TECHNICAL UNIVERSITY OF CLUJ-NAPOCA

ACTA TECHNICA NAPOCENSIS

Series: Applied Mathematics, Mechanics, and Engineerir
Vol. 60, Issue IV, November, 2017

A METHOD FOR THE STUDY OF THE VIBRATION OF MECHANICAL BARS SYSTEMS WITH SYMMETRIES

Sorin VLASE, Cristi NĂSTAC, Marin MARIN, Mircea MIHĂLCICĂ

Abstract: *The paper presents a method to simplify the calculus of the eigenmodes of a mechanical system with bars in order to obtain more information concerning the deformations and the loads in the elements of the system using a semi-analytical model. Finite element method, currently used in engineering applications offers a set of standard results and sometime is necessary to process the dataset to obtain useful results. The method is applied to a symmetrical structure, this kind of system being discussed in previous papers. In this paper we aim to study a symmetrical system of beams that presents vibrations perpendicular on the plane of the system. The determination of properties of such systems would decrease the computational time and effort and offers more information about system.*

Key words: *vibrations, symmetrical beams, finite element method, eigenmodes*

1. INTRODUCTION

A standard analysis of the vibration of a mechanical system with elastic elements consists in an eigenvalues and eigenmodes analysis. To obtains eigenvalues numerical procedures are well developed. To determine the eigenmodes is necessary only to solve a linear system. The models used for such analysis cause the form of the characteristic equations. In the case when we have a mechanical system with bars is possible to use an analytical model to obtain this characteristic equation. But to solve this equation is a very difficult tusk even in simple cases of a single bar. A numerical method as FEM (Finite Element Method) offers the eigenvalues and eigenmodes using simplifying assumptions. Unfortunately the results of the calculus are offered in a standard form and can be difficult to make a study of the system using these results. It can be necessary to make a new program and to use the output results offers by the FEM as input data for our own soft. The mechanical systems made of bars are frequently used in engineering practice and there a lot of applications using this kind of elements. We will focus on the systems with symmetries, studied in previous papers [1]-[6].

The basic idea of the paper is to use the benefits of analytical representations in a study of such system avoiding the disadvantages involved in calculating eigenvalues. To obtain the eigenvalues can be used a standard Finite Element software and, using these, to determine the eigenmodes using the analytical models. The formulas obtain are easy to use and is possible to obtain analytic representations as the variation of the forces and moments in the system's elements.

2. ANALYTICAL MODEL OF THE MECHANICAL SYSTEM WITH BARS

We will present the method for a mechanical system composed of 2 beams, considered in the plane. The symmetry of the system can be useful to reduce the dimension of the problem [2]-[6].

The transverse vibrations are considered to take place normal to the plane defined by the beams of the structure. Torsional vibrations will be also considered. Some coupling occurs between the torsional and transverse vibration, this being determined by the rigid link between the beams.

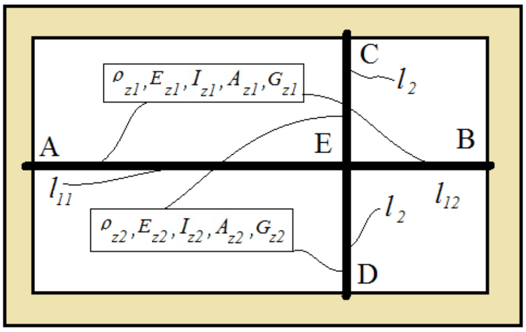

Fig. 1. A mechanical structure consisting of two identical beams

Let us consider a mechanical structure (Fig.1) which consists of two identical beams, AB and CD, rigidly fixed at right angles (for example by welding or soldering in the case of an engineering system). The beams may present vibrations in a direction normal to the plan of bars and, at the same time, also torsional vibrations. A, B, C and D are clamped ends, and from this results that the displacement, torsion angle and tangential slope are zero (12 Boundary Conditions). For the point E we have several conditions. First, the displacement of the point E for all four bars must be equal (three conditions). Second, the tangential slope for the bar AE must be equal with the tangential slope for EB and with the torsional angle in E for the bars CE and DE (there conditions). Third, the tangential slope for CE and De in E must be equal and equal too with the torsional angle of the bars AE and EB in E (three conditions). Finally, if we consider the equilibrium of the point E, the shear forces, the bending moments and the torsional moments must be in equilibrium in E (three more conditions). We have together 24 conditions to determine 24 constants for an analytical model presented in the following.

3. EQUATIONS OF TRANSVERSE AND TORSIONAL VIBRATIONS

In the following we will consider a continuous beam with constant section. For this situation, the transverse vibration of the beams are described, if there are no forces distributed along the length of the beam, by the well-known equations [7],[12],[13],[14]

$$\frac{\partial^4 v}{\partial x^4}+\frac{\rho A}{EI_z}\frac{\partial^2 v}{\partial x^2}=0 \qquad (1)$$

In equation (1) we denoted with: v – deflection of the beam, A - the sectional area of the beam, ρ - the density of the material, E – Young 'modulus, Iz – the second moment of the area with respect to the *z* axis in the center of mass and x - the distance from the left end of the beam to the point where *v* is the deflection.

To solve the differential equation above we search for a solution like:

$$v(x,t)=\Phi(x)\sin(pt+\theta) \qquad (2)$$

If we set the condition that (2) verifies (1) at any moment in time, we obtain:

$$\frac{\partial^4 \Phi}{\partial x^4}-p^2\frac{\rho A}{EI_z}\Phi=0 \qquad (3)$$

We denote:

$$\lambda^4=\frac{\rho A}{EI_z} \qquad (4)$$

In this situation, (1) becomes:

$$\frac{\partial^4 \Phi}{\partial x^4}-p^2\lambda^4\Phi=0, \qquad (5)$$

where Φ is the function which gives the deformed beam (the eigenmode) which will vibrate with the pulsation *p*. We have four such bars and we can write four such solutions:

For the first segment, AE:

$$\frac{\partial^4 \Phi_{AE}}{\partial x^4}-p^2\frac{\rho_1 A_1}{E_1 I_{z1}}\Phi_{AE}=0 \qquad (5.1)$$

For the second segment, EB:

$$\frac{\partial^4 \Phi_{BE}}{\partial x^4}-p^2\frac{\rho_1 A_1}{E_1 I_{z1}}\Phi_{BE}=0 \qquad (5.2)$$

For the segment CE:

$$\frac{\partial^4 \Phi_{CE}}{\partial x^4}-p^2\frac{\rho_2 A_2}{E_2 I_z 2}\Phi_{PR}=0 \qquad (5.3)$$

For the segment DE:

$$\frac{\partial^4 \Phi_{DE}}{\partial x^4}-p^2\frac{\rho_2 A_2}{E_2 I_z 2}\Phi_{DE}=0 \qquad (5.4)$$

If we note:

$$\lambda_1^4=\frac{\rho_1 A_1}{E_1 I_{z1}} \quad ; \quad \lambda_2^4=\frac{\rho_2 A_2}{E_2 I_{z2}} \quad ,$$

the solution to the system of equations of 4^{th}

degree (5.1), (5.2) şi (5.3) will be given by [8]-[12]:

$$\Phi_{AE}(x) = \alpha_1^{AE} \sin\lambda_1\sqrt{p}x + \alpha_2^{AE} \cos\lambda_1\sqrt{p}x + \alpha_3^{AE} sh\lambda_1\sqrt{p}x + \alpha_4^{AE} ch\lambda_1\sqrt{p}x \quad (6.1)$$

$$\Phi_{BE}(x) = \alpha_1^{BE} \sin\lambda_1\sqrt{p}x + \alpha_2^{BE} \cos\lambda_1\sqrt{p}x + \alpha_3^{BE} sh\lambda_1\sqrt{p}x + \alpha_4^{BE} ch\lambda_1\sqrt{p}x \quad (6.2)$$

$$\Phi_{CE}(x) = \alpha_1^{CE} \sin\lambda_1\sqrt{p}x + \alpha_2^{CE} \cos\lambda_1\sqrt{p}x + \alpha_3^{CE} sh\lambda_1\sqrt{p}x + \alpha_4^{CE} ch\lambda_1\sqrt{p}x \quad (6.3)$$

$$\Phi_{DE}(x) = \alpha_1^{DE} \sin\lambda_1\sqrt{p}x + \alpha_2^{DE} \cos\lambda_1\sqrt{p}x + \alpha_3^{DE} sh\lambda_1\sqrt{p}x + \alpha_4^{DE} ch\lambda_1\sqrt{p}x \quad (6.4)$$

The torsional vibrations of the beams are given by:

$$\frac{\partial^2\varphi}{\partial x^2} - \frac{\rho}{E}\frac{\partial^2\varphi}{\partial t^2} = 0 \quad (7)$$

where φ is the torsional angle of the beam, x is the ordinate of the point with the displacement φ. In order to solve the equation, we will consider the solution φ in the form of:

$$\varphi(x,t) = \Psi(x)\sin(pt+\theta) \quad (8)$$

Bringing it into (7), we obtain:

$$\frac{\partial^2\Psi}{\partial x^2} + p^2\delta^2\Psi = 0 \quad (9)$$

where:

$$\delta^2 = \frac{\rho}{E} \quad (10)$$

If we make these notations, the solution of the system of equations obtained by applying (9) for the beams AE, BE,CE and DE looks like:

$$\Psi_{AE}(x) = \alpha_5^{AE} \sin\delta_1 px + \alpha_6^{AE} \cos\delta_1 px \quad (11.1)$$

$$\Psi_{BE}(x) = \alpha_5^{BE} \sin\delta_1 px + \alpha_6^{BE} \cos\delta_1 px \quad (11.2)$$

$$\Psi_{CE}(x) = \alpha_5^{CE} \sin\delta_1 px + \alpha_6^{CE} \cos\delta_1 px \quad (11.3)$$

$$\Psi_{DE}(x) = \alpha_5^{DE} \sin\delta_1 px + \alpha_6^{DE} \cos\delta_1 px \quad (11.4)$$

The equations (6.1)-(6.4), (11.1)-(11.4) describe the free vibrations of the studied mechanical system. We observe that in these systems we have 24 constants of integration: $\alpha_i^{AE}, \alpha_i^{BE}, \alpha_i^{CE}, \alpha_i^{DE}, \quad i=\overline{1,6}$, which are determined considering the Boundary Conditions.

4. EIGENVALUES AND EIGENMODES

The Boundary Conditions at the clamped ends A, B, C and D can be written as:

For the AE beam:

$$\Phi_{AE}(0) = 0 \quad ; \quad \left.\frac{\partial\Phi_{AE}}{\partial x}\right|_{x=0} = 0 \quad ; \quad \Psi_{AE}(0) = 0$$

For the BE beam

$$\Phi_{BE}(0) = 0 \quad ; \quad \left.\frac{\partial\Phi_{BE}}{\partial x}\right|_{x=0} = 0 \quad ; \quad \Psi_{BE}(0) = 0$$

For the CE beam:

$$\Phi_{CE}(0) = 0 \quad ; \quad \left.\frac{\partial\Phi_{CE}}{\partial x}\right|_{x=0} = 0 \quad ; \quad \Psi_{CE}(0) = 0$$

For the DE beam:

$$\Phi_{DE}(0) = 0 \quad ; \quad \left.\frac{\partial\Phi_{DE}}{\partial x}\right|_{x=0} = 0 \quad ; \quad \Psi_{DE}(0) = 0$$

so, in the end, we will have **12 conditions**. If we introduce these conditions in (6) and (11) we obtain:

$$\alpha_2^{AE} + \alpha_4^{AE} = 0 \quad ; \quad \alpha_1^{AE} + \alpha_3^{AE} = 0 \quad ; \quad \alpha_6^{AE} = 0 \quad ;$$

$$\alpha_2^{BE} + \alpha_4^{BE} = 0 \quad ; \quad \alpha_1^{BE} + \alpha_3^{BE} = 0 \quad ; \quad \alpha_6^{BE} = 0 \quad ;$$

$$\alpha_2^{CE} + \alpha_4^{CE} = 0 \quad ; \quad \alpha_1^{CE} + \alpha_3^{CE} = 0 \quad ; \quad \alpha_6^{CE} = 0 \quad ;$$

$$\alpha_2^{DE} + \alpha_4^{DE} = 0 \quad ; \quad \alpha_1^{DE} + \alpha_3^{DE} = 0 \quad ; \quad \alpha_6^{DE} = 0 \quad ;$$

The slope of the E point must be equal for all the four bars. It means that:

$$v_{AE}\big|_{x=l_{11}} = v_{BE}\big|_{x=l_{12}} = v_{CE}\big|_{x=l_2} = v_{DE}\big|_{x=l_2} \quad .$$

If we use the rel (2) and (6) it results:

$$\alpha_1^{AE} \sin\lambda_1\sqrt{p}l_{11} + \alpha_2^{AE} \cos\lambda_1\sqrt{p}l_{11} + \alpha_3^{AE} sh\lambda_1\sqrt{p}l_{11} + \alpha_4^{AE} ch\lambda_1\sqrt{p}l_{11} =$$

$$= \alpha_1^{BE} \sin\lambda_1\sqrt{p}l_{12} + \alpha_2^{BE} \cos\lambda_1\sqrt{p}l_{12} + \alpha_3^{BE} sh\lambda_1\sqrt{p}l_{12} + \alpha_4^{BE} ch\lambda_1\sqrt{p}l_{12} =$$

$$= \alpha_1^{CE} \sin\lambda_1\sqrt{p}l_2 + \alpha_2^{CE} \cos\lambda_1\sqrt{p}l_2 + \alpha_3^{CE} sh\lambda_1\sqrt{p}l_2 + \alpha_4^{CE} ch\lambda_1\sqrt{p}l_2 =$$

$$= \alpha_1^{DE} \sin \lambda_1 \sqrt{p} l_2 + \alpha_2^{DE} \cos \lambda_1 \sqrt{p} l_2 + \alpha_3^{DE} sh \lambda_1 \sqrt{p} l_2 + \alpha_4^{DE} ch \lambda_1 \sqrt{p} l_2$$

If we note M^b the bending moment of a beam in the section x, *T* the shear force and *S* the axial force, in the following, we will write these conditions in analytical form. The bending moment, shear force and axial force can be written using:

$$M^b(x) = -EI_z \frac{\partial^2 v(x)}{\partial x^2} \quad ; \quad T(x) = EI_z \frac{\partial^3 v}{\partial x^3} \quad ; \quad M^t(x) = GI_p \frac{\partial \varphi(x)}{\partial x} \tag{12}$$

By successive differentiations we obtain:

$$\frac{\partial v(x)}{\partial x^2} = \Phi'(x) \sin(pt + \theta) =$$

$$= \lambda \sqrt{p} \begin{bmatrix} \alpha_1 \cos \lambda \sqrt{p} x - \alpha_2 \sin \lambda \sqrt{p} x + \\ \alpha_3 ch \lambda \sqrt{p} x + \alpha_4 sh \lambda \sqrt{p} x \end{bmatrix} \sin(pt + \theta) \tag{13}$$

$$\frac{\partial^2 v(x)}{\partial x^2} = \Phi''(x) \sin(pt + \theta) =$$

$$= \lambda^2 p \begin{bmatrix} -\alpha_1 \sin \lambda \sqrt{p} x - \alpha_2 \cos \lambda \sqrt{p} x + \\ \alpha_3 sh \lambda \sqrt{p} x + \alpha_4 ch \lambda \sqrt{p} x \end{bmatrix} \sin(pt + \theta) \tag{14}$$

$$\frac{\partial^3 v(x)}{\partial x^3} = \Phi'''(x) \sin(pt + \theta) =$$

$$= \left(\lambda \sqrt{p}\right)^3 \left[\alpha_1 \cos \lambda \sqrt{p} x + \alpha_2 \sin \lambda \sqrt{p} x + \right.$$

$$\left. + \alpha_3 ch \lambda \sqrt{p} x + \alpha_4 sh \lambda \sqrt{p} x\right] \sin(pt + \theta) \tag{15}$$

$$\frac{\partial \varphi(x)}{\partial x} = \Psi'(x) \sin(pt + \theta) = \tag{16}$$

$$\delta p \left(\alpha_5 \cos \delta p x - \alpha_6 \sin \delta p x\right) \sin(pt + \theta)$$

In E the tangent slope of the bar AE is equal (and opposite) with the tangent slope of the bar BE, with torsion of CE and opposite torsion of DE. We have:

$$\lambda \sqrt{p} \left[\alpha_1^{AE} \cos \lambda \sqrt{p} l_{11} - \alpha_2^{AE} \sin \lambda \sqrt{p} l_{11} + \alpha_3^{AE} ch \lambda \sqrt{p} l_{11} + \alpha_4^{AE} sh \lambda \sqrt{p} l_{11}\right] =$$

$$= -\lambda \sqrt{p} \left[\alpha_1^{BE} \cos \lambda \sqrt{p} l_{12} - \alpha_2^{BE} \sin \lambda \sqrt{p} l_{12} + \alpha_3^{BE} ch \lambda \sqrt{p} l_{12} + \alpha_4^{BE} sh \lambda \sqrt{p} l_{12}\right] =$$

$$= \left(\alpha_5^{CE} \sin \delta p l_2 - \alpha_6^{CE} \cos \delta p l_2\right) = -\left(\alpha_5^{DE} \sin \delta p l_2 - \alpha_6^{DE} \cos \delta p l_2\right)$$

In a similar manner we have In E the tangent slope of the bar CE is equal with the tangent slope of the bar DE and with torsion of AE and BE. It results:

$$\lambda \sqrt{p} \left[\alpha_1^{CE} \cos \lambda \sqrt{p} l_2 - \alpha_2^{CE} \sin \lambda \sqrt{p} l_2 + \alpha_3^{CE} ch \lambda \sqrt{p} l_2 + \alpha_4^{CE} sh \lambda \sqrt{p} l_2\right] =$$

$$= -\lambda \sqrt{p} \left[\alpha_1^{DE} \cos \lambda \sqrt{p} l_2 - \alpha_2^{DE} \sin \lambda \sqrt{p} l_2 + \alpha_3^{DE} ch \lambda \sqrt{p} l_2 + \alpha_4^{DE} sh \lambda \sqrt{p} l_2\right] =$$

$$= \delta p \left(\alpha_5^{AE} \cos \delta p l_{11} - \alpha_6^{AE} \sin \delta p l_{11}\right) =$$

$$= -\delta p \left(\alpha_5^{BE} \cos \delta p l_{12} - \alpha_6^{BE} \sin \delta p l_{12}\right)$$

The continuity of the moment offers us the condition that the bending moment in E of the bar AE is equal and opposite with the bending moment of the beam BE, with the torsion moment of the beam CE and opposite with the torsion moment of the beam DE. These means:

$$\lambda^2 p \left[-\alpha_1^{AE} \sin \lambda \sqrt{p} l_{11} - \alpha_2^{AE} \cos \lambda \sqrt{p} l_{11} + \alpha_3^{AE} sh \lambda \sqrt{p} l_{11} + \alpha_4^{AE} ch \lambda \sqrt{p} l_{11}\right] =$$

$$= \lambda^2 p \left[-\alpha_1^{BE} \sin \lambda \sqrt{p} l_{12} - \alpha_2^{BE} \cos \lambda \sqrt{p} l_{12} + \alpha_3^{BE} sh \lambda \sqrt{p} l_{12} + \alpha_4^{BE} ch \lambda \sqrt{p} l_{12}\right] =$$

$$= \delta p \left(\alpha_5^{CE} \cos \delta p l_2 - \alpha_6^{CE} \sin \delta p l_2\right) =$$

$$= \delta p \left(\alpha_5^{DE} \cos \delta p l_2 - \alpha_6^{DE} \sin \delta p l_2\right)$$

In a similar way we have:

$$\lambda^2 p \left[-\alpha_1^{CE} \sin \lambda \sqrt{p} l_2 - \alpha_2^{CE} \cos \lambda \sqrt{p} l_2 + \alpha_3^{CE} sh \lambda \sqrt{p} l_2 + \alpha_4^{CE} ch \lambda \sqrt{p} l_2\right] =$$

$$= \lambda^2 p \left[-\alpha_1^{DE} \sin \lambda \sqrt{p} l_2 - \alpha_2^{DE} \cos \lambda \sqrt{p} l_2 + \alpha_3^{DE} sh \lambda \sqrt{p} l_2 + \alpha_4^{DE} ch \lambda \sqrt{p} l_2\right] =$$

$$= \delta p \left(\alpha_5^{AE} \cos \delta p l_{11} - \alpha_6^{AE} \sin \delta p l_{11}\right) =$$

$$= \delta p \left(\alpha_5^{BE} \cos \delta p l_{12} - \alpha_6^{BE} \sin \delta p l_{12}\right)$$

To determine the constants which provide the imposed end conditions we must solve the homogeneous linear system, in order to determine constants $\alpha_1^{AE}, \alpha_2^{AE}, \ldots, \alpha_6^{AE}, \alpha_1^{BE}, \alpha_2^{BE}, \ldots, \alpha_6^{BE}, \alpha_1^{CE}, \alpha_2^{CE}, \ldots, \alpha_6^{CE}, \alpha_1^{DE}, \alpha_2^{DE}, \ldots, \alpha_6^{DE}$.

In the following, we will note:

$$\{\alpha\} = \left[\alpha_1^{AE} \quad \alpha_2^{AE} \quad \alpha_3^{AE} \quad \alpha_4^{AE} \quad \alpha_5^{AE} \quad \alpha_6^{AE} \quad \alpha_1^{BE} \quad \alpha_2^{BE} \quad \cdots \quad \alpha_6^{DE}\right]^T$$

being the vector of the integration constants.

If we set the condition that the system should have a zero determinant, we will now get the natural frequencies (the eigenvalues) of the mechanical system [15]-[20]:

$$[S]\{C\} = \{0\} \tag{17}$$

The condition that the system should have a zero determinant is: $\{C\}$

$$\det(S) = 0 \tag{18}$$

and solving this equation will give us the eigenvalues of the system of differential equations (7.1)-(7.18).

It is very clear that to solve the characteristic equations (18) is practically impossible due to the complicated algebraic form of the equation (a very complex expression with the function cos, sin, ch, sh). To applied numerical procedure to solve the equation is very difficult because of the complex form of the eq. (18). On the other hand the solutions (6) and (11) are very useful in order to make a study of such mechanical structure. It is very convenient to have such simple and continuous form the solutions. FEM procedure offer us the eigenvalues and the eigenmodes but in a numerical and rigid form. But to determine the eigenvalues the FEM procedures have a long and strong validation. Consequently, FEM procedures can be applied to determine the eigenvalues and the obtained results can be introduced in the eq. (17) to obtain the constant vector $\{C\}$ and as a consequence the continuous solution functions express by the rel. (6), (11).

5. CONCLUSIONS

In the paper a semi-analytical method to simplify the calculus of the eigenmodes of a mechanical system with bars is presented. The method is justified by the needs to obtain more information concerning the deformations and the loads in the elements of the system. Finite element method, currently used in engineering applications offers a set of standard results and sometime is necessary to process the dataset to obtain useful results. The method is applied to a symmetrical structure, this kind of system being discussed in previous papers. In this paper we aim to study a symmetrical system of beams that presents vibrations perpendicular on the plane of the system. The determination of properties of such systems would decrease the computational time and effort and offers more information about system. In this way, a device or a machine used in industry can be designed and executed faster and cheaper. The paper examined the particular case of a mechanical system consisting of two beams in plane but can be extend to more complicated systems.

6. REFERENCES

[1] Meirovitch, L., *Principles and Techniques of Vibrations,* Pearson (1996).

[2] Mangeron, D., Goia, I., Vlase, S., *Symmetrical Branched Systems Vibrations,* Scientific Memoirs of the Romanian Academy, Bucharest, Serie IV, Tom XII, Nr.1, 1991,p.232-236.

[3] Vlase, S., Chiru, A*., Simmetry in the study of the vibration of some engineering mechanical systems,* Proceedings of the 3rd International Conference on Experiments/ Process/ System Modeling/ Simulation/ Optimization (3rd IC-EpsMsO), Athens, Greece, 8-11 July, 2009.

[4] Shi, C.Z., Parker, R.G., *Modal structure of centrifugal pendulum vibration absorber systems with multiple cyclically symmetric groups of absorbers*, Journal of Sound and Vibration. ISSN 0022-460X, DI 10.1016/j.jsv.2013.03.009

[5] Paliwal, D.N., Pandey,R.K., *Free vibrations of circular cylindrical shell on Winkler and Pasternak foundations,* International Journal of Pressure Vessels and Piping, Volume 69, Issue 1, November 1996, Pages 79-89

[6] Celep,Z., *On the axially symmetric vibration of thick circular plates*, Ingenieur-Archiv. November 1978, Volume 47, Issue 6, pp 411-420

[7] Buzdugan, Gh., Fetcu, L., Rades, M., *Mechanical vibrations*, Ed. Did. si Ped., Bucharest, 1982

[8] Douglas, Th., *Structural Dynamics and Vibrations in Practice: An Engineering Handbook*, CRC Press, 2012

[9] Timoshenko, P.S., M. Gere, J.M., *Theory of Elastic Stability,* McGraw-Hill,New York, London, 2nd Edition, 2009

[10] Tyn Myint-U, *Ordinary Differential Equations*, Elsevier, 1977

[11] Henderson, J., Luca,R., Boundary *Value Problems for Systems of Differential,*

Difference and Fractional Equations: Positive Solutions, Elsevier, 2016

[12] Sharma, K., Marin, M., *Effect of distinct conductive and thermodynamic temperatures on the reflection of plane waves in micropolar elastic half-space*, U.P.B. Sci. Bull., Series A-Appl. Math. Phys., 75(2), 121-132, 2013

[13] Marin,M., *On weak solutions in elasticity of dipolar bodies with voids*, J. Comp. Appl. Math., vol. 82 (1-2), 291-297, 1997

[14] Marin,M., *Harmonic vibrations in thermoelasticity of microstretch materials*, J. Vibr. Acoust., ASME, vol. 132(4), 044501-044501-6, 2010

[15] Vlase, S., Paun, M., *Vibration Analysis of a Mechanical System consisting of Two Identical Parts,* Ro. J. Techn. Sci. - Appl. Mechanics, Vol. 60, No 3, pp. 216-230, Bucharest, 2015.

[16] Vlase,S., *Finite Element Analysis of the Planar Mechanisms: Numerical Aspects. In Applied Mechanics – 4,* Elsevier, pp.90-100, 1992.

[17] Vlase S, Danasel C, Scutaru ML, Mihalcica M., *Finite Element Analysis of a Two-Dimensional Linear Elastic Systems with a Plane “Rigid Motion”*, Rom. Journ. Phys., Vol. 59, Nos. 5–6, pp. 476–487, Bucharest, 2014

[18] Vlase,S., Teodorescu,P.P., Itu,C., Scutaru, M.L., *Elasto-Dynamics of a Solid with a General "Rigid" Motion Using FEM Model. Part II. Analysis of a Double Cardan Joint*, Romanian Journal of Physics, 58 (7-8), 2013

[19] Vlase, S., *A Method of Eliminating Lagrangian-Multipliers from the Equation of Motion of Interconnected Mechanical Systems*, Journal of Applied Mechanics-Transaction of the ASME, vol. 54, Issue:1, pp.235-237,1987

[20] Vlase,S., Munteanu,M.V., Scutaru, M.L., *On the Topological Description of the Multibody Systems*, Annals of DAAAM, Book Series, pp.1493-1494, 2008

O metodă pentru studiul sistemelor mecanice de bare cu simetrii

Lucrarea prezintă o metodă de simplificare a calculului modurilor proprii ale unui sistem mecanic cu bare pentru a obține mai multe informații referitoare la deformările și sarcinile din elementele sistemului folosind un model semianalitic. Metoda cu elemente finite, utilizată în prezent în aplicații inginerești, oferă un set de rezultate standard și uneori este necesară procesarea setului de date pentru a obține rezultate utile. Metoda se aplică unei structuri simetrice, acest tip de sistem fiind discutat în lucrările anterioare. În lucrare ne propunem să studiem un sistem simetric de grinzi care prezintă vibrații perpendiculare pe planul sistemului. Determinarea proprietăților acestor sisteme ar reduce timpul și efortul de calcul și ar oferi mai multe informații despre sistem.

Sorin VLASE, Prof. PhD, Transylvania University of Brașov, Department of Mechanical Engineering, svlase@unitbv.ro, Tel. +40 268 413000 / 168, Str. Politehnicii 1, Brașov 500024, Romania

Cristi NĂSTAC, Lect. PhD, Transylvania University of Brașov, Department of Civil Engineering, ndcproiect@gmail.com

Marin MARIN, Prof. PhD, Transylvania University of Brașov, Department of Mathematics and Computer Science, m.marin@unitbv.ro

Mircea MIHĂLCICĂ, Lect. PhD, Transylvania University of Brașov, Department of Mechanical Engineering, mihalcica.mircea@unitbv.ro, Tel. +40 268 413000 / 168, Str. Politehnicii 1, Brașov 500024, Romania