TECHNICAL UNIVERSITY OF CLUJ-NAPOCA

ACTA TECHNICA NAPOCENSIS

Series: Applied Mathematics, Mechanics, and Engineering
Vol. 60, Issue III, September, 2017

GEOMETRICAL MODELLING USING MATRIX EXPONENTIAL FUNCTIONS FOR A SERIAL ROBOT STRUCTURE

Claudiu SCHONSTEIN, Iuliu NEGREAN, Nicolae PANC

Abstract: *In the paper, on the basis of formulations based on matrix exponential will be established the direct and inverse geometry equations. The matrix exponentials functions in robotics will be applied for a serial robot structure. Unlike classical algorithms, the application of the exponential matrix functions presents some advantages in determining the direct geometry equations. The results obtained in geometrical modeling, are representing input data for kinematical modeling, all those being important data in the study of dynamic behavior of any mechanical robot system.*
Key words: *Matrix exponentials functions, mathematical modeling, geometry, control.*

1. INTRODUCTION

Nowadays, significant resources are being spent in the development of mechanical robotic systems. A part of this development is based on mathematical studies, having the role of using new approaches for establishing and studying the parameters which are defining the dynamic behavior of any mechanical system.

Transfer matrix equations, specific to a serial kinematic chain, having rotation and translation kinematic joint, from the robot's mechanical structure, can be expressed by using Exponential Matrix Functions (ME), [1]-[4].

2. THE GEOMETRY EQUATIONS

In this section, will be presented the geometry equations for a serial structure, by using the matrix exponentials used for determine the locating matrices, according to [1]-[4].

2.1 Establishing of Locating Matrices by using of Matrix Exponentials

The matrix exponentials and their associated transformations are included in the algorithm of matrix exponentials *(MEG)* devoted to direct geometry equations, according to [2] and [4]. The main steps of the algorithm are presented in the following.

- The matrix of the nominal geometry $M_{vn}^{(0)}$, corresponding to configuration $\bar{\theta}^{(0)}$ is known:

$$M_{vn}^{(0)} = \underset{[(n+1)\times 6]}{Matrix}\left\{\left[\bar{p}_i^{(0)T} \quad k_i^{(0)T}\right] i = 1 \rightarrow n+1\right\}^T . \quad (1)$$

- On the basis of this matrix is represented the kinematical structure of the robot, taken in study.
- The matrix of the nominal geometry is completed with the screw parameters $\left\{\bar{k}_i^{(0)} ; \bar{v}_i^{(0)}\right\}$ also named the homogeneous coordinates. The new matrix is symbolized by $M_{vn}^{(0)**}$, according to:

$$M_{vn}^{(0)} = \underset{[(n+1)\times 6]}{Matrix}\left\{\left[\bar{p}_i^{(0)T} \quad k_i^{(0)T}\right] i = 1 \rightarrow n+1\right\}^T . \quad (2)$$

- In keeping with MEG Algorithm, an outward loop is opened $(i = 1 \rightarrow n)$. The matrix exponentials devoted to direct geometry equations *(DGM equations)* are bellow obtained.
- The differential matrix A_i has the same expression for the both configurations $\bar{\theta}^{(0)}$ and $\bar{\theta}$. Considering [2] and [4], this matrix shows as:

$$\left\{\begin{array}{c} A_i = \begin{bmatrix} \left\{\bar{k}_i^{(0)} \times\right\}\Delta_i & \bar{v}_i^{(0)} \\ 0 \;\; 0 \;\; 0 & 0 \end{bmatrix} = \\ \begin{bmatrix} \left\{\bar{k}_i^{(0)} \times\right\}\Delta_i & \left\{\bar{p}_i^{(0)} \times\right\}\bar{k}_i^{(0)} \cdot \Delta_i + (1-\Delta_i)\cdot \bar{k}_i^{(0)} \\ 0 \;\; 0 \;\; 0 & 0 \end{bmatrix} \end{array}\right\} \quad (3)$$

where $\{\bar{k}_i = \bar{z}_i ; \bar{v}_i = \{\bar{p}_i \times\} \bar{z}_i \cdot \Delta_i + (1-\Delta_i)\cdot \bar{z}_i\}$ are the screw parameters or homogeneous coordinates of the driving axis (i), according to [5] and [6], which by generalization becomes

$$\{\bar{k}_i = \{\bar{x}_i;\bar{y}_i;\bar{z}_i\};\ \bar{v}_i = \{\bar{p}_i \times\}\bar{k}_i \cdot \Delta_i + (1-\Delta_i)\cdot \bar{k}_i\} \qquad (4)$$

➢ The exponential of rotation matrix is:

$$\left\{\begin{array}{l} exp\left\{\left\{\bar{k}_i^{(0)}\times\right\}\cdot q_i\cdot \Delta_i\right\} \equiv R\left(\bar{k}_i; q_i\cdot \Delta_i\right)\equiv \\ \equiv \left\{I_3\cdot c(q_i\cdot \Delta_i)+\left\{\bar{k}_i^{(0)}\times\right\}s(q_i\cdot \Delta_i)+\right. \\ \left. +\ \bar{k}_i^{(0)}\cdot \bar{k}_i^{(0)T}\left[1-c(q_i\cdot \Delta_i)\right]\right\}\equiv {}^{i-1}_{\ i}[R] \end{array}\right\}. \quad (5)$$

➢ The inverse of the exponential of rotation matrix is also expressed, in keeping with the next:

$$\left\{\begin{array}{l} exp\left\{-\left\{\bar{k}_i^{(0)}\times\right\}q_i\cdot \Delta_i\right\} \equiv R^T\left(\bar{k}_i; q_i\cdot \Delta_i\right)\equiv \\ \equiv \left\{I_3\cdot c(q_i\cdot \Delta_i)+\left\{\bar{k}_i^{(0)}\times\right\}s(q_i\cdot \Delta_i)+\right. \\ \left. +\ \bar{k}_i^{(0)}\cdot \bar{k}_i^{(0)T}\left[1-c(q_i\cdot \Delta_i)\right]\right\}^T \equiv {}^{i-1}_{\ i}[R]^T \end{array}\right\}. \quad (6)$$

➢ The defining expression for the column vector $\bar{b}_i$, is established with the following:

$$\left\{\begin{array}{l} \bar{b}_i = \left\{I_3\cdot q_i+\left\{\bar{k}_i^{(0)}\times\right\}\left[1-c(q_i\cdot \Delta_i)\right]+\right. \\ \left. +\ \bar{k}_i^{(0)}\cdot \bar{k}_i^{(0)T}\cdot\left[q_i - s(q_i\cdot \Delta_i)\right]\right\}\cdot \bar{v}_i^{(0)} \end{array}\right\}. \quad (7)$$

➢ Another matrix exponential, having a great significance for locating transformation, shows as:

$$\left\{\begin{array}{l} e^{A_i q_i} = exp\left(\begin{bmatrix}\left\{\bar{k}_i^{(0)}\times\right\} & \bar{v}_i^{(0)} \\ 0\ \ 0\ \ 0 & 0\end{bmatrix} q_i\right) = \\ = \begin{bmatrix} exp\left\{\left\{\bar{k}_i^{(0)}\times\right\}q_i\cdot \Delta_i\right\} & \bar{b}_i \\ 0\ \ \ 0\ \ \ 0 & 1\end{bmatrix}\end{array}\right\}; \quad (8)$$

$$exp\left\{\sum_{j=0}^{i} A_j\cdot q_j\right\} = \begin{bmatrix} exp\{R\} & exp\{p\} \\ 0\ 0\ 0 & 1\end{bmatrix};$$

$$where\quad exp\{R\} = \prod_{j=0}^{i} exp\left\{\left\{\bar{k}_j^{(0)}\times\right\}q_j\cdot \Delta_j\right\}; \qquad (9)$$

$$exp\{p\} = \sum_{j=0}^{i}\left\{\prod_{k=0}^{i} exp\left\{\left\{\bar{k}_k^{(0)}\times\right\}q_k\cdot \Delta_k\right\}\right\}\cdot \bar{b}_{j+1}.$$

➢ The inverse of the matrix exponential (8) is:

$$\left\{\begin{array}{l} exp\left\{-\left\{\sum_{j=i}^{0} A_j\cdot q_j\right\}\right\} = \begin{bmatrix} exp\{R\} & exp\{\bar{p}\} \\ 0\ 0\ 0 & 1\end{bmatrix} \\ where\quad exp\{R\} = \prod_{j=i}^{0} exp\left\{-\left\{\bar{k}_j^{(0)}\times\right\}q_j\, \Delta_j\right\} \\ exp\{\bar{p}\} = -\sum_{j=i}^{0}\left\{\prod_{k=i}^{j} exp\left\{-\left\{\bar{k}_k^{(0)}\times\right\}q_k\, \Delta_k\right\}\right\}\bar{b}_j \end{array}\right\}. \quad (10)$$

➢ The exponentials expressions for the locating matrices, which define the position and orientation of the $\{n\}$ and $\{n+1\}$ with respect to fixed frame $\{0\}$, are obtained as follows:

$$\left\{\begin{array}{l} T_{x0} = \prod_{i=1}^{x} T_{ii-1} = \begin{bmatrix} R_{x0} & \bar{p} \\ 0\ 0\ 0 & 1\end{bmatrix} \\ where\quad x = \{n; n+1\}\end{array}\right\}; \qquad (11)$$

$$T_{x0} = \prod_{i=1}^{n}\left(e^{A_i\cdot q_i}\right)\cdot T_{x0}^{(0)} = exp\left\{\sum_{i=1}^{n} A_i\cdot q_i\right\}\cdot T_{x0}^{(0)};$$

$$where\quad R_{x0} = exp\left\{\sum_{i=1}^{n}\left\{\bar{k}_i^{(0)}\times\right\}q_i\cdot \Delta_i\right\}\cdot R_{x0}^{(0)};$$

$$\bar{p} = \sum_{i=1}^{n}\left\{\left\{exp\left\{\sum_{j=0}^{i-1}\left\{\bar{k}_i^{(0)}\times\right\}q_j\, \Delta_j\right\}\right\}\bar{b}_i + \right. \qquad (12)$$

$$\left. +exp\left\{\sum_{i=1}^{n}\cdot\left\{\bar{k}_i^{(0)}\times\right\}q_i\, \Delta_i\right\}\bar{p}^{(0)}\,\delta_x\right\};$$

$$and\ \delta_x = \{\{0;\ x=n\};\{1;\ x=n+1\}\}.$$

➢ The inverse of the locating matrix (11) is:

$$\left\{T_{x0}^{-1} = \prod_{i=x}^{1} T_{ii-1}^{-1} = \begin{bmatrix} R_{x0}^T & -R_{x0}^T\cdot \bar{p} \\ 0\ 0\ 0 & 1\end{bmatrix}\right\}; \quad (13)$$

$$\left\{\begin{array}{l} T_{x0}^{-1} = \left\{T_{x0}^{(0)}\right\}^{-1}\cdot \prod_{i=n}^{1} exp(-A_i\cdot q_i) = \\ = \left\{T_{x0}^{(0)}\right\}^{-1}\cdot exp\left\{-\sum_{i=n}^{1} A_i\cdot q_i\right\}\end{array}\right\}; \quad (14)$$

and:

$$R_{x0}^T = \left\{R_{x0}^{(0)}\right\}^T\cdot exp\left\{-\left\{\sum_{i=n}^{1}\left\{\bar{k}_i^{(0)}\times\right\}q_i\Delta_i\right\}\right\}; \quad (15)$$

$$\left\{\begin{array}{l} -R_{x0}^T\cdot \bar{p} = -\sum_{i=n}^{1}\left\{exp\left\{\sum_{j=i-1}^{0}\left\{\bar{k}_i^{(0)}\times\right\}q_j\Delta_j\right\}\right\}\bar{b}_i - \\ -exp\left\{-\left\{\sum_{i=n}^{1}\left\{\bar{k}_i^{(0)}\times\right\}q_i\Delta_i\right\}\right\}\bar{p}^{(0)}\right\}\cdot \delta_x \end{array}\right\}. \quad (16)$$

Remark: *The MEG Algorithm*, due to computational advantages and independent of the reference can be applied for any robot structure.

3. APPLICANTION

Further, in the paper, for the exemplification of the MEG algorithm, there is considered a serial structure, of 2TR type, presented in the Figure 1.

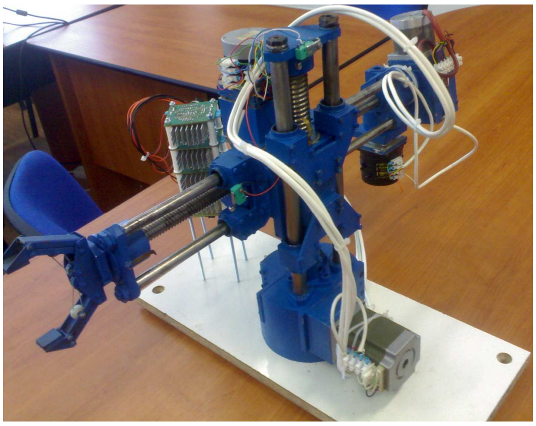

Figure 1 The serial robot structure 2TR

3.1 Direct Geometry Modeling of 2TR robot

For the proposed structure, according to, [1]-[4], algorithms based on the matrix calculation with exponential functions will form the basis of the homogeneous transformations in the Direct Geometry Modeling (DGM), and the exponential of the location matrix between the systems $\{0\}$® $\{4\}$ is determined first. These are characterizing the position and orientation (or location) of the final effector relative to the system attached to the fixed base of the robot. Thus, according to the MEG algorithm, the following steps are performed to determine the position and orientation of the end-effector.

- There is determined the nominal geometry matrix, $M_{vn}^{(0)}$, which is describing the $\overline{q}^{(0)}$ configuration of the robot, which for the mechanical system taken into study is presented in Table 1.

$M_{vn}^{(0)**}$ Î $2TR$ Table 1

Joint	$\overline{k}_i^{(0)T}$			$\overline{p}_i^{(0)T}$			$\overline{v}_i^T = \left\{ \begin{matrix} \Delta_i \cdot \left(\overline{p}_i^{(0)} \times \overline{k}_i^{(0)}\right) + \\ +(1-\Delta_i)\cdot \overline{k}_i^{(0)} \end{matrix} \right\}^T$		
$i = 1 \rightarrow 4$	$k_{ix}^{(0)T}$	$k_{iy}^{(0)T}$	$k_{iz}^{(0)T}$	$x_i^{(0)T}$	$y_i^{(0)T}$	$z_i^{(0)T}$			
1	0	0	1	0	0	l_1	0	0	1
2	1	0	0	0	$-a_1$	l_1	1	0	0
3	1	0	0	l_2	$-a_1$	l_1	0	l_1	$-a_1$
4	-	-	-	$l_2 + a_2$	$-a_1$	l_1	-	-	-

As it can be observed from the Table 1, the screw parameters $\{\overline{k}_i^{(0)}; \overline{v}_i^{(0)}\}$ are contained in the nominal geometry matrix . [7]

- Is represented the kinematic schema of the 2TR type robot, in Figure 2, according to Table 1.

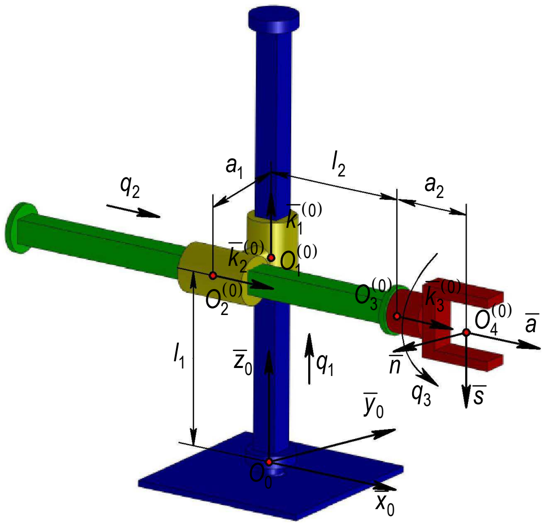

Figure 2 Kinematic schema of the 2TR serial robot

- According to the algorithm, is opened an external cycle for $(i = 1$® $3)$. Within the cycle is determined the matrices and exponential functions, specific to the direct geometry modeling, in accordance to (3)-(8).

For the first element, representing a prismatic joint, there are determined the following expressions for exponential matrix functions:

$$A_1 = \begin{bmatrix} \{\overline{k}_1^{(0)} \times\} & \overline{v}_1^{(0)} \\ 0\ \ 0\ \ 0 & 0 \end{bmatrix} \equiv \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}; \qquad (17)$$

$$\left\{ \begin{matrix} e^{\{\overline{k}_1^{(0)} \times\} \cdot q_1 \cdot \Delta_1} \\ \exp\left\{\{\overline{k}_1^{(0)} \times\} \cdot q_1 \cdot \Delta_1\right\} \end{matrix} \right\} \equiv \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I_3 \qquad (18)$$

$$\left\{ \begin{matrix} \overline{b}_1 \\ I_3 \cdot q_1 \cdot \overline{v}_1^{(0)} \end{matrix} \right\} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \cdot q_1 = \begin{bmatrix} 0 \\ 0 \\ q_1 \end{bmatrix} \qquad (19)$$

$$\left\{ \begin{matrix} e^{A_1 \cdot \Delta_1} \\ \exp\left(\begin{bmatrix} \{\overline{k}_1^{(0)} \times\} & \overline{v}_1^{(0)} \\ 0\ \ 0\ \ 0 & 0 \end{bmatrix} \cdot q_1 \right) \end{matrix} \right\} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & q_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (20)$$

According to Figure 2, the second joint of the 2TR structure, is also a prismatic joint, so for the kinetic element, the matrix exponentials are:

$$A_2 = \begin{bmatrix} \left\{\overline{k}_2^{(0)}\times\right\} & \overline{v}_2^{(0)} \\ 0\ 0\ 0 & 0 \end{bmatrix} \equiv \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (21)$$

$$\left\{ \begin{array}{c} e^{\left\{\overline{k}_2^{(0)}\times\right\}\cdot q_2\cdot\Delta_2} \\ \exp\left\{\left\{\overline{k}_2^{(0)}\times\right\}\cdot q_2\cdot\Delta_2\right\} \end{array} \right\} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \equiv I_3; \quad (22)$$

$$\left\{ \begin{array}{c} \overline{b}_2 \\ I_3\cdot q_2\cdot\overline{v}_2^{(0)} \end{array} \right\} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \cdot q_2 = \begin{bmatrix} q_2 \\ 0 \\ 0 \end{bmatrix} (23)$$

$$\left\{ \begin{array}{c} e^{A_2\cdot q_2} = \exp\{A_2\cdot q_2\} \\ \exp\left(\begin{bmatrix} \left\{\overline{k}_2^{(0)}\times\right\} & \overline{v}_2^{(0)} \\ 0\ 0\ 0 & 0 \end{bmatrix} \cdot q_2 \right) \end{array} \right\} = \\ = \begin{bmatrix} R\left(\overline{k}_2^{(0)}; q_2\right) & \overline{b}_2 \\ 0\ \ 0\ \ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & q_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad (24)$$

For the third kinetic element, consisting of a rotation joint, are determined the following expressions:

$$A_3 = \begin{bmatrix} \left\{\overline{k}_3^{(0)}\times\right\} & \overline{v}_3^{(0)} \\ 0\ 0\ 0 & 0 \end{bmatrix} \equiv \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & l_1 \\ 0 & 0 & 0 & a_1 \\ 0 & 0 & 0 & 0 \end{bmatrix}; \quad (25)$$

$$\left\{ \begin{array}{c} e^{\left\{\overline{k}_3^{(0)}\times\right\}\cdot q_3} \\ \exp\left\{\left\{\overline{k}_3^{(0)}\times\right\}\cdot q_3\right\} \end{array} \right\} = \\ = I_3 + \left\{\overline{k}_3^{(0)}\times\right\}\cdot sq_3 + \left\{\overline{k}_3^{(0)}\times\right\}^2\cdot(1-cq_3) \quad (26)$$

$$e^{\left\{\overline{k}_3^{(0)}\times\right\}\cdot q_3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cq_3 & -sq_3 \\ 0 & sq_3 & cq_3 \end{bmatrix}; \quad (27)$$

$$\overline{b}_3 = \left\{ \begin{array}{c} I_3\cdot q_3 + \left\{\overline{k}_3^{(0)}\times\right\}\cdot(1-cq_3) + \\ + \left\{\overline{k}_3^{(0)}\times\right\}^2\cdot(q_3 - sq_3) \end{array} \right\} \cdot \overline{v}_3^{(0)} = \\ = \begin{bmatrix} 0 \\ a_1\cdot cq_3 - a_1 + l_1\cdot sq_3 \\ l_1 + a_1\cdot sq_3 - l_1\cdot cq_3 \end{bmatrix} ;(28)$$

$$\left\{ \begin{array}{c} e^{A_3\cdot q_3} = \exp\{A_3\cdot q_3\} \\ \exp\left(\begin{bmatrix} \left\{\overline{k}_3^{(0)}\times\right\} & \overline{v}_3^{(0)} \\ 0\ 0\ 0 & 0 \end{bmatrix} \cdot q_3 \right) \end{array} \right\} = \begin{bmatrix} R\left(\overline{k}_3^{(0)}; q_3\right) & \overline{b}_3 \\ 0\ \ 0\ \ 0 & 1 \end{bmatrix} = \\ = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & cq_3 & -sq_3 & a_1\cdot cq_3 - a_1 + l_1\cdot sq_3 \\ 0 & sq_3 & cq_3 & l_1 + a_1\cdot sq_3 - l_1\cdot cq_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (29)$$

For the initial configuration $\overline{\theta}^{(0)}$, the homogeneous transformation matrix between $\{0\}\rightarrow\{4\}$ reference frames, is determined as:

$$\left\{ T_{40}^{(0)} \equiv \begin{bmatrix} R_{40}^{(0)} & \overline{p}^{(0)} \\ 0\,0\,0 & 1 \end{bmatrix} \right\} = \begin{bmatrix} 0 & 0 & 1 & l_2 + a_2 \\ -1 & 0 & 0 & -a_1 \\ 0 & -1 & 0 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} (30)$$

In keeping with (9)-(10), the situation matrix between $\{0\}\rightarrow\{4\}$ reference frames, which expresses the position and orientation of the end effector, is expressed as follows:

$$\left\{ \begin{array}{c} {}_4^0[T] = \prod_{i=1}^{4} T_{ii-1}(q_i) \\ \begin{bmatrix} {}_4^0[R] & \overline{p}^{(0)} \\ 0\,0\,0 & 1 \end{bmatrix} \end{array} \right\} = e^{\sum_{i=1}^{3} A_i\cdot q_i} \cdot T_{40}^{(0)} \quad (31)$$

where:

$${}_4^0[R] = \left\{ \exp\left\{ \sum_{i=1}^{3} \left\{\overline{k}_i^{(0)}\times\right\}\cdot q_i\cdot\Delta_i \right\} \right\} \cdot R_{40}^{(0)} \quad (32)$$

$$\overline{p} = \sum_{i=1}^{3} \left\{ e^{\left\{ \sum_{j=0}^{i-1} \left\{\overline{k}_j^{(0)}\times\right\}\cdot q_j\cdot\Delta_j \right\}} \right\} \cdot \overline{b}_i \cdot e^{\sum_{i=1}^{3}\left\{\overline{k}_i^{(0)}\times\right\}\cdot q_i\cdot\Delta_i} \cdot \overline{p}^{(0)} \quad (33)$$

Therefore, on the basis of (31)-(33) by performing matrix calculations, the following expressions are obtained for the 2TR structure:

$$ {}^0_4[T]=\begin{Bmatrix} {}^0_4[R] \\ [\bar{n}\;\;\bar{s}\;\;\bar{a}] \end{Bmatrix}=e^{\left\{\bar{k}_3^{(0)}\times\right\}q_3}\cdot R_{40}^{(0)}=\begin{bmatrix} 0 & 0 & 1 \\ -cq_3 & sq_3 & 0 \\ -sq_3 & -cq_3 & 0 \end{bmatrix} \quad (34) $$

$$ \bar{p}_4^{(0)}=\begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix}=\bar{b}_1+e^{\left\{\bar{k}_1^{(0)}\times\right\}q_1}\cdot\left(\bar{b}_2+\bar{b}_3\right)+ $$
$$ +e^{\left\{\bar{k}_1^{(0)}\times\right\}q_1}\cdot e^{\left\{\bar{k}_3^{(0)}\times\right\}q_3}\cdot\bar{p}^{(0)}=\begin{bmatrix} a_2+d_2+q_2 \\ -a_1 \\ l_1+q_1 \end{bmatrix} \quad (35) $$

which represents the resultant matrix and the position vector, these being included in the following expression of the column vector of the operational variables [4]:

$$ {}^0\bar{X}=\begin{bmatrix} \bar{p} \\ \hline \bar{\theta} \end{bmatrix}=\begin{bmatrix} \left(p_x\;\;p_y\;\;p_z\right)^T \\ \hline \left(\alpha_z\;\;\beta_y\;\;\gamma_z\right)^T \end{bmatrix} \quad (36) $$

known as direct geometry equations (DGM equations).

According to [4], in order to establish the orientation angles, $\alpha_z,\ \beta_y,\ \gamma_z$ for exact determination of the values, there is used the trigonometric function $A\tan 2$, defined by:

$$ x=A\tan 2(s\alpha;c\alpha)=\begin{Bmatrix} \{\alpha;[s\alpha\geq 0;c\alpha>0]\}; \\ \{\pi/2+\alpha;[s\alpha>0;c\alpha<0]\} \\ \{\pi+\alpha;[s\alpha<0;c\alpha<0]\}; \\ \{-\pi/2+\alpha;[s\alpha<0;c\alpha\geq 0]\} \end{Bmatrix} \quad (37) $$

Hence, in keeping with (37), results:

$$ \begin{gathered} \alpha_z=A\tan 2(s\alpha_z;c\alpha_z)=A\tan 2(1;0)=\pi/2; \\ \beta_y=\pi/2; \\ \gamma_z=A\tan 2(s\gamma_z;c\gamma_z)=A\tan 2(-sq_3;-cq_3)=\pi+q_3 \end{gathered} \quad (38) $$

The column vector of the operational coordinates, defined by (36), becomes [7]:

$$ {}^0\bar{X}=\begin{bmatrix} \bar{p} \\ \cdots \\ \bar{\psi} \end{bmatrix}=\begin{bmatrix} \left(p_x\;\;p_y\;\;p_z\right)^T \\ \cdots\cdots \\ \left(\alpha_z\;\;\beta_y\;\;\gamma_z\right)^T \end{bmatrix}= $$
$$ =\left\{\begin{bmatrix} a_2+d_2+q_2 \\ -a_1 \\ l_1+q_1 \end{bmatrix}\quad [\pi/2\;\;\pi/2\;\;\pi+q_3]^T\right\}^T \quad (39) $$

and characterizes the direct geometric modeling of the 2TR type robot taking into account.

3.2 Inverse Geometry Equations of 2TR serial structure

The inverse geometric modeling (MGI), or modeling of geometric control functions, consists of determining the column vector of generalized coordinates (q_1, q_2, q_3), which are characterizing the movement in each kinetic link, expressed as:

$$ \begin{gathered} \left\{\bar{\theta}\equiv[q_1\;\;q_2\;\;q_3]^T\right\}= \\ =f^{-1}\left({}^0\bar{X}\right)\equiv f^{-1}\left[\left(p_x\;\;p_y\;\;p_z\;\;\alpha_x\;\;\beta_y\;\;\gamma_z\right)^T\right] \end{gathered} \quad (40) $$

In inverse geometric modeling, the position of the mobile system attached to the final effector in the characteristic point is known by numerical values. As an observation, the number of operational parameters varies, but cannot exceed the number of degrees of freedom. In keeping with (39), results:

$$ \begin{Bmatrix} q_2=p_x-a_2-d_2 \\ q_1=p_z-l_1 \end{Bmatrix} \quad (41) $$

According to previous expression, the displacement on $\bar{y}_0$ axis is absent, namely:

$$ p_y=-a_1=cst. \quad (42) $$

From the same relations belonging to the direct geometry, the third generalized coordinate is determined as being:

$$ q_3=\gamma_z-\pi \quad (43) $$

The expressions (41) and (43) are the geometric control functions corresponding to the input data relating to the position of the characteristic point in the Cartesian space.

4. CONCLUSION

According to the paper, there can be established the Direct Geometry Equations on the basis of algorithms, which are using matrix exponential functions. As can be seen from the analysis, the use of the algorithms for determining the geometric control functions of a robot implies the application of some mathematical methods that are helping to establish the connection between the elements that determine the location of the end effector in the Cartesian space. Thus, by applying matrix methods based on matrix exponentials, were presented the direct geometry equations, which express the position and orientation of the characteristic point of the end effector with respect to the fixed reference system attached to the robot base.

As an important remark, is that the approach of geometry with exponential functions leads to essential advantages on the one hand, due to the writing in a compact form, easy to visualize in geometrical form, on the other hand it is noted as an essential advantage the lack of reference systems which introduces geometric restrictions, and in the precision study could lead to additional geometric errors.

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Modelare geometrică cu funcţii exponenţiale de matrice pentru o structură serială de robot

Rezumat: În lucrare, pe baza formulărilor bazate pe exponenţiale de matrice se vor stabili ecuaţiile geometriei directe şi inverse. Funcţiile exponenţiale de matrice în robotică vor fi aplicate în modelarea matematică a unei structuri de robot serial. Spre deosebire de algoritmii clasici, aplicarea funcţiilor exponenţiale de matrice prezintă câteva avantaje în determinarea ecuaţiilor geometriei directe. Rezultatele obţinute în modelarea geometrică, reprezintă date de intrare pentru modelarea cinematică, toate acestea fiind date importante în studiul comportării dinamice a oricărui sistem mecanic robotizat.

Claudiu SCHONSTEIN, Lecturer, Ph.D, Eng, Technical University of Cluj-Napoca, Department of Mechanical Systems Engineering, E-mail: schonstein_claudiu@yahoo.com, No.103-105, Muncii Blvd., Office C103A, 400641, Cluj-Napoca.

Iuliu NEGREAN, Professor, Ph.D., Head of Department of Mechanical Systems Engineering, Technical University of Cluj-Napoca, iuliu.negrean@mep.utcluj.ro, No.103-105, Muncii Blvd., +40-264-401616.

Nicolae PANC, Lecturer, Ph.D, Eng, Technical University of Cluj-Napoca, Department of Manufacturing Engineering, E-mail: nicolae.panc@tcm.utcluj.ro, No.103-105, Muncii Blvd. Office G19B, 400641, Cluj-Napoca.