Vertical damping constant along a second axis of the buttocks	c_{4V}	Ns/m
16.	Damping constant along the symmetry axis of the buttocks	c_{4H}	Ns/m
17.	Mass of the region of buttocks	m_4	kg
18.	Vertical elasticity constant between buttocks and platform	k_{5V}	N/m
19.	Vertical damping constant between buttocks and platform	c_{5V}	Ns/m
20.	The active force over the region of buttocks	$F_2(t)$	N
21.	Mass of the abdomen	m_5	kg
22.	Elasticity constant along a second axis of abdomen	k_{6V}	N/m
23.	Elasticity constant along the symmetry axis of the abdomen	k_{6H}	N/m
24.	Damping constant along a second axis of the abdomen	c_{6V}	Ns/m
25.	Damping constant along the symmetry axis of the abdomen	c_{6V}	Ns/m
26.	Mass of the thorax	m_6	kg
27.	Elasticity constant along a second axis of the thorax	k_{7V}	N/m
28.	Elasticity constant along the symmetry axis of the thorax	k_{7H}	N/m
29.	Damping constant along a second axis of the thorax	c_{7V}	Ns/m
30.	Damping constant along the symmetry axis of the thorax	c_{7V}	Ns/m

3.2. Mechanical Schemes of the Segmentation

The mechanical schemes [1], [2] of the segmentation are given in the following figures.

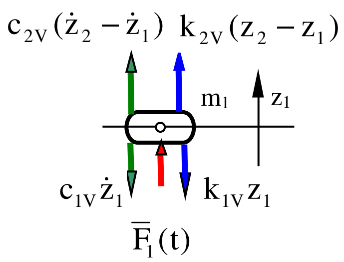

Fig. 4. Mechanical scheme of the foot

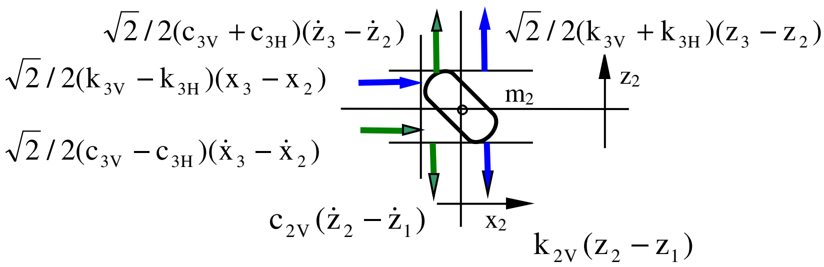

Fig. 5. Mechanical scheme of the shank

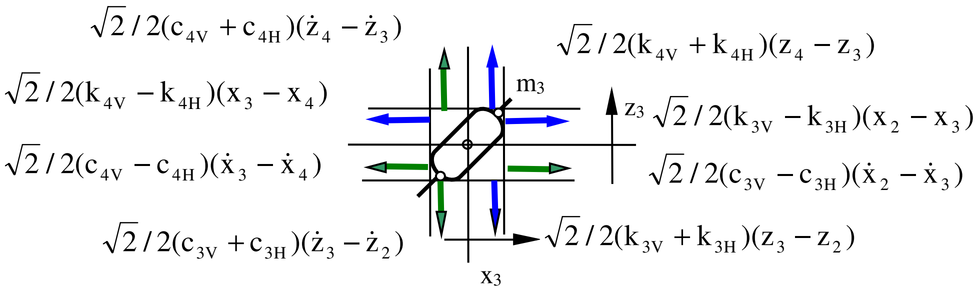

Fig. 6. Mechanical scheme of the femur

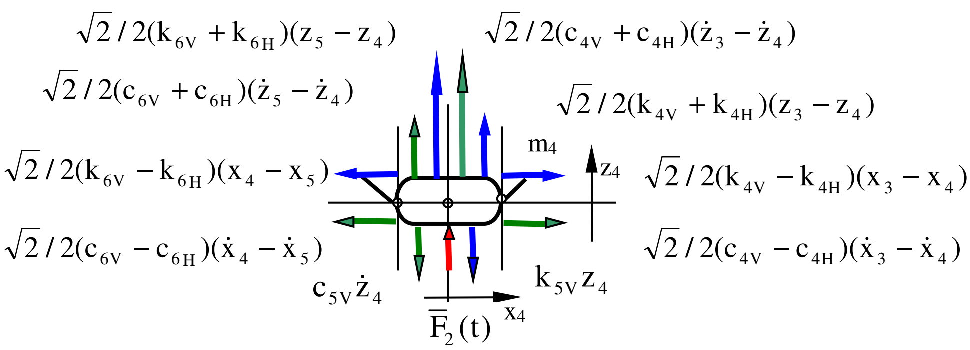

Fig. 7. Mechanical scheme of the buttocks

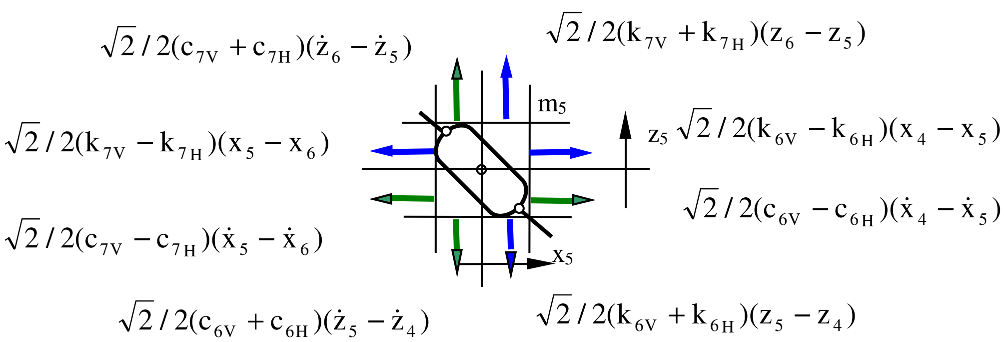

Fig. 8. Mechanical scheme of the abdomen

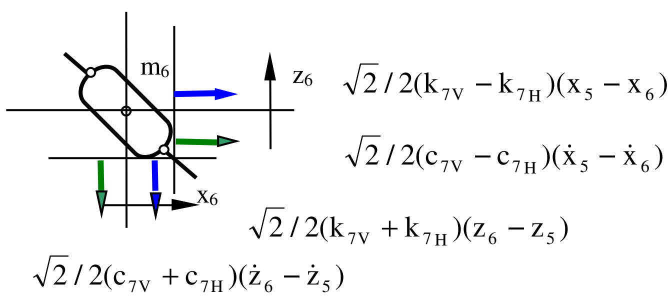

Fig. 9. Mechanical scheme of the thorax

4. DIFFERENTIAL EQUATIONS

Using the mechanical schemes made in the figures between 4 and 9, they can be write the differential governing equations, for the system assimilated human body located on a vibrating platform.

4.1. Dorsal Region Leg

In the Figure 4 is the mechanical scheme of the dorsal region leg (foot), and the dynamics equilibrium is given by the central motion theorem.

$$m_1\ddot{z}_1 = -c_{1V}\dot{z}_1 - k_{1V}z_1 + c_{2V}(\dot{z}_2 - \dot{z}_1) + \\ + k_{2V}(z_2 - z_1) + F_1(t) \tag{1}$$

4.2. Crurale Region of the Previous Leg

The crurale region of the previous leg, named and the calf of the leg, has its mechanical scheme in the Figure 5, and the dynamics equilibrium is given by the central motion theorem, into two differential dynamics equilibrium, because the shank has two perpendicular motions.

$$m_2\ddot{z}_2 = -c_{2V}(\dot{z}_2 - \dot{z}_1) - -k_{2V}(z_2 - z_1) + \\ + \frac{\sqrt{2}}{2}(c_{3V} + c_{3H})\cdot(\dot{z}_3 - \dot{z}_2) + \frac{\sqrt{2}}{2}(k_{3V} + k_{3H})\cdot(z_3 - z_2) \tag{2}$$

$$m_2\ddot{x}_2 = \frac{\sqrt{2}}{2}(c_{3V} - c_{3H})\cdot(\dot{x}_3 - \dot{x}_2) + \frac{\sqrt{2}}{2}(k_{3V} - k_{3H})\cdot(x_3 - x_2) \tag{3}$$

4.3. Femoral Region Leg

The femoral region leg (femur) is presented in the Figure 6, and its differential equations into two perpendicular directions are given in the following system.

$$m_3\ddot{z}_3 = -\frac{\sqrt{2}}{2}(c_{3V} + c_{3H})\cdot(\dot{z}_3 - \dot{z}_2) - \frac{\sqrt{2}}{2}(k_{3V} + k_{3H})\cdot(z_3 - z_2) + \\ + \frac{\sqrt{2}}{2}(c_{4V} + c_{4H})\cdot(\dot{z}_4 - \dot{z}_3) + \frac{\sqrt{2}}{2}(k_{4V} + k_{4H})\cdot(z_4 - z_3) \tag{4}$$

$$m_3\ddot{x}_3 = \frac{\sqrt{2}}{2}(c_{3V} - c_{3H})\cdot(\dot{x}_2 - \dot{x}_3) + \frac{\sqrt{2}}{2}(k_{3V} - k_{3H})\cdot(x_2 - x_3) - \\ - \frac{\sqrt{2}}{2}(c_{4V} - c_{4H})\cdot(\dot{x}_3 - \dot{x}_4) - \frac{\sqrt{2}}{2}(k_{4V} - k_{4H})\cdot(x_3 - x_4) \tag{5}$$

4.4. Leg Buttocks Region

In the Figure 7 is given the mechanical scheme of the buttocks, and it has the motions in vertical direction for the active force excitation, but, in same time it has the motion in horizontal direction due to the linkages with the segments of the body.

The differential equations of the buttocks motions are in the next system.

$$m_4\ddot{z}_4 = \frac{\sqrt{2}}{2}(c_{4V} + c_{4H})\cdot(\dot{z}_3 - \dot{z}_4) + \frac{\sqrt{2}}{2}(k_{4V} + k_{4H})\cdot(z_3 - z_4) + \\ + \frac{\sqrt{2}}{2}(c_{6V} + c_{6H})\cdot(\dot{z}_5 - \dot{z}_4) + \frac{\sqrt{2}}{2}(k_{6V} + k6_{4H})\cdot(z_5 - z_4) - \\ - c_{5V}\dot{z}_4 - k_{5V}z_4 + F_2(t) \tag{6}$$

$$m_4\ddot{x}_4 = \frac{\sqrt{2}}{2}(c_{4V} - c_{4H})\cdot(\dot{x}_3 - \dot{x}_4) + \frac{\sqrt{2}}{2}(k_{4V} - k_{4H})\cdot(x_3 - x_4) - \\ - \frac{\sqrt{2}}{2}(c_{6V} - c_{6H})\cdot(\dot{x}4 - \dot{x}_5) - \frac{\sqrt{2}}{2}(k_{6V} - k_{6H})\cdot(x_4 - x_5) \tag{7}$$

4.5. Abdomen

The abdomen is considered as a segment of the body situated on the vibrating platform, and it has the two different dynamics equilibrium equations.

$$m_5\ddot{z}_5 = -\frac{\sqrt{2}}{2}(c_{6V} + c_{6H})\cdot(\dot{z}_5 - \dot{z}_4) - \frac{\sqrt{2}}{2}(k_{6V} + k_{6H})\cdot(z_5 - z_4) + \\ + \frac{\sqrt{2}}{2}(c_{7V} + c_{7H})\cdot(\dot{z}_6 - \dot{z}_5) + \frac{\sqrt{2}}{2}(k_{7V} + k_{7H})\cdot(z_6 - z_5) \tag{8}$$

$$m_5\ddot{x}_5 = \frac{\sqrt{2}}{2}(c_{6V} - c_{6H})\cdot(\dot{x}_4 - \dot{x}_5) + \frac{\sqrt{2}}{2}(k_{6V} - k_{6H})\cdot(x_4 - x_5) - \\ - \frac{\sqrt{2}}{2}(c_{7V} - c_{7H})\cdot(\dot{x}_5 - \dot{x}_6) - \frac{\sqrt{2}}{2}(k_{7V} - k_{7H})\cdot(x_5 - x_6) \tag{9}$$

In the Figure

4.6. Thorax

The thorax is last segment in this study, and it has the dynamics equations, as:

$$m_6\ddot{z}_6 = -\frac{\sqrt{2}}{2}(c_{7V} + c_{7H})\cdot(\dot{z}_6 - \dot{z}_5) - \frac{\sqrt{2}}{2}(k_{7V} + k_{7H})\cdot(z_6 - z_5) \tag{10}$$

$$m_6\ddot{x}_6 = \frac{\sqrt{2}}{2}(c_{7V} - c_{7H})\cdot(\dot{x}_5 - \dot{x}_6) + \frac{\sqrt{2}}{2}(k_{7V} - k_{7H})\cdot(x_5 - x_6) \tag{11}$$

The system of differential equations has eleven unknown quantities, and can be write in an orderly form after the unknown.

4.7. Differential Equations System

The assembly of scalar differential equations is a second-order differential system, scratchy (not homogeneous), with constant coefficients which can be written in the form of orderly after the unknown, for integration in relation to the independent variable - the time.

$$
\begin{aligned}
& m_1\ddot{z}_1 + (c_{1V} + c_{2V})\dot{z}_1 - c_{2V}\dot{z}_2 + (k_{1V} + k_{2V})z_1 - k_{2V_2} = F_1(t) \\
& m_2\ddot{z}_2 - c_{2V}\dot{z}_1 - k_{2V}z_1 + \left[c_{2V} + \frac{\sqrt{2}}{2}(c_{3V} + c_{3H})\right]\dot{z}_2 + \left[k_{2V} + \frac{\sqrt{2}}{2}(k_{3V} + k_{3H})\right]z_2 - \\
& \qquad - \frac{\sqrt{2}}{2}(c_{3V} + c_{3H})\dot{z}_3 - \frac{\sqrt{2}}{2}(k_{3V} + k_{3H})z_3 = 0 \\
& m_2\ddot{x}_2 + \frac{\sqrt{2}}{2}(c_{3V} - c_{3H})\dot{x}_2 + \frac{\sqrt{2}}{2}(k_{3V} - k_{3H})x_2 - \frac{\sqrt{2}}{2}(c_{3V} - c_{3H})\dot{x}_3 - \frac{\sqrt{2}}{2}(k_{3V} - k_{3H})x_3 = 0 \\
& m_3\ddot{z}_3 - \frac{\sqrt{2}}{2}(c_{3V} + c_{3H})\dot{z}_2 - \frac{\sqrt{2}}{2}(k_{3V} + k_{3H})z_2 + \frac{\sqrt{2}}{2}(c_{3V} + c_{3H} + c_{4V} + c_{4H})\dot{z}_3 + \\
& \qquad + \frac{\sqrt{2}}{2}(k_{3V} + k_{3H} + k_{4V} + k_{4H})z_3 - \frac{\sqrt{2}}{2}(c_{4V} + c_{4H})\dot{z}_4 - \frac{\sqrt{2}}{2}(k_{4V} + k_{4H})z_4 = 0 \\
& m_3\ddot{x}_3 - \frac{\sqrt{2}}{2}(c_{3V} - c_{3H})\dot{x}_2 - \frac{\sqrt{2}}{2}(k_{3V} - k_{3H})x_2 + \frac{\sqrt{2}}{2}(c_{3V} - c_{3H} + c_{4V} - c_{4H})\dot{z}_3 + \\
& \qquad + \frac{\sqrt{2}}{2}(k_{3V} - k_{3H} + k_{4V} - k_{4H})z_3 - \frac{\sqrt{2}}{2}(c_{4V} - c_{4H})\dot{x}_4 - \frac{\sqrt{2}}{2}(k_{4V} - k_{4H})x_4 = 0 \\
& m_4\ddot{z}_4 - \frac{\sqrt{2}}{2}(c_{4V} + c_{4H})\dot{z}_3 - \frac{\sqrt{2}}{2}(k_{4V} + k_{4H})z_3 + \left[\frac{\sqrt{2}}{2}(c_{4V} + c_{4H} + c_{6V} + c_{6H}) + c_{5V}\right]\dot{z}_4 + \\
& \qquad + \left[\frac{\sqrt{2}}{2}(k_{4V} + k_{4H} + k_{6V} + k_{6H}) + k_{5V}\right]z_4 - \frac{\sqrt{2}}{2}(c_{6V} + c_{6H})\dot{z}_5 - \frac{\sqrt{2}}{2}(k_{6V} + k_{6H})z_5 = F_2(t) \\
& m_4\ddot{x}_4 - \frac{\sqrt{2}}{2}(c_{4V} - c_{4H})\dot{x}_3 - \frac{\sqrt{2}}{2}(k_{4V} - k_{4H})x_3 + \left[\frac{\sqrt{2}}{2}(c_{4V} - c_{4H} + c_{6V} - c_{6H}) - c_{5V}\right]\dot{x}_4 + \\
& \qquad + \frac{\sqrt{2}}{2}(k_{4V} - k_{4H} + k_{6V} - k_{6H})x_4 - \frac{\sqrt{2}}{2}(c_{6V} - c_{6H})\dot{x}_5 - \frac{\sqrt{2}}{2}(k_{6V} - k_{6H})x_5 = 0 \qquad (12) \\
& m_5\ddot{z}_5 - \frac{\sqrt{2}}{2}(c_{6V} + c_{6H})\dot{z}_4 - \frac{\sqrt{2}}{2}(k_{6V} + k_{6H})z_4 + \frac{\sqrt{2}}{2}(c_{6V} + c_{6H} + c_{7V} + c_{7H})\dot{z}_5 + \\
& \qquad + \frac{\sqrt{2}}{2}(k_{6V} + k_{6H} + k_{7V} + k_{7H})z_5 - \frac{\sqrt{2}}{2}(c_{7V} + c_{7H})\dot{z}_6 - \frac{\sqrt{2}}{2}(k_{7V} + k_{7H})z_6 = 0 \\
& m_5\ddot{x}_5 - \frac{\sqrt{2}}{2}(c_{6V} - c_{6H})\dot{x}_4 - \frac{\sqrt{2}}{2}(k_{6V} - k_{6H})x_4 + \frac{\sqrt{2}}{2}(c_{6V} - c_{6H} + c_{7V} - c_{7H})\dot{x}_5 + \\
& \qquad + \frac{\sqrt{2}}{2}(k_{6V} - k_{6H} + k_{7V} - k_{7H})x_5 - \frac{\sqrt{2}}{2}(c_{7V} - c_{7H})\dot{x}_6 - \frac{\sqrt{2}}{2}(k_{7V} - k_{7H})x_6 = 0 \\
& m_6\ddot{z}_6 - \frac{\sqrt{2}}{2}(c_{7V} + c_{7H})\dot{z}_5 - \frac{\sqrt{2}}{2}(k_{7V} + k_{7H})z_5 + \frac{\sqrt{2}}{2}(c_{7V} + c_{7H})\dot{z}_6 + \frac{\sqrt{2}}{2}(c_{7V} + c_{7H})z_6 = 0 \\
& m_6\ddot{x}_6 - \frac{\sqrt{2}}{2}(c_{7V} - c_{7H})\dot{x}_5 - \frac{\sqrt{2}}{2}(k_{7V} - k_{7H})x_5 + \frac{\sqrt{2}}{2}(c_{7V} - c_{7H})\dot{x}_6 + \frac{\sqrt{2}}{2}(c_{7V} - c_{7H})x_6 = 0
\end{aligned}
$$

5. CONCLUSIONS

In this work it is proposed a new biomechanical model of human body subjected to connections on a vibrating platform.

The human body is divided into six different segments of it, and it sat in an inclined position towards the platform with 45 degrees. Two segments are situated on the platform (foot and buttocks), and over them acts the active force having the distribution about them taking into account the masses of the segments.

The system of scalar differential equations, that governs the mechanical system motion has 11 equations with eleven unknown quantities, and it is a second-order differential system, scratchy (not homogeneous), with constant coefficients which can be written in the form of orderly after the unknown.

In this work the aim was to determine the system of differential equations that are of great complexity and required a different approach, compared to our previous studies and those that are still in the specialized literature in the field.

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Dinamica corpului uman supus la vibratii pe o placa vibratoare

Rezumat: *Lucrarea prezinta un studiu al dinamicii corpului uman, supus la vibratii pe o platforma vibratoare, fata de care corpul uman este asezat in pozitia sezand, inclinata fata de directia de actionare a vibratiilor. Sistemul astfel conceput este o dezvoltare originala a studiilor anterioare. Dinamica sistemului material asimilat corpului uman corespunde, astfel, la solicitarile uzuale ale corpului uman, situat in camp vibrational.In lucrare sunt stabilite ecuaţiile diferenţiale scalare, care guverenează dinamica corpului uman*

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