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THE WORKSPACE AND THE SINGULARITIES OF THE 3KTK SPATIAL PARALLEL MANIPULATOR

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Abstract*: In this article is studied the graphical representation using the meshing method (based on input-output equations and on a program designed in AutoLISP - AutoCAD) for the workspace and for the singularities of the 3KTK manipulator with 3 degrees of freedom in translation. Is calculated the areas of the different plane sections (Zp = constant), and the workspace volume. It shows the influence of the the constructive parameters on the workspace of the manipulator.*

Key words: *parallel manipulator, the meshing method, the Jacobian matrix, the input - output equations, the singularities, degrees of freedom.*

1. INTRODUCTION

The figure 1 shows the kinematic scheme of the 3KTK spatial parallel manipulator having three degrees of freedom in translation and three identical kinematic chains (KTK - subsequent joints type of kinematic chain from the base to the final element K - cardan, T - translation).

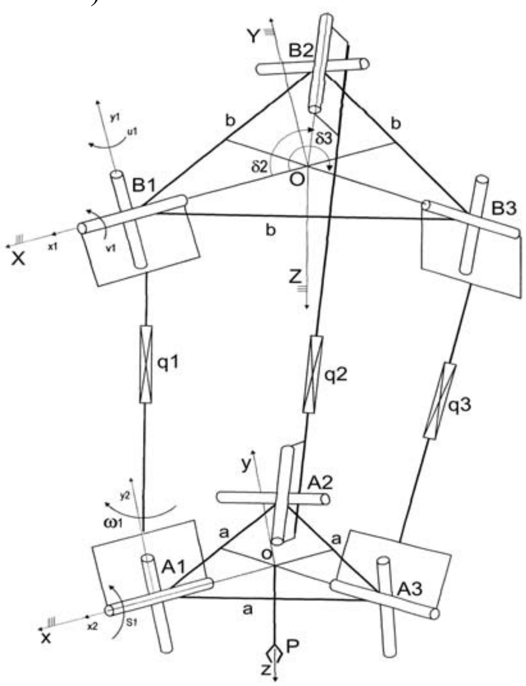

Fig. 1 The kinematic scheme of the manipulator

Only an arrangement of the kinematic chains in the three joints according to fig. 1 leads to a spatial parallel mechanism with three degrees of freedom in translation [1], [2].

According to some Korean researchers [3], the clearances of the cardan joints must be less than $0,05^0$.

2. THE GRAPHICAL REPRESENTATION FOR THE WORKSPACE OF THE 3KTK MANIPULATOR USING THE MESHING METHOD

For the 3KTK parallel manipulator the question arises it will determine the workspace in translation because the manipulator's platform executes only spatial translations. Obtaining workspace is achieved using the computer [4], [5], [6], [7].

It is starts from analytical solution of the inverse problem of the positions deduced in [1] and [2].

$$q_i = \pm\sqrt{\left[X_P + (r-R)\cos\delta_i\right]^2 + \left[Y_P + (r-R)\sin\delta_i\right]^2 + (Z_P - h)^2} \qquad (1)$$

By requiring Z_P coordinate of the characteristic point P, the parameters that specify the position of the manipulated object in the workspace are the coordinates X_P and Y_P. The position of point P in the plane Z_P = constant may be given in cartesian coordinates:

$$\begin{cases} X_P = X_P^{\min} + h_X(i-1) \\ Y_P = Y_P^{\min} + h_Y(i-1) \end{cases} \quad (2)$$

where i=1, 2,..., $h_{X(Y)}$, and $h_{X(Y)}$ is the maximum number of steps.

For the given values of parameters h_X, h_Y, using the inverse geometric model, shall be checked if the point P belongs to the working section, according to restrictions imposed on driving devices.

$$q_{i\,\min} \le q_i \le q_{i\,\max} \quad i = 1, 2, 3 \quad (3)$$

The variation range of the h_X and h_Y parameters ensure scanning ensure scanning of the plan Z_P = constant. The points that satisfy the conditions (3) determines a section of the workspace as discreet.

By changing the value of Z_P shall be determined a sequence of sections for which borders are the isohypses of the workspace, having a constant orientation.

To achieve the 3D workspace and its various sections, was developed a program written in AutoLISP under AutoCAD.

Can be accessed the following options:

- the 3D representation of the workspace with its rotation and the possibility of choosing a desired viewing angle.
- the graphical representation of the sectional planes: X = ct., Y = ct., Z = ct.
- the graphical representation of a plane containing the Z axis and which makes an angle α (between 0^0 şi 180^0) to the OX axis.

For the 3KTK manipulator having the following constructive data: b=400 mm, a=200 mm, h=50 mm $\delta_1 = \delta_1^{'} = 0°$, $\delta_2 = \delta_2^{'} = 120°$, $\delta_3 = \delta_3^{'} = 240°$, $q_{i,min}$ = 200mm, $q_{i,max}$ = 400mm and the input data: h_x = 2 mm; h_y = 2 mm; Z_P = 100 ÷ 430 mm, have been obtained in the figures 2, 3, 4, 5, the 3D workspace which is comprised of plane sections Zp = ct. (the red ones) between the quotas Z = 100 mm, Z = 430 mm viewed from points (X = 4, Y = -5, z = 0.75), (x = 2, y = 0.5, z = 0.5) respectively (X = 4, Y = -3, Z = 1), the difference consisting as shown in the figure 2; there are a vertical section (the black) that contains the Z axis and makes the angle α=0^0 with OX axis. In the figures 3 and 4 the same vertical section that contains the Z axis make an angle α =60^0 with OX axis. In the figure 5 is shown the projection of the image from figure 4, in the XOY plan. The four figures provide a complete description of the workspace. It is observed that for small values of the Z coordinate the sections are deformed rings that lower in size and are filled with increasing Z.

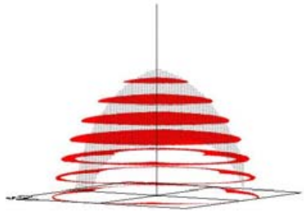

Fig. 2. 3D workspace (Z_p = ct, α = 0^0)

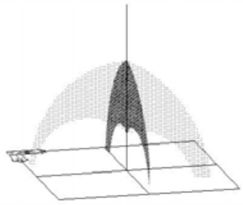

Fig. 3. 3D workspace (α = 60^0, α = 0^0)

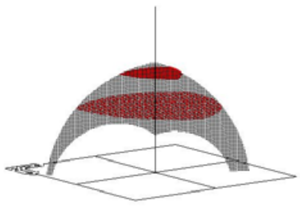

Fig. 4. 3D workspace (Z_p = ct, α = 60^0)

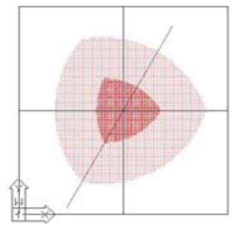

Fig. 5. 2D workspace (Z_p = ct, α = 60^0)

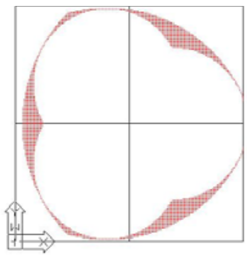

Fig. 6. 2D workspace (Zp = 100 mm)

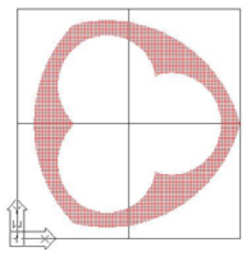

Fig. 7. 2D workspace (Zp = 200 mm)

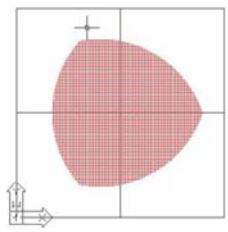

Fig. 8. 2D workspace (Z_p = 300 mm)

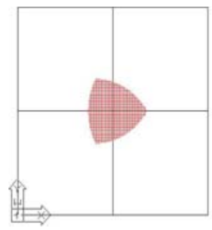

Fig. 9. 2D workspace (Z_p = 400 mm)

The table 1 shows the values of the different planar sections Zp = ct. (between Z = 100 mm and Z = 430 mm), and in figure 10 are plotted these areas.

Also was conceived a program that calculates the volume of the 3KTK manipulator's workspace. For the construction data shown above, the volume of the manipulator's workspace is: Vol. 3KTK = 13,106,750 mm^3

Table 1.

The values of the different planar sections

Zp	The surface area (Zp=ct) [mm^2]	Zp	The surface area Zp=ct [mm^2]
200	3400	320	89775
210	7000	330	87350
220	13625	340	83275
230	32325	350	77000
240	57600	360	68375
250	93025	370	56900
260	93025	380	45550
270	93025	390	34200
280	93025	400	23450
290	92825	410	13500
300	92350	420	5400

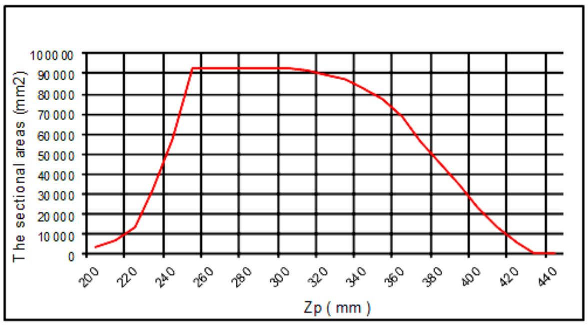

Fig. 10. The areas of the planar sections for different Z_p coordinates

In the same way, with slight modifications, can be determined the workspaces for the other constructive and for the input data.

3. THE INFLUENCE OF THE CONSTRUCTIVE PARAMETERS ON THE 3KTK MANIPULATOR'S WORKSPACE

For the 3KTK manipulator is important how the variation of constructive parameters "b" and "a" contributes to changing the volume of the manipulator's workspace.

Based on the method presented in the previous paragraph were determined the volumes of the 3KTK parallel manipulator's workspace, having the following constructive data: b = 400 mm, a = 100; 200; 300; 400 mm.

For each of the four possible combinations (b, a), while retaining the same range of variation of $q_i \in (200, 400)$ mm, i = 1, 2, 3, have been calculated the volumes of 4 manipulators and the values were listed in the Table 2.

It is found that the maximum volume of the workspace is achieved for a/b = 1, that is when the dimensions of the mobile platform are equal to the dimensions of the fixed base the same as for other different structures studied in the literature.

Table 2

The volumes of the 4 manipulators

a / b	The workspace volume *(mm^3)*
1 / 4	10150125
1 / 2	13106750
3 / 4	16131500
1	19304625

4. THE SINGULARITIES STUDY FOR THE 3KTK MANIPULATOR

It starts from the input – output equations of the manipulator set forth in [2], equations wherein the variables X_P, Y_P, Z_P şi q_i are functions of time.

$$\left(X_P + r\cos\delta_i' - R\cos\delta_i\right)^2 + \left(Y_P + r\sin\delta_i' - R\sin\delta_i\right)^2 + \left(Z_P - h\right)^2 = q_i^2 \qquad (4)$$

Differentiating these equations relative to time, we get by grouping of terms:

$$\left[X_P + (r-R)\cos\delta_i\right]\dot{X}_P + \left[Y_P + (r-R)\sin\delta_i\right]\dot{Y}_P + (Z_P - h)\dot{Z}_P = q_i\dot{q}_i \qquad (5)$$

The equations (5) can be put in matrix form:

$$[A]\cdot\dot{q} = [B]\cdot\dot{q}_P \qquad (6)$$

The matrix equation (6) can be detailed written in the form:

$$\begin{bmatrix} q_1 & 0 & 0 \\ 0 & q_2 & 0 \\ 0 & 0 & q_3 \end{bmatrix}\cdot\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} = \begin{bmatrix} X_P + (r-R)\cos\delta_1 & Y_P + (r-R)\sin\delta_1 & Z_P - h \\ X_P + (r-R)\cos\delta_2 & Y_P + (r-R)\sin\delta_1 & Z_P - h \\ X_P + (r-R)\cos\delta_3 & Y_P + (r-R)\sin\delta_1 & Z_P - h \end{bmatrix}\cdot\begin{bmatrix} \dot{X}_P \\ \dot{Y}_P \\ \dot{Z}_P \end{bmatrix} \qquad (7)$$

Based on equations (6) and (7) can be pointed three types of singularities. The first type of singularity occurs when the determinant of the matrix A = 0, the second type of singularity occurs when the determinant of the matrix B = 0. and the third type of singularity occurs when the determinant of the matrix A = 0 and the determinant of the matrix B = 0, simultaneously. The general characteristics of each type of singularities ware presented in [1].

In the case of 3KTK manipulator is interesting to study the singularity of type II.

The matrix [B] is a function of variables X_P, Y_P, Z_P. The values of variables X_P, Y_P, Z_P, in the workspace of the manipulator that make the determinant of the matrix [B] tend to 0 brings the manipulator in an undesirable situation.

It is through the entire workspace smallest step possible, observing that in each point of the workspace, the value of the determinant of the matrix [B] take the very high values. The lowest value of the determinant B is det B = 5196152, for the point defined by: X = -156 mm point, Y = - 135 mm, Z = 200 mm.

Therefore there is no question to cancel the determinant B in the manipulator workspace.

5. CONCLUSIONS

The conclusions stemming from this study on are:

• The horizontal sections form Zp = constant is sufficiently convenient from the point of view of size and shape (Fig. 6-9).

• The values of planar sectional areas are large (~ 90.00 mm^2) and can be kept relatively constant over a wide range of variation of the Z coordinate (Fig. 10).

• The 3D representation of the workspace of suggests a large volume of it (~ 13. Mln mm^3) and a convenient layout (Figure 2-4).

• The maximum volume of the workspace is achieved when the base and the mobile platform has the same shape and the same size (Table 2).

• The Jacobian matrices [A] and [B] are obtained relatively easy and have simple expressions.

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Analiza spaţiului de lucru şi a singularităţilor manipulatorului paralel spaţial 3KTK

Rezumat: *În acest articol se studiază reprezentarea grafică prin metoda discretizării (pe baza ecuaţiilor de intrare-ieşire si a unui program conceput în AutoLISP - AutoCAD) spaţiul de lucru şi singularităţile manipulatorului 3KTK care are 3 grade de libertate în translaţie. Se calculează ariile diferitelor secţiuni plane Zp = cst. şi volumul spaţiului de lucru. Se prezintă influenţa parametrilor constructivi asupra spaţiului de lucru a manipulatorului.* ***Key words:*** *manipulator paralel, metoda discretizării, matrice Jacobiană, singularităţi, grade de libertateive.*

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