TECHNICAL UNIVERSITY OF CLUJ-NAPOCA

ACTA TECHNICA NAPOCENSIS

Series: Applied Mathematics, Mechanics, and Engineerir
Vol. 60, Issue IV, November, 2017

A GENERALIZATION OF THE MINIMUM PRINCIPLE ENERGY FOR COSSERAT POROUS MATERIALS

Marin MARIN, Sorin VLASE, Lavinia CODARCEA-MUNTEANU, Adina CHIRILĂ

Abstract: *Our study is a generalization of the minimum principle energy obtained by Ieșan and Quintanilla for microstretch elastic bodies. By this extension we wish to cover the theory of Cosserat bodies with voids. In this new context we formulate the boundary value problem and we demonstrate through an accessible method a uniqueness result for the solution to this problem. As a main result, we also prove an extension of the principle of minimum potential energy..*

Key words: *elastostatics, dipolar bodies, stretch, minimum principle*

1. INTRODUCTION

Eringen and his co-workers introduced for the first time the micromorphic elastic material (see [1], [2]). After that, in [3] they generalized this theory in order to cover the theory of thermo-microstretch elastic materials. The basic principle of this theory is the fact that the material points can stretch and contract independently of their translations and rotations. We can find some applications of such kind of materials in [3]. Other approaches to bodies with microstructure can be found in [6]-[9].

In our present study we consider, first of all, the basic equations and conditions for Cosserat elastic materials with voids (pores) in the context of the general theory. Namely, we consider basic boundary value problems of Elastostatics and, for such kind of problems, we first generalize a relation of Betti's type, that is, a relation of reciprocity. Next, for the same problems, we prove the basic qualitative results, i.e. the existence and the uniqueness of the solution. We conclude our considerations through formulation and demonstration of the principle of minimum potential energy which is a generalization of this principle from classical elasticity.

2. PRELIMINARIES

Suppose that our elastic Cosserat solid with voids occupies at time $t = 0$ a properly regular region B of the three-dimensional Euclidean space R^3. As usual, the boundary of the domain B, denoted by ∂B, is assumed to be a closed and bounded surface and it is sufficiently smooth to admit the application of the divergence theorem. The components of the outward unit normal to the surface ∂B will be denoted by n_i. Let us denote by $\bar{B}$ the closure of the domain B, which means $\bar{B} = B \cup \partial B$.

In the following we refer the evolution of the continuum to a fixed system of rectangular Cartesian axes Ox_i $(i = 1, 2, 3)$ and adopt the Cartesian tensor notation. Points in B are denoted by x_i and $t \in (0, \infty)$ is the temporal variable.

The Italic indices will always have the values $1, 2, 3$, whereas the Greek indices will range over the values $1, 2$. As usual, a superposed dot stands for the material derivative, while a comma followed by a subscript denotes partial differentiation with respect to the respective Cartesian coordinate. Einstein convention regarding the summation over repeated indices is also used. By convention, the spatial argument and the time argument of a function will be omitted when there is no likelihood of confusion.

The notations and the terminology chosen are similar to those of the paper [5]. In all that follows, we consider an anisotropic and homogeneous Cosserat elastic solid with pores.

The motion of this material will be described by the following independent variables:

- $u_i = u_i(x,t)$ - the components of the displacement fields from the reference configuration;
- $\varphi_i = \varphi_i(x,t)$ - the components of the microrotation fields from the reference configuration;
- $\phi = \phi(x,t)$ - a scalar function that characterizes the voids. It is called the change in volume fraction.

With the aid of a procedure similar to that used by Eringen in [3], we obtain the following equations and conditions for the time independent behavior of an elastic material (see also [6] and [7]):

- the equations of equilibrium

$$t_{ij,j} + F_i = 0, m_{ij,j} + \varepsilon_{ijk} t_{jk} + G_i = 0; \quad (1)$$

- the balance of the equilibrated forces:

$$\lambda_{k,k} - g + H = 0; \quad (2)$$

- the constitutive equations:

$$t_{ij} = A_{ijmn}\varepsilon_{mn} + B_{ijmn}\mu_{mn} + D_{ijm}\gamma_m + a_{ij}\phi,$$

$$m_{ij} = B_{mnij}\varepsilon_{mn} + C_{ijmn}\mu_{mn} + E_{ijm}\gamma_m + b_{ij}\phi,$$

$$\lambda_i = D_{mni}\varepsilon_{mn} + E_{mni}\mu_{mn} + g_{im}\gamma_m + d_i\phi, \quad (3)$$

$$g = a_{mn}\varepsilon_{mn} + b_{mn}\mu_{mn} + d_m\gamma_m + m\phi.$$

Here $\varepsilon_{ij}, \mu_{ij}$ and γ_i are measures of deformation and are defined by the following geometric equations:

$$\varepsilon_{ij} = u_{j,i} + \varepsilon_{ijk}\varphi_k, \mu_{ij} = \varphi_{j,i}, \gamma_i = \phi_{,i}. \quad (4)$$

All the above equations are considered on the domain B and we used the following notations:

- t_{ij}, m_{ij} - the components of the stress tensors;
- λ_i - the components of the microstress vector;
- F_i - the components of the body force per unit mass;
- G_i - the components of the body couple per unit mass;
- H - the extrinsic equilibrated body forces;
- g - the intrinsic equilibrated forces;
- λ_i - the components of the equilibrated microstress;
- $\varepsilon_{ij}, \mu_{ij}, \gamma_i$ - the components of the kinematic characteristics of the strain tensors.

Finally, the tensors $A_{ijmn}, B_{ijmn}, \ldots, D_{ijm}, E_{ijm}, \ldots, a_{ij}, b_{ij}$, the vector of components d_i and the scalar coefficient m represent the characteristic functions of the material, that is, the constitutive coefficients and they obey the following symmetry relations:

$$A_{ijmn} = A_{mnij}, C_{ijmn} = C_{mnij}, g_{im} = G_{mi}. \quad (5)$$

For the sake of systematization, we will impose from the beginning the following regularity hypotheses on the considered functions:

- all the constitutive coefficients are functions of class C^2 on B;
- the body loads F_i, G_i and H are continuous functions on B.

We say that the ordered array (u_i, φ_i, ϕ) is an admissible process on $\bar{B} = B \cup \partial B$ provided that $u_i, \varphi_i, \phi \in C^1(\bar{B}) \cap C^2(B)$. Also, the ordered array of functions $(t_{ij}, m_{ij}, \lambda_i)$ is an admissible system of stress on $\bar{B}$ if $t_{ij}, m_{ij}, \lambda_i \in C^1(B) \cap C^0(\bar{B})$ and $t_{ij,i}, m_{ij,i}, \lambda_{k,k} \in C^0(\bar{B})$.

Finally, we assume that the components of the strain $\varepsilon_{ij}, \mu_{ij}, \gamma_i \in C^1(B) \cap C^0(\bar{B})$. To the system of field equations (1) - (4) we adjoin the following boundary conditions:

$$u_i = \tilde{u}_i \text{ on } \partial B_1, \varphi_i = \tilde{\varphi}_i \text{ on } \partial B_2, \phi = \tilde{\phi} \text{ on } \partial B_3,$$

$$t_{ij}n_j = \tilde{t}_i \text{ on } \partial B_1^c, m_{ij}n_j = \tilde{m}_i \text{ on } \partial B_2^c, \lambda_i n_i = \tilde{h} \text{ on } \partial B_3^c, \quad (6)$$

Also, the surfaces $\partial B_1, \partial B_2$ and ∂B_3 with the respective complements $\partial B_1^c, \partial B_2^c$ and ∂B_3^c are subsets of the boundary ∂B such that:

$$\partial B_1 \cup \partial B_1^c = \partial B_2 \cup \partial B_2^c = \partial B_3 \cup \partial B_3^c = \partial B$$

$$\partial B_1 \cap \partial B_1^c = \partial B_2 \cap \partial B_2^c = \partial B_3 \cap \partial B_3^c = \emptyset.$$

An ordered array $L = (F_i, G_i, H, \tilde{u}_i, \tilde{\varphi}_i, \tilde{t}_i, \tilde{m}_i, \tilde{h})$ is an external data system on $\bar{B}$ if the following conditions are met

- the functions $F_i, G_i, H, \tilde{u}_i, \tilde{\varphi}_i$ and $\tilde{\phi}$ are continuous on their domains;
- the functions $\tilde{t}_i, \tilde{m}_i$ and $\tilde{h}$ are piecewise regular on their domains.

In conclusion, we say that the boundary value problem of the equilibrium theory for elastic Cosserat bodies with voids consists in finding the functions (u_i, φ_i, ϕ) that satisfy the equations (1) - (4) and the boundary conditions (6). Of course, the solution (u_i, φ_i, ϕ) corresponds to the loads (F_i, G_i, H).

3. MAIN RESULTS

In order to obtain first our main result, we shall consider that our Cosserat elastic body with voids is under the action of two external data systems:

$$\mathcal{L}^{(\alpha)} = (F_i^{(\alpha)}, G_i^{(\alpha)}, H^{(\alpha)}, \tilde{u}_i^{(\alpha)}, \tilde{\varphi}_i^{(\alpha)}, \tilde{\phi}^{(\alpha)}, \tilde{t}_i^{(\alpha)}, \widetilde{m}_i^{(\alpha)}, \tilde{h}^{(\alpha)}), \alpha = 1,2.$$

These external loads will bring out two corresponding elastic states, that is:

$$\mathcal{A}^{(\alpha)} = \left(u_i^{(\alpha)}, \varphi_i^{(\alpha)}, \phi^{(\alpha)}, \varepsilon_{ij}^{(\alpha)}, \mu_{ij}^{(\alpha)}, \gamma_i^{(\alpha)}, t_{ij}^{(\alpha)}, m_{ij}^{(\alpha)}, \lambda_i^{(\alpha)}, g^{(\alpha)}\right), \alpha = 1,2.$$

The next theorem is a theorem of Betti's type and is a counterpart of the reciprocity theorem of classical elasticity.

Theorem 1. *Let $\mathcal{A}^{(\alpha)}$ be two elastic states $(\alpha = 1,2)$ corresponding to two external data systems □$^{(\alpha)}$. Then the next equality takes place:*

$$\int_B \left(F_i^{(1)} u_i^{(2)} + G_i^{(1)} \varphi_i^{(2)} + H^{(1)} \phi^{(2)}\right) dV + \int_{\partial B} \left(t_i^{(1)} u_i^{(2)} + m_i^{(1)} \varphi_i^{(2)} + h^{(1)} \phi^{(2)}\right) dA \quad \blacksquare$$
$$= \int_B \left(F_i^{(2)} u_i^{(1)} + G_i^{(2)} \varphi_i^{(1)} + H^{(2)} \phi^{(1)}\right) dV + \int_{\partial B} \left(t_i^{(2)} u_i^{(1)} + m_i^{(2)} \varphi_i^{(1)} + h^{(2)} \phi^{(1)}\right) dA, \quad (7)$$

where:

$$t_i^{(\alpha)} = t_{ji}^{(\alpha)} n_j, m_i^{(\alpha)} = m_{ij}^{(\alpha)} n_j, h^{(\alpha)} = \lambda_i^{(\alpha)} n_i, \alpha = 1,2. \quad (8)$$

Proof. For beginners, we will introduce the notations

$$2I_{\alpha\beta} = t_{ji}^{(\alpha)} \varepsilon_{ij}^{(\beta)} + m_{ij}^{(\alpha)} \mu_{ij}^{(\beta)} + \lambda_i^{(\alpha)} \gamma_i^{(\beta)}. \quad (9)$$

Taking into account the constitutive equations (3), we can write $I_{\alpha\beta}$ in the following form:

$$2I_{\alpha\beta} = A_{ijmn} \varepsilon_{ij}^{(\alpha)} \varepsilon_{mn}^{(\beta)} + B_{ijmn} \left(\mu_{ij}^{(\alpha)} \varepsilon_{mn}^{(\beta)} + \mu_{ij}^{(\beta)} \varepsilon_{mn}^{(\alpha)}\right) + C_{ijmn} \mu_{ij}^{(\alpha)} \mu_{mn}^{(\beta)} + D_{ijm} \left(\gamma_m^{(\alpha)} \varepsilon_{ij}^{(\beta)} + \gamma_m^{(\beta)} \varepsilon_{ij}^{(\alpha)}\right) + E_{ijm} \left(\gamma_m^{(\alpha)} \mu_{ij}^{(\beta)} + \gamma_m^{(\beta)} \mu_{ij}^{(\alpha)}\right) + g_{im} \gamma_i^{(\alpha)} \gamma_m^{(\beta)} + a_{ij} \left(\phi^{(\alpha)} \varepsilon_{ij}^{(\beta)} + \phi^{(\beta)} \varepsilon_{ij}^{(\alpha)}\right) + b_{ij} \left(\phi^{(\alpha)} \mu_{ij}^{(\beta)} + \phi^{(\beta)} \mu_{ij}^{(\alpha)}\right) + d_i \left(\phi^{(\alpha)} \gamma_i^{(\beta)} + \phi^{(\beta)} \gamma_i^{(\alpha)}\right) + m \phi^{(\alpha)} \phi^{(\beta)}. \quad (10)$$

By using the symmetry relations (5), from (10) we get:

$$I_{\alpha\beta} = I_{\beta\alpha}. \quad (11)$$

By introducing the geometrical equations (4) into the equations of equilibrium (1) and into the balance of the equilibrated forces (2), from (10), we can write:

$$2I_{\alpha\beta} = F_i^{(\alpha)} u_i^{(\beta)} + G_i^{(\alpha)} \varphi_i^{(\beta)} + H^{(\alpha)} \phi^{(\beta)} + [t_{ji}^{(\alpha)} u_j^{(\beta)} + m_{ij}^{(\alpha)} \varphi_{ij}^{(\beta)} + \lambda_i^{(\alpha)} \phi^{(\beta)}]_{,i}. \quad (12)$$

We shall integrate in (12) so that if we use the divergence theorem, it is easy to obtain the relation:

$$2 \int_B I_{\alpha\beta} dV = \int_B \left(F_i^{(\alpha)} u_i^{(\beta)} + G_i^{(\alpha)} \varphi_i^{(\beta)} + H^{(\alpha)} \phi^{(\beta)}\right) dV + \int_{\partial B} \left(t_i^{(\alpha)} u_i^{(\beta)} + m_i^{(\alpha)} \varphi_i^{(\beta)} + h^{(\alpha)} \phi^{(\beta)}\right) dA. \quad (13)$$

Taking into account (11), from (13) we obtain the desired reciprocity relation (7) and the proof of theorem is concluded.

Now, we intend to obtain an existence result for the solution of our boundary value problem. To this end, we first give another formulation of the field equations. Consider that $u = (u_i, \varphi_i, \phi)$ is an admissible process on B and $(\varepsilon_{ij}(u), \mu_{ij}(u), \gamma_i(u))$ are the characteristics of the strain associated with u.

In addition, we will consider that $(t_{ij}(u), m_{ij}(u), \lambda_i(u), g(u)$ is the admissible system of stresses on $\bar{B}$ corresponding to u. Let us introduce the operators L_i, defined on the domain $\bar{B}$ by:

$L_i(u) = -t_{ij,i}(u), L_{3+i}u = -m_{ij,j}(u) - \varepsilon_{ijk}t_{jk}(u), L_7u = -\lambda_{i,i}(u) + g(u).$ (14)

If we use the tensor notations:

$Lu = (L_iu, L_{3+i}u, L_7u), f = (F_i, G_i, H)$ (15)

then the equations of equilibrium can be rewritten in the following form

$Lu = f.$ (16)

Let us denote by M the set of all admissible processes on B. The loads at regular points of ∂B corresponding to the admissible process $u \in M$ are given by:

$t_i(u) = t_{ij}(u)n_j, m_i(u) = m_{ij}(u)n_j, h(u) = \lambda_i(u)n_i.$ ■ (17)

Considering the procedure of Eringen from [3] or Ieșan and Quintanilla from [5], we obtain that the internal energy density corresponding to $u \in M$ is given by:

$e(u) = \frac{1}{2}A_{ijmn}\varepsilon_{ij}(u)\varepsilon_{mn}(u) + B_{mnij}\varepsilon_{mn}(u)\mu_{ij}(u) + \frac{1}{2}C_{ijmn}\mu_{ij}(u)\mu_{mn}(u) + D_{ijm}\varepsilon_{ij}(u)\gamma_m(u) + E_{ijm}\mu_{ij}(u)\gamma_m(u) + \frac{1}{2}g_{im}\gamma_i(u)\gamma_m(u) + a_{ij}\varepsilon_{ij}(u)\varphi + b_{ij}\kappa_{ij}(u)\varphi + d_i\gamma_i(u)\varphi + \frac{1}{2}m\varphi^2.$ (18)

As such, the strain energy $\mathcal{E}(u)$ corresponding to $u \in M$ will be defined as follows

$\mathcal{E}(u) = \int_B e(u)dV.$ (19)

The functional $\mathcal{E}$ generates the following bilinear form:

$\mathcal{E}(u,v) = \frac{1}{2}\int_B [\![\{A_{ijmn}\varepsilon_{ij}(u)\varepsilon_{mn}(v)]\!] + B_{mnij}[\varepsilon_{mn}(u)\mu_{ij}(v) + \varepsilon_{mn}(v)\mu_{ij}(u)] + C_{ijmn}\mu_{ij}(u)\mu_{mn}(v) + D_{ijm}[\varepsilon_{ij}(u)\gamma_m(v) + \varepsilon_{ij}(v)\gamma_m(u)] + E_{ijm}[\mu_{ij}(u)\gamma_m(v) + \mu_{ij}(v)\gamma_m(u)] + g_{im}\gamma_i(u)\gamma_m(v) + a_{ij}[\varepsilon_{ij}(u)\varphi + \varepsilon_{ij}(v)\psi] + b_{ij}[\mu_{ij}(u)\varphi + \mu_{ij}(v)\psi] + d_i[\gamma_i(u)\varphi + \gamma_i(v)\psi] + m\varphi\psi\}dV$

(20)

Where $u, v \in M, u = (u_i, \varphi_i, \phi), v = (v_i, \psi_i, \Psi)$.

In view of the uniqueness result, we will prove now an auxiliary result.

Theorem 2. *The bilinear form $\mathcal{E}(u,v)$ can be written in the form:*

$2E(u,v) = \int_B (v_iL_iu + \psi_iL_{3+i}u + \Psi L_7u)dV + \int_{\partial B}[v_it_i(u) + \psi_im_i(u) + \Psi h(u)]dA, \forall u, v \in M.$ (21)

Proof. We start by using relation (10) where we take:

$$\left(u_i^{(1)}, \varphi_i^{(1)}, \phi^{(1)}\right) \to (u_i, \varphi_i, \phi)$$
$$\left(u_i^{(2)}, \varphi_i^{(2)}, \phi^{(2)}\right) \to (v_i, \psi_i, \Psi)$$

and then the bilinear functional $\mathcal{E}(u,v)$ from (20) receives the form

$\mathcal{E}(u,v) = \int_B I_{12}dV.$ (22)

Finally, if we use relation (12), from (19) we are led to the desired result (21) and Theorem 2 is concluded.

Relation (21) is the basis of the following uniqueness result. This is a counterpart of the uniqueness result which was established by Eringen in the case of isotropic micromorphic bodies.

Theorem 3. *Any two solutions of the boundary value problem in the context of Cosserat bodies with voids, are equal up to a rigid displacement, provided the internal energy density is a positive definite form. Moreover, if ∂B_1 and ∂B_2 are both non-empty, then the above mentioned problem has at most one solution.*

Proof. For any $u \in M$, from (19) and (21) we obtain:

$2\int_B \sigma(u)dV = \int_B (u_iL_iu + \varphi_iL_{3+i}u + \phi L_7u)dV + \int_{\partial B}[u_it_i(u) + \varphi_im_i(u) + \phi h(u)]dA.$ (23)

We will denote by u' and u'' two solutions of our boundary value problem and consider

$$u^0 = u' - u'', u^0 = (u_i^0, \varphi_i^0, \phi^0).$$

Based on the linearity of our problem, we have $u^0 \in M$ and $Lu^0 = 0$ on B. Moreover, we have

$u_i^0t_i(u^0) + \varphi_i^0m_i(u^0) + \phi^0h(u^0) = 0 \; on \; \partial B.$

Above we assumed that $e(u^0)$ is positive definite such that taking into account expression (18) of e, from (23) we deduce that

$\varepsilon_{ij}(u^0) = 0, \mu_{ij}(u^0) = 0, \phi^0 = 0.$ (24)

Now, we take into account the geometrical equations (4). First from (4)2 we deduce that

$$\varphi^0_{j,i} = 0 \Longrightarrow \varphi^0_j = c_j = constants,$$

and then, in view of (4)1, we have:

$u^0_{j,i} + \varepsilon_{ijk}\varphi^0_k = 0 \Longrightarrow u^0_{j,i} = b_{ij} = constants \Longrightarrow$
$u^0_i = b_{ij}x_j + a_i, a_i = constants.$

Finally, if ∂B_1 and ∂B_2 are both non-empty, then $a_i = 0$ and $b_{ij} = 0$, that is $u^0 = 0$, such that $u' = u''$ and the proof is completed.

At the end of our study, we will formulate the principle of minimum potential energy for elastostatics of Cosserat bodies with voids as a counterpart of the similar principle of the classical elastostatics. To this end, we will denote by K the set of all admissible processes $u = (u_i, \varphi_i, \phi)$ on $\overline{B}$ that satisfy the boundary conditions:

$$u_i = \tilde{u}_i \ on\ \partial B_1, \varphi_i = \tilde{\varphi}_i \ on\ \partial B_2, \phi = \tilde{\phi} \ on\ \partial B_3.$$

On the set K we define the functional Λ by

$$\Lambda(u) = E(u) - \int_B (F_i u_i + G_i\varphi_i + H\phi)dV - \int_{\partial B_1^c} \tilde{t}_i u_i dA - \int_{\partial B_2^c} \tilde{m}_i\varphi_i dA - \int_{\partial B_3^c} \tilde{h}\phi dA \quad (25)$$

for every $u = (u_i, \varphi_i, \phi) \in K$.

This is necessary for the next results regarding the minimum principle.

Theorem 4. *Suppose that the internal energy density (18) is positive definite. If we denote by* u^* *a solution for the boundary value problem in our context, then the next estimate takesplace*

$$\Lambda(u^*) \leq \Lambda(w) \quad (26)$$

for every $w \in K$ *and the equality holds only if* u^* *is equal to* w *up to a rigid displacement.*

Proof. Since u^* is a solution for the above boundary value problem, then we have $u^* = (u^*_i, \varphi^*_i, \phi^*), u^* \in K$. Also, $w = (w_i, \psi_i, \Psi^*), w \in K$. We define $u'_i = w_i - u^*_i, \varphi'_i = \psi_i - \varphi^*_i, \phi' = \Psi - \phi^*$, that is,

$$u' = (u'_i, \varphi'_i, \phi'), u' = w - u^*.$$

Based on the linearity of the problem, we deduce that $u' \in M$. Moreover,

$$u'_i = 0 \ on\ \partial B_1, \varphi'_i = 0 \ on\ \partial B_2, \phi' = 0 \ on\ \partial B_3. \quad (27)$$

Taking into account relations (25), (21) and (27), we deduce that:

$$\Lambda(w) = \Lambda(u^*) + E(u') + \int_B \left[(L_i u^* - F_i)u'_i + (L_{3+i}u^* - G_i)\varphi'_i + (L_7 u^* - H)\phi'\right]dV + \int_{\partial B_1^c}[t_i(u^*) - \tilde{t}_i]u'_i dA + \int_{\partial B_2^c}[m_i(u^*) - \tilde{m}_i]\varphi'_i dA + \int_{\partial B_3^c}[h(u^*) - \tilde{h}]\phi' dA. \quad (28)$$

But, since u^* is a solution of the boundary value problem, we deduce that all previous integrals are zero, so that relation (28) becomes

$$\blacksquare\ \Lambda(w) = \Lambda(u^*) + E(u').$$

By hypothesis, the internal energy density is positive definite so that the last equality becomes

$$\Lambda(w) \geq \Lambda(u^*)$$

that is, we obtain the desired inequality (26), that concludes the proof of Theorem 4.

Remark. It is easy to observe that the minimum principle demonstrated in Theorem 4 is a generalization of the minimum principle proved by Wilkes in [10].

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O generalizare a principiului de minim al energiei pentru materialele Cosserat poroase

Studiul nostrum este o generalizare a principiului de minim pentru energie obtinut de Iesan si Quintanilla pentru mediile elastice cu microstructura. Prin aceasta extensie ne-am propus sa acoperim mediile Cosserat cu goluri (pori). In acest context nou noi am formulat problema cu date pe frontiera si am demonstrat printr-o metoda accesibila un rezultat de unicitate a solutiei problemei formulate. In rezultatul nostru principal am demonstrat extensia principiului de minim.

Marin MARIN, Prof. PhD, Transylvania University of Brașov, Department of Mathematics and Computer Science, m.marin@unitbv.ro

Sorin VLASE, Prof. PhD, Transylvania University of Brașov, Department of Mechanical Engineering, svlase@unitbv.ro, Tel. +40 268 413000 / 168, Str. Politehnicii 1, Brașov 500024, Romania

Lavinia CODARCEA-MUNTEANU, Prof. PhD, Transylvania University of Brașov, Department of Mathematics and Computer Science, lavinia.codarcea.munteanu@unitbv.ro

Adina CHIRILĂ, Prof. PhD, Transylvania University of Brașov, Department of Mathematics and Computer Science, adina.chirila@unitbv.ro