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STUDY OF GRAVITATIONAL DISPLACEMENTS OF A MOBILE LOAD ON ROPEWAY CABLE

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Abstract: *There are few publications which bring up the subject of the ropeway cable under moving in-service load. Our main goal is to establish the so-called differential equation of a moving load on the ropeway cable of a zip line entertainment facility. The unknown is its movement speed as a function of time, but the direct integration of this equation is not possible. This impediment is bypassed by changing the variable "time", with the variable "angle" swept by the radius vector.*
However, the new first order constant coefficient differential nonlinear equation can be easily solved. The solution obtained was applied in a real case study and a comparative study is presented plotting speed variation and acceleration of mobile load.
Key words: *mobile load, steel wier rope, wind action, nonlinear differential equation, study case*

1. INTRODUCTION

The "catenary" is the curve assumed of a flexible inextensible wire or chain with uniform density hanging freely from two fix points, acted on by gravity. Finding out the equation of this "chain-curve" is related to the beginning of calculus. This problem has been launched by Jacob Bernoulli in 1690, first to Gottfried Leibniz, then to the scientific community. The equation was obtained by Leibniz, Christiaan Huygens and Johann Bernoulli in 1691, in Acta Eruditorum [1].

Nowadays, the emergence of high-strength materials, along with the expansion of cable transport solutions, required increased need precision for design and exploitation, aimed at ensuring a high level of security. Geometric and material nonlinearity, dynamic behavior and environmental conditions in operation (temperature, wind) are permanent and ongoing topics for researchers. An example is a recent book that uses FEM in dynamic modeling, starting from the constitutive equations and taking into account the compressibility of the chains or cables [2]. It is well known that a stretched cable has an equilibrium shape that approximates a parabola. This model will be used and refined in this work.

The purpose of the paper is to provide designers an advanced model for design work.

Cable transport systems are usually in rugged mountain areas in the form of funiculars for transporting materials, cableways and cable cranes. In contrast, for recreation, there are used installations commonly called "zip-line" or "Tyrolean traverse" where the moving load moves gravitationally based on the level difference between the departure and arrival stations. The zip lines openings are usually lower than the ones mentioned above, rarely exceeding one kilometer.

Zip-line particular problem is posed by controlling and limiting the input speed of the mobile load in the downstream station (destination). If this speed exceeds a certain value, a braking system is required whose price, compared to the price of the whole plant, is more than significant. On the contrary, if the difference in level between the two stations is small, or if the cable is not stretched enough, the mobile load may not reach the arrival station, stopping on the way. The wind having the opposite travel direction increases this danger.

An approximate solution to the problem is considered the displacement of the load of the chord line corresponding to the carrier cable, to be identical to the movement on the incline plane. Taking into account the aerodynamic drag resistance, the problem is mathematically formalized by a Riccati differential equation, whose analytical solution could be found [3].

In relation to the inclined plane displacement having a constant base angle, the displacement on the curve of the carrier rope (approximated by a parabola) has a variable slope. This is the difficulty of the problem whose approach and solution are presented in this paper.

2. DIFFERENTIAL EQUATION OF MOTION

The differential equation of motion results from the dynamic equilibrium condition of the moving load on an infinitesimal element of its trajectory curve *ds* (Fig. 1). This is an equation of a parabola [4]:

$$y(x) = f_x - x\cdot tg\beta = \frac{x(l-x)}{2H}\left(\frac{q}{\cos\beta}+\frac{2Q}{l}\right) - x\cdot tg\beta \qquad (1)$$

where: q is the distributed load from the weight of the carrier cable; Q is the mobile load, H - the horizontal tension component of the cable, and f_x it is his sag; the other notations result from the figure.

Forces in dynamic equilibrium are: the load due to the mobile payload, $G=Mg$ in which: M is the mass of the load, and g is the gravitational acceleration; the centrifugal force, $F_C=M\cdot v^2/\rho$ where: v is the instantenous speed of the displacement, and ρ - the radius of curvature of the trajectory at the considered point; the normal reaction of the carrier cable, N; the aerodynamic drag resistance force, $F_a=k_a\cdot v^2$, k_a is the aerodynamic drag coefficient; the rolling resistance, $W= w\cdot(Mg\cdot\cos\alpha+F_c)$, w is the rolling resistance coefficient; the active weight of mobile load, $M\cdot g\cdot\sin\alpha$; the inertial force (or dynamic resistance), $F_i=M\cdot a$.

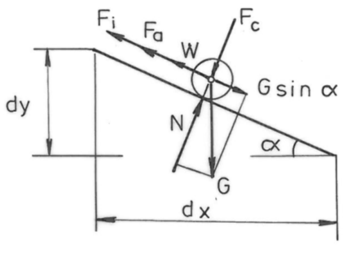

Fig. 1. The forces acting on the mobile load

The equilibrium equation of forces in the direction of the tangent to the trajectory, after the forces are replaced by their expressions takes the form

$$M\cdot\dot{v} + k_a\cdot v^2 + w\cdot\frac{M\cdot v^2}{\rho} = M\cdot g\cdot(\sin\alpha - w\cdot\cos\alpha) \qquad (2)$$

If we divide (2) by M and denote

$$k = \frac{k_a}{M} + \frac{w}{\rho} \qquad (3)$$

then we get

$$\dot{v} + k\cdot v^2 = (\sin\alpha - w\cdot\cos\alpha)\,g \qquad (4)$$

The difficulty of approaching this equation is that it contains two unknown functions of time, namely v (load velocity), and α (angle of the tangent to the trajectory). It is possible to overcome this difficulty taking as the independent variable the radius vector angle φ to the horizontal, as shown in Figure 2.

The two angles α and φ are complementary

$$\alpha = 0{,}5\pi - \varphi \qquad (5)$$

In what concerns the expression of the velocity depending on the angle φ, it is based on relationship

$$\dot{v} = \frac{dv}{dt} = \frac{dv}{d\varphi}\cdot\frac{d\varphi}{dt} = \frac{dv}{d\varphi}\cdot\omega = \frac{dv}{d\varphi}\cdot\frac{v}{\rho} = \frac{1}{2\rho}\cdot\frac{d\left(v^2\right)}{d\varphi} \qquad (6)$$

where ω represents instantaneous angular velocity. Thus (1) becomes

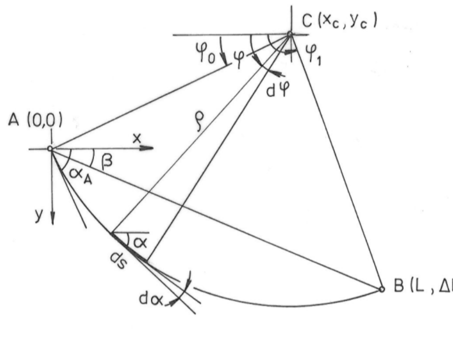

Fig. 2. The geometric scheme of the carrier cable

$$\frac{1}{2\rho}\cdot\frac{d}{d\varphi}\left(v^2\right)+k\cdot v^2=\left(\cos\varphi-w\cdot\sin\varphi\right)\cdot g$$

If we use the notation

$$2\rho\cdot k=n \tag{7}$$

we get the final form of the motion equation whose unknown is $v(\alpha)$

$$\frac{d}{d\alpha}\left(v^2\right)+n\cdot v^2=2\rho g\cdot\left(\cos\varphi-w\cdot\sin\varphi\right) \tag{8}$$

3. SOLUTION OF THE EQUATION OF MOTION

Based on the following substitution

$$u=v^2 \tag{9}$$

Equation (8) is transformed into a first-order nonhomogeneous differential equation

$$\frac{du}{d\varphi}+n\cdot u=2\rho g\left(\cos\varphi-w\cdot\sin\varphi\right) \tag{10}$$

An analytical solution to this equation is probably impossible, or at least very difficult to find solution because the radius of curvature of the trajectory that is not constant, appears both in the expression of n (implicitly), and explicit in the right-hand member. Therefore, an approximate solution is proposed, assuming

$$\rho\left(\alpha\right)\square\ ct=R \tag{11}$$

This equates to the approximation of the parabolic trajectory of the load with a circular one. This approximation results in a lower error as the carrier cable is stretched and therefore has a greater radius of curvature, which corresponds to the usual installations. The case study in the following paragraph offers more clarification on this point.

The general solution of equation (10) is the sum of the general solution of the homogeneous equation and the particular solution of the whole nonhomogeneous equation.

$$u(\varphi)=u_o(\varphi)+u_p(\varphi)=$$

$$Ce^{-n\varphi}+\frac{2\rho g}{n^2+1}\left[\left(1-nw\right)\sin\varphi-\left(n+w\right)\cos\varphi\right]$$

The constant C is now determined by applying initial conditions: at the time $t=0$, the initial velocity $v(0)=0$ and the angle $\varphi=\varphi_0$. Based on (9), results

$$\begin{cases}u(0)=0\\ \varphi=\varphi_0\end{cases} \tag{12}$$

By making appropriate calculations and grouping terms is obtained

$$u(\varphi)=\frac{2\rho g}{n^2+1}\begin{Bmatrix}\left(1-nw\right)\left[\sin\varphi-e^{n(\varphi_0-\varphi)}\sin\varphi_0\right]+\\ +\left(n+w\right)\left[\cos\varphi-e^{n(\varphi_0-\varphi)}\cos\varphi_0\right]\end{Bmatrix}$$

and

$$v(\varphi)=\sqrt{u(\varphi)} \tag{13}$$

The expression of acceleration results from the derivation of $u(\varphi)$. Indeed:

$$\begin{aligned}a(t)&=\frac{dv}{dt}=\frac{dv}{d\varphi}\cdot\frac{d\varphi}{dt}=\frac{dv}{d\varphi}\cdot\omega=\\ &=\frac{dv}{d\varphi}\cdot\frac{v}{\rho}=\frac{1}{2\rho}\cdot\frac{d(v^2)}{d\varphi}=\frac{1}{2\rho}\cdot\frac{du}{d\varphi}\end{aligned} \tag{14}$$

Thus

$$a(\varphi)=\frac{g}{n^2+1}\begin{Bmatrix}\left(1-nw\right)\cdot\left[\cos\varphi+ne^{n(\varphi_0-\varphi)}\sin\varphi_0\right]-\\ -\left(n+w\right)\left[\sin\varphi-ne^{n(\varphi_0-\varphi)}\cos\varphi_0\right]\end{Bmatrix}$$

The distance covered is obtained in the form

$$s=R(\varphi_1-\varphi_0) \tag{15}$$

and it corresponds to the length of the circle arch that approximates the parabola. The real distance is the length of the parabolic arc, and it can be calculated by knowing the expression (1) of the trajectory of the moving load.

Finally, the time for which the trajectory is traveled by the mobile load can be determined as follows:

$$v(t)=\frac{ds}{dt}=\frac{\rho\cdot d\varphi}{dt}\ \Rightarrow\ dt=\frac{\rho\cdot d\varphi}{v} \tag{16}$$

$$t_p=\int_{\varphi_0}^{\varphi_1}\frac{\rho\cdot d\varphi}{v(\varphi)} \tag{17}$$

4. EFFECTS OF WIND ACTION

Wind action in the direction of movement increases the speed of the mobile load and reduces it, if it acts in the opposite direction. In this way it produces a transport movement at speed $\pm v_v$.

At the same time, the aerodynamic drag resistance force is proportional to the square of the relative velocity, namely $v\pm v_v$ and the centrifugal force is proportional to the square of the absolute velocity v.

Finally, if the wind speed v_v is assumed to be constant, the acceleration is

$$dv/dt = d\left(v \mp v_v\right) = \dot{v}$$

and does not depend on the wind.

Therefore the second term of (2) must be replaced by

$$k_a \cdot (v \mp v_v)^2$$

By developing the square of relative velocity, it appears an additional term in v, as well as another term in v_v^2.

Appling substitution (9), the resulting differential equation is non-linear. The problem can be solved by neglecting the effect of centrifugal force because it is negligible compared to the normal component of the payload $Mg\cos\alpha$. In this case, (8) retains its shape (instead of v it will be $v \pm v_v$). Also the solution (13) thereof are maintained.

Thus:

$$v \mp v_v = \sqrt{u} \text{ wherefrom } v = \sqrt{u} \pm v_v \quad (18)$$

which means that the wind provides the transport component of the absolute velocity.

5. CASE STUDY

The case study highlights the conclusions of using the mathematical formalism above, which led to the solution (13), compared to the solution deduced from the simplified model of moving the mobile load on the real trajectory [3]. We will use the data of the planned installation to be installed in the Buzau locality in the tourist traveling zone "*Vulcanii Noroioşi*" (translated: "*Mud Volcaneous*") in Romania, for which: the opening of the carrier cable: L=391m; the drop: Δh=51m; the payload: Q=125kg; the linear mass of the cable: m=0,48kg/m; the cable sag (horizontal tension) at the ambient temperature of θ=8°C: H=42277N; the air density at the average altitude at wich the installation is located ρ^H=1,182kg/m^3; the travel resistance coefficient w=0,025 N/N; the exposed aria of mobile load A=1,2m^2 and the aerodynamic coefficient of the mobile load c_a=1,1.

The radius of the circle that approximates the trajectory of the moving load, and the coordinates of its center, were determined in the following conditions:

(*a*) The circle passes through the start anchor and end anchor of the carrier cable. The two link points of the carrier cable have the coordinates A(0,0) and B(391m, 51m).

(*b*) The tangent to the circle at the starting point of the mobile load has the same slope with the parabolic trajectory. We have obtained the results: R = 3966m, x_C = 707,86m and y_C = -3903m.

In order to highlight the influence of cable tension, it was also considered the situation $H_1 \cong H/2$=21050N, the other data remaining unchanged. The results are presented as graphs of variation with the angle φ of velocity and acceleration of the mobile load.

The following Figures 3 and 4 will be commented comparatively, considering the moving of the mobile load on the rectilinear trajectory (on the real parabola string).

Studying the allure of the graphs in Figures 3*a* and 3*b*, we can observe:

1. The speed of the mobile load increases up to a certain maximum value on the steeper portion of the trajectory, then decreases on the less steep portion.

2. Reducing the tension of the carrier cable leads to an increase in its sag, and the maximum speed value increases (Fig. 3*b*). This is in agreement with the increase of the downward trajectory slope, but the final speed of the mobile load (the input speed thereof at the arrival station) is reduced. The reduction is from 10,714m/s to 8,062m/s, i.e. around 20% when the horizontal component of tension in the cable is halved and its maximum sag doubles. This second observation also indicates the path of reducing the input speed of the mobile load at the arrival station: relaxing the initial tension of the carrier cable.

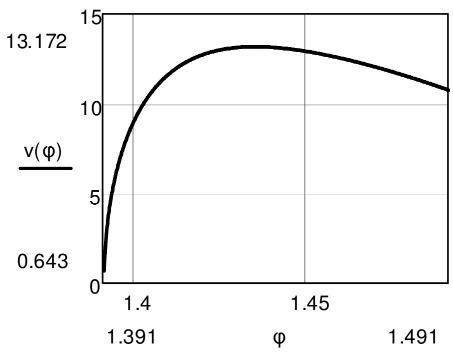

Fig. 3*a*: Graph of variation of mobile load speed in the assumption H=42277N

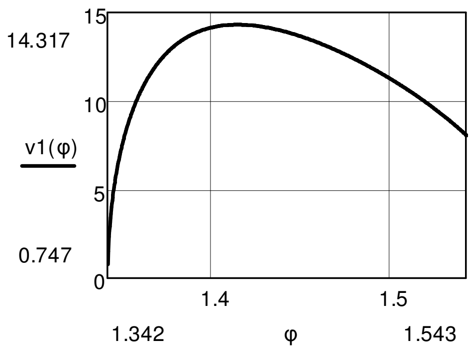

Fig. 3*b*. Graph of variation of mobile load speed in the assumption H=21050N

In the real case, the speed goes through a maximum, but the movement is not stabilize at this value; the speed decreases as the trajectory slope decreases as the mobile load approaches the arrival station (Fig. 3*a*).

In what concerns the acceleration graph (Fig. 4*a*), it shows a monotonic decrease in acceleration, and also highlights where it has been canceled and then becomes negative.

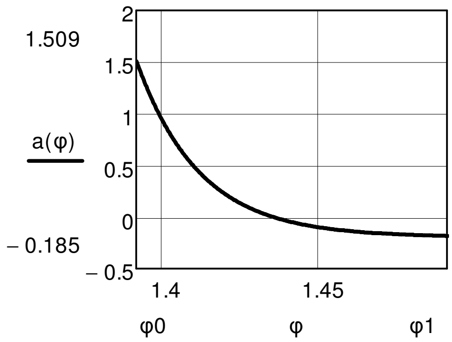

Fig. 4*a*. Graph of variation of mobile load acceleration in the assumption H=42277N

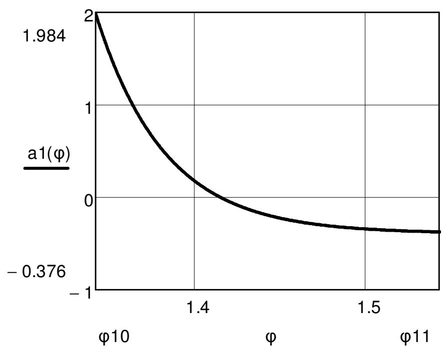

Fig. 4*b*. Graph of variation of mobile load acceleration in the assumption H=21050N

In the simplified theory, the acceleration does not take negative values, but only tends to zero if the trajectory is long enough for the speed to reach the limit value.

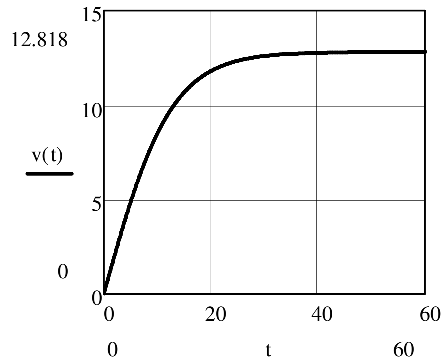

Fig. 5*a*. Graph of variation of mobile load speed in the simplified theory (movement of the mobile load on the string of the real trajectory).

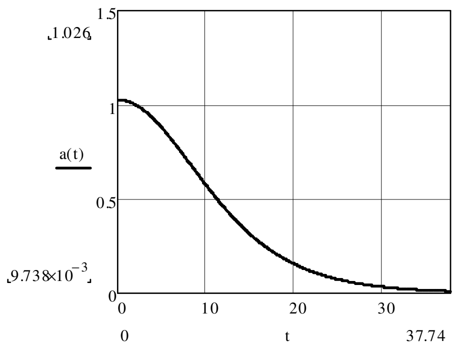

Fig. 5*b*. Graph of variation of mobile load acceleration in the simplified theory (movement of the mobile load on the string of the real trajectory).

The velocity graph in the simplified theory (movement of the mobile load on the real trajectory), reveals that mobile load speed increases continuously with time and tends to a limit determined by the aerodynamic drag (Fig. 5*a*). In what concerns the maximum speed it is found to be higher than in the case of movement on the arc of curve, than the movement on the chord. This can be explained by the fact that at the starting, the angle of the tangent to the trajectory is greater than the angle of the chord.

6. CONCLUSIONS

1. The change of variable (6) has solved the problem of determining the kinematic quantities of the gravitational displacement of the loads on carrier cables. This variable change was made according to the position on the trajectory, not by time. We believe this is the idea that this work is desirable.

2. The obtained solution (13) and (14) substantially improves the solution based on the consideration of the load displacement on the actual trajectory, both as an allure of varying aspect, as well as the resulting values.

3. In addition, we can make a very useful comment for the designer. As long as it results: $u(\varphi_1)$ in a positive value, the input speed of the mobile load at the arrival station results in a real value; if $u(\varphi 1) = 0$ it results that the mobile load stops by itself at the arrival station, and if $u(\varphi 1) < 0$, an imaginary value is obtained for the input speed at the arrival station, meaning that the mobile load stops on the way, before it arrives at the station of arrival.

Then, of course, the load will move back to the point where the trajectory has the minimum quota. Therefore, the solution is likely to signal the wrong design.

4. The solution obtained is not accurate, at least in theory, as it assumes that the radius of curvature of the trajectory is constant, so that the trajectory of the load is a circle arc.

However, given the large radius of curvature as well as the conditions considered in establishing this circle, the solution offers a high degree of accuracy, as can be seen from the qualitative interpretations and comments made on the case study.

The deviation of the circle from the parabolic trajectory (1) can be evaluated by plotting the two curves on the same graph. In the case study considered this deviation is approx. 30*mm*, that it is 0.6% from the maximum sag equals 5*m*.

5. It is expected that, due to the energy consumption to overcome the friction between the carrier cable due to its local bending under the load weight, the speeds and the acceleration of the displacement of the load will be somewhat computing tool that has been lacking so far, at least according to our knowledge.

6. We think, therefore, that the designers of such cableway facilities, have been provided with the computing tool that has been lacking so far, at least according to our knowledge.

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Studiul deplasării gravitaţionale a sarcinilor pe cabluri purtătoare

În lucrare se stabileşte ecuaţia diferenţială a mişcării sub acţiunea gerutăţii proprii a sarcinilor pe cabluri purtătoare, având ca necunoscută viteza de deplasare a acesteia ca funcţie de timp. Integrarea directă a acestei ecuaţii nu este posibilă. Dificultatea a fost ocolită prin trecerea de la variabila timp la variabila unghi al razei vectoare. Astfel, prin considerarea în primă aproximaţie că raza de curbură a traiectoriei sarcinii nu variază, s-a reuşit reducerea ecuaţiei la o ecuaţie diferenţială de ordinul întâi cu coeficienţi constanţi care a fost rezolvată cu uşurunţă. Soluţia a fost aplicată într-un studiu de caz real al unei de instalaţii de agrement (tiroliene); au fost reprezentate grafic variaţiile vitezei şi a acceleraţiei sarcinii mobile, care au fost interpretate comparativ cu cazul simplificat a deplasării sarcinii pe coarda traectoriei reale, des utilizat în practica inginerească. Acest studiu a prilejuit unele comentarii, interpretări şi concluzii de interes practic.

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