TECHNICAL UNIVERSITY OF CLUJ-NAPOCA

ACTA TECHNICA NAPOCENSIS

Series: Applied Mathematics, Mechanics, and Engineering
Vol. 60, Issue IV, November, 2017

STRAIN AMPLITUDE DEPENDENT INTERNAL FRICTION AND THE YOUNG'S MODULUS DEFECT IN DAMAGED SOLIDS

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Abstract: The paper investigates the nonlinear effects in the propagation of acoustic waves in damaged materials having the property that the distance between grains is comparable with the Burgers vector magnitude. The main ideea is to unify the explanation at the atomic level of the following properties of the damaged material:

- dependence of the strain amplitude on the internal friction (ADIF) and the Young's modulus defect (YMD);
- the behavior of ADIF over the range of low, moderate and high strain amplitudes, in terms on temperature and frequency;
- the correlation between ADIF and YMD via the dislocation motion.

Key words: Damaged material, Burger vector, strain amplitude dependent internal friction, Young's modulus defect;.

1. INTRODUCTION

Interaction of dislocations with defects are traditionally considered as basic to the ADIF and YMD [1]. The experimental investigations of ADIF are less well understood or analysed in the literature. For example, the origin of low strain amplitude internal friction background, existence of several stages in ADIF and YMD at different temperatures and in different frequency ranges in a two component array of point obstacles, are not very well understand and analysed [2, 3].

We consider a damaged material asociated with mesoscopic defects, such as dislocations, voids, and cracks. While dislocations and their mutual interactions determine the material strength in the absence of other internal defects, they tend to self-organize in the certain form, resulting into a heterogeneous field of strain at the microscale although the overall macroscopic field is thought to be homogeneous. The main difficulty in modeling the nonlinearity lies in the fact that the length scale of these phenomena is not large enough to treat them within the classical continuum mechanics framework not small enough to view these phenomena within the mechanics of dislocations [4-7].

In this paper, both thermally activated and athermal contributions to dislocation mesoplasticity in damaged materials are investigated. The theory is based of dislocation motion in a two component array of point defects. The change of the strength of one type of defect with temperature anf frequency lead to a change of the dislocation segment lengh.

2. BASIC THEORY

The effect of internal defects, such as dislocations, voids, microcracks, etc., on the material behavior and the manner they influence material properties is modeled through a set of internal variables and corresponding phenomenological evolution equations. The material response is measured in terms the macroscopic strain rate tensor D and its relation to the stress tensor σ.

On the macroscale level, it is assumed that the material obeys the basic laws of continuum mechanics, i.e. the linear momentum balance:

$$\mathrm{div}\sigma = \rho \dot{v}\,, \tag{1}$$

and the coupled heat equation

$$\rho c_v \dot{T} = k\nabla^2 T + \sigma D^p\,, \tag{2}$$

where $v = \dot{u}$ is the particle velocity, u, ρ, c_v and k are the displacement vector field, mass density, specific heat and thermal conductivity respectively, and D^p the plastic part of the rate of strain tensor D which is defined as the symmetric part of the velocity gradient

$$D_{ij} = \frac{1}{2}(v_{i,j} + v_{j,i})\,. \tag{3}$$

For elasto-viscoplastic behavior, D is decomposed into an elastic part D^e and a plastic part D^p such that

$$D = D^e + D^p\,. \tag{4}$$

The associated spin W is defined as the antisymmetric part of the velocity gradient, analogous to the vorticity in fluids

$$W_{ij} = \frac{1}{2}(v_{i,j} - v_{j,i})\,. \tag{5}$$

For elasto-viscoplastic behavior, W is decomposed also into an elastic part W^e and a plastic part W^p such that

$$W = W^e + W^p\,. \tag{6}$$

It is usufull to write also the strain field $\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})$ as a sum between an elastic part ε^e and a plastic part ε^p. For most damaged materials the elastic response is linear and can be expressed by the incremental form of Hooke's law for large strain and material rotation such that

$$\tilde{\sigma} = C \cdot D^e - \gamma \Delta T\,, \tag{7}$$

where γ is the temperature coefficient of stress at constant strain, $\tilde{\sigma}$ is an objective rate of the Cauchy stress tensor σ, and C a fourth-order elastic constants tensor. There are several choises for $\tilde{\sigma}$. The most common one is the rotational rate of Cauchy stress which is referred to as the Jaumann stress-rate tensor [7]

$$\tilde{\sigma} = \dot{\sigma} + \omega \cdot \sigma - \sigma \cdot \omega\,, \tag{8}$$

where ω is the spin of the cell and is given as the difference between the material spin W and plastic spin W^p.

$$\omega = W - W^p\,. \tag{9}$$

Combining (7) with (4) leads to

$$\tilde{\sigma} = C(D^e - D^p) - \gamma \Delta T\,. \tag{10}$$

We need the constitutive laws for D^p and W^p. The macroplastic strain and the plastic spin are produced by the dislocation motion. These laws should be based on the underlying microstructure, mainly dislocations, i.e. bridging the continuum scale and the discrete dislocation scale.

Dislocations are discretized into pure screw and edge segments of length restricted to be integer multiples of a minimum size. The analysis is based on explicit evaluation of dislocation motion and its interaction with defects. Curved dislocations interact among themselves and with any applied stresses. The stress induced by an arbitrary dislocation loop at an arbitrary field point $P(x)$ can be computed by the Peach-Koehler equation given by the following line integral [8]

$$\begin{aligned} \sigma_{\alpha\beta}(P) = & -\frac{\mu}{8\pi}\int_C b_m \varepsilon_{im\alpha} \frac{\partial}{\partial x'_i} \nabla'^2 R \,\mathrm{d}x'_\beta - \\ & -\frac{\mu}{8\pi}\int_C b_m \varepsilon_{im\beta} \frac{\partial}{\partial x'_i} \nabla'^2 R \,\mathrm{d}x'_\alpha - \\ & -\frac{\mu}{4\pi(1-\nu)}\int_C b_m \varepsilon_{imk} \left(\frac{\partial^3 R}{\partial x'_i \partial x'_\alpha \partial x'_\beta} \right. \\ & \left. -\delta_{\alpha\beta} \frac{\partial}{\partial x'_i} \nabla'^2 R \right) \mathrm{d}x'_k \end{aligned} \tag{11}$$

where b_i is the i-th component of the Burgers vector, $\varepsilon_{im\alpha}$ is the permutation symbol, μ is the shear modulus, ν is Poisson's ratio and $R = \| x - x' \|$ is the radius vector norm between the loop and the point P. This integral can be evaluated numerically by meshing the curve into set of nodal points and perform piecewise integration so that we have

$$\sigma_{\alpha\beta}(P)=\sum_{loops}\sum_{j=1}^{m-1}[-\frac{\mu}{8\pi}\int_{j}^{j+1}b_k\varepsilon_{ik\alpha}\frac{\partial}{\partial x_i'}\nabla'^2 R\mathrm{d}x_\beta' -$$
$$-\frac{\mu}{8\pi}\int_{j}^{j+1}b_k\varepsilon_{ik\beta}\frac{\partial}{\partial x_i'}\nabla'^2 R\mathrm{d}x_\alpha' - \tag{12}$$
$$-\frac{\mu}{4\pi(1-\nu)}\int_{j}^{j+1}b_k\varepsilon_{ikl}\Big(\frac{\partial^3 R}{\partial x_i'\partial x_\alpha'\partial x_\beta'}$$
$$-\delta_{\alpha\beta}\frac{\partial}{\partial x_i'}\nabla'^2 R\Big)\mathrm{d}x_l']$$

where m is the total number of nodal points in a given loop or curve. Eq. (12) reduces to the approximate form

$$\sigma(P)=\sum_{j=1}^{N}\sigma_{j,j+1}\ , \tag{13}$$

where N is the total number of nodes from all loops and curves and $\sigma_{j,j+1}=\sigma_{j+1}-\sigma_j$.

In order to compute the dynamics of the dislocation curve we evaluate the stress distribution and the driving force and velocity distribution along the entire curve. Using (13), the Peach-Kohler force can be computed directly on node i

$$F_i=\sum_{j=1}^{N-1}[\sigma_{j,j+1}(P)+\sigma^a(P)]b_i\times\xi_i\ , \tag{14}$$

where b_i is the Burgers vector and ξ_i is the line sense. The governing equation of slide motion for each dislocation node combined with (14) leads to

$$m_i\dot{v}_i+\frac{1}{M}\dot{v}_i=F_i\ ,\ i=1,2,...N\ , \tag{15}$$

where v_i are the slip velocities, m *is* the effective mass per unit dislocation length and M is the dislocation mobility, F_i are given by (14). For the effective mass m we use the following expressions derived in [8, 9] for the screw dislocation

$$m=\frac{W_0}{v^2}(-\frac{1}{\gamma}+\frac{1}{\gamma^3})\ , \tag{16}$$

for the screw dislocation, and for the edge dislocation

$$m=\frac{W_0 v_P^2}{v^4}\Big(-16\gamma_l-\frac{40}{\gamma_l}+\frac{8}{\gamma_l^3}+$$
$$14\gamma+\frac{50}{\gamma}-\frac{22}{\gamma^3}+\frac{6}{\gamma^5}\Big) \tag{16}$$

where $\gamma_l=\sqrt{1-\frac{v^2}{v_P^2}}$, $\gamma=\sqrt{1-\frac{v^2}{v_S^2}}$, v_P is the longitudinal sound velocity and v_S the transverse sound velociy, and W_0 is the rest energy for the screw.

The set of equations (15) are coupled and highly nonlinear. They were solved simultaneously using the finite difference method which turns out to be stable provided that the integration time step is small enough, which is the case here as dictated by the short-range interaction.

The motion of each dislocation segment contributes to the overall macroscopic plastic strain D^p and plastic spin W^p via the relations

$$D^p=\sum_{i=1}^{N}\frac{l_i\hat{v}_i}{2V}(n_ib_i+b_in_i)\ , \tag{17}$$

$$W^p=\sum_{i=1}^{N}\frac{l_i\hat{v}_i}{2V}(n_ib_i-b_in_i)\ , \tag{18}$$

$$\dot{n}=\omega n\ ,$$

where l_i is the segment length, n_i is a unit normal to the slip plane, $\hat{v}_i$ is the magnitude of the slip velocity field solutions of the equations (15), V is the volume of the cell, and N is the total number of segments. The relations (17) and (18) provide the most rigorous connection between the dislocation motion and the macroscopic plastic strain, with its dependence on strength and applied stress being explicitly embedded in the calculation of the velocity of each dislocation.

3. RESULTS AND DISCUSSION

The visco-plastic model described in this paper give rise to energy dissipation or internal friction, as a loss of energy. The energy dissipated per cycle, $\Delta\tilde{E}$ is calculated from

$\Delta\tilde{E} = \int_{o}^{2\pi/\omega} \sigma D \mathrm{d}t$. The energy loss is expressed in terms of a loss factor Q^{-1}

$$Q^{-1} = \frac{1}{2\pi}\frac{\Delta\tilde{E}}{\tilde{E}} = \sin\delta_0 \ . \qquad (19)$$

Writing the constitutive law in term of the complex modulus $\sigma = M^{*}\varepsilon$ we can calculate the phase angle δ_0 and the relaxation time τ.

The relaxation time depends on temperature according to $\tau = \exp\frac{U}{kT}$, where U is the activation energy. The frequency at which maximum attenuation occurs is also dependent $\tilde{\omega} = \tilde{\omega}_0 \exp(-\frac{U}{kT})$, where $\tilde{\omega}_0$ and $\tilde{\omega}$ are the resonant and applied angular frequencies.

The decrement δ is defined as the ratio of the energy loss per cycle, $\Delta\tilde{E}$, to twice the total vibrational energy $\tilde{E}_0$, stored in the structure, that is $\delta = \frac{\Delta\tilde{E}}{2\tilde{E}_0}$. We define the Young's modulus defect as

$$\frac{\Delta E}{E} = \frac{E - E_0}{E_0} = \frac{\varepsilon_{11}^{p}}{\varepsilon_{11}^{e}} \ , \qquad (20)$$

where E is the instantaneous value of Young' modulus and E_0 is its value at amplitudes less than critical value of exceeded σ_{cr}, ε_{11}^{p} is the plastic strain amplitude, and ε_{11}^{e} the elastic strain amplitude in the direction x.

The critical stress σ_{cr} is frequency and temperature dependent, and for the stationary thermally activated dislocation motion over an array of short-range obstacles and is given by [9]

$$\sigma_{cr} = \frac{F_0}{Lb}[1 - \frac{kT}{U}\ln\frac{\tilde{\omega}_0}{\tilde{\omega}}]^{\alpha}, \qquad (21)$$

where U_0 and F_0 are the maximum values of the activation energy and interaction force between a dislocation and a short-range obstacle, L and b are the average dislocation segment length and the magnitude of the Burgers vector, and $1 < \alpha < 2$ is a constant [7].

In the following we consider 60 joints. We analyze the dependence of the decrement δ on the strain amplitude ε_{11}. Fig. 1 shows the strain amplitude-dependent decrement at the low temperature 10 K and high frequency 100 kHz. The curve increases and decreases as indicated by arrows. At low temperature, the IF shows no amplitude-independent part. The dependence is moderate at the lowest strain amplitudes $\approx 10^{-6}$ and strong at the moderate and high strain amplitudes. At low strains dislocations overcome only weak obstacles. The strong obstacles are not penetrable at low temperature but can be overcome at higher strain amplitudes.

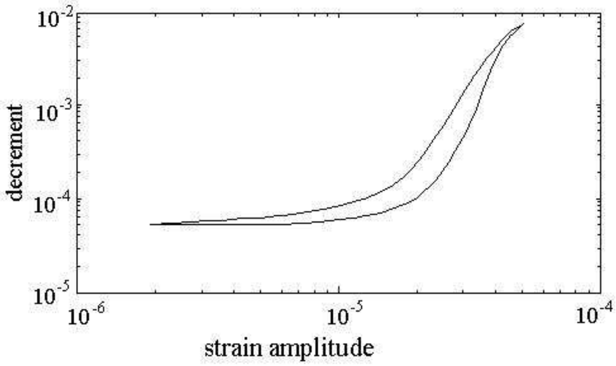

Fig.1. Strain amplitude ε_{11} dependence of the decrement δ at low temperature 10 K and high frequency 100 kHz.

In Fig. 2 the strain amplitude-dependent decrement on temperature for lowest frequency $\tilde{\omega}_0$=7 kHz I shown. This is the resonant frequency at the lowest strain amplitude. We observe that the slope is likely to be the same in the range 10-300 K at the lowest strain amplitudes $\approx 10^{-6}$. Deviation from this regularity is observed at the strain amplitude of $\approx 10^{-5}$ at elevated temperatures 100-300 K.

This is explained by the fact that the decrement for lowest resonant frequency $\tilde{\omega}_0$= 7 kHz may be written as a sum of two components: 1) one dependent on the strain amplitude and independent of temperature, which is responsible for the constant slope at different temperatures and, 2) one amplitude-dependent and temperature-dependent which is responsible for the deviations.

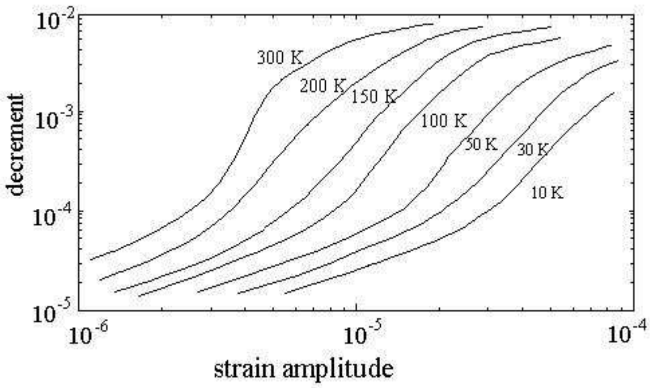

Fig. 2. Strain amplitude dependence of the decrement at different temperatures for lowest resonant frequency f_0 =7 kHz.

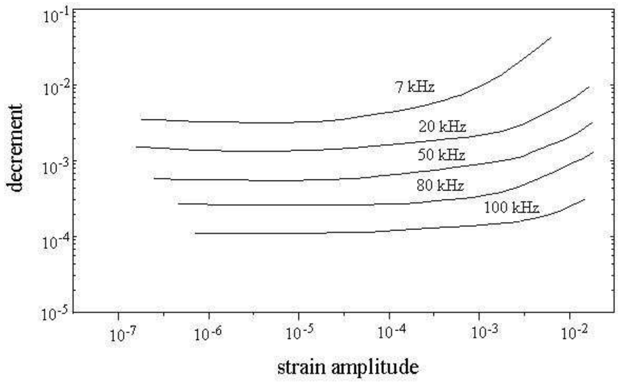

Fig. 3. Strain amplitude ε_{11} dependence of the decrement δ at 10 K for different frequencies.

Fig. 3 shows the influence of frequency on the strain amplitude ε_{11} dependence of the decrement δ at 10 K. The decrement is higher at a frequency around 7 kHz than in the 100 kHz range.

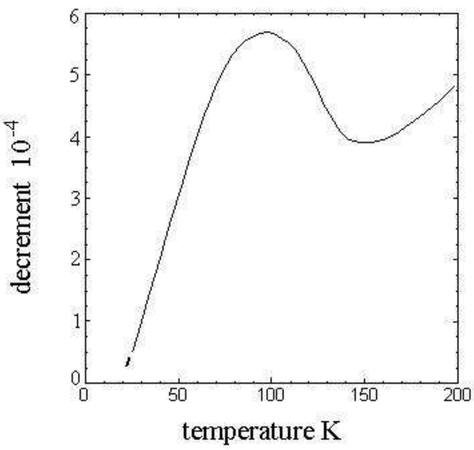

Fig. 4 Temperature dependence of the strain amplitude-independent component of internal friction.

A comparison between the decrements at 7 kHz and at 100kHz shows that the internal friction is still higher in the sonic range than at ultrasonic frequencies.

Fig. 4 clarifies the nature of the strain amplitude-independent component of the internal friction. The amplitude-independent component increases continuously with temperature. A maximum is observed near 100 K. This is an indication of the dislocation nature of the amplitude-independent internal friction component.

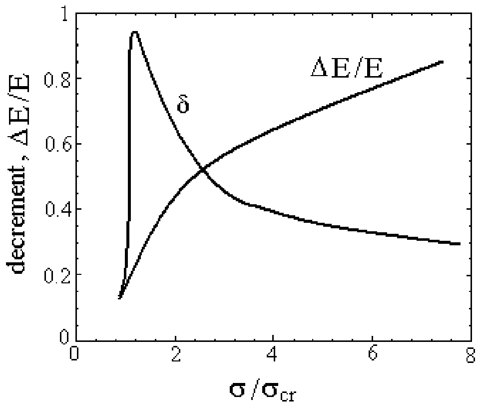

Fig. 5. The dependence of the decrement and the Young's modulus defect on the stress ratio σ / σ_{cr}

The dependence of the decrement and the Young's modulus defect on the stress ratio σ / σ_{cr} is shown in Fig.5. The Young's modulus defect is increasing with increasing stress, and the decrement relaxes with increasing stress.

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AMPLITUDINEA SOLICITARII DEPENDENTA DE FRECAREA INTERNA SI MODULUL LUI YOUNG DEFECT IN SOLIDE DETERIORATE

Abstract: Lucrarea investighează efectele neliniare ale propagarii undelor acustice in materiale deteriorate având proprietatea că distanța dintre granule este comparabilă cu magnitudinea vectorului Burgers. Ideea principală este a explica in mod unitary la nivel atomic următoarele proprietăți ale materialului deteriorat:
- dependenta amplitudinii deformatiei de frecarea interna (ADIF) și defectul modulului Young (YMD);
- comportamentul ADIF la amplitudini mici, moderate și inalte ale deformatiei, în functie de temperatură și frecvență;
- corelația dintre ADIF și YMD prin dislocatii.

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