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NUMERICAL MODEL BASED ON A BERNOULLI INTEGRAL EXTENSION FOR PRESSURE ANALYZE ON A MILLIMETER DEVICE IN A TURBULENT FLOW

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Abstract: *There massive calculus resources are needed for computing a RANS fluid motion and a piezoelectric model, as we already noticed in our previous models . The matched and verified hypothesis about the device response and qualitative behavior make a new approach to be necessary. Therefore, some theoretical and field observations on PDEs together with the applied genetically algorithms immerge for the problem's convergent solution. The aim of the paper is to validate a model computed with a fluid pressure depending on fluid speed and device' body dimensions for optimal electric response.*

Key words: *turbulence flow, Bernoulli integration extension, pressure analyze*

1. INTRODUCTION

We observed the dependence of the pressure applied on the device body faces with the square speed of the fluid. The applied pressure is in charge of the piezo-response through its internal stress tensor, which further outputs an electrical signal [1],[2],[3].

Here the device is a detector of a fluid speed ,fluid that can have different viscosities.

The mathematical model has three subsystems, such as the turbulent fluid motion, the solid mechanic and the piezoelectric components, matched by the boundary and initial conditions.

The physical device consists in a piezoelectric probe and a foam mounted between two metal electrodes [4],[6], witch allowes the piezoelectric potential to be measured, as we presented in our previous paper[7].

So we search for a most simplified numerical model to investigate the qualitative behavior of the piezoelectric device and the turbulent flow.

This model has to be validated by comparison with the results from our previous research, to further investigate another proposed device.

2. A BERNOULLI INTEGRAL EXTENSION

We assume that the pressure applied on the millimeter device depends with the square speed of the fluid. Let's first prove this assumption:

We search for the pressure equation of the model built on our previous study [7].

As a general rule, the product $u_j \dfrac{\partial u_i}{\partial x_j}$ can be written as the addition between a gradient and a vector product among a rotor and the vector fluid field:

$$u_j \frac{\partial u_i}{\partial x_j} = \frac{\partial}{\partial x_i}\left(\frac{1}{2}u_j u_j\right) + [(rot\vec{u})\times\vec{u}]_i ,$$

In the later assumption the velocity field is an irrotational field ($rot\vec{u} = 0$), so we obtained the relationship for the pressure.

$$p = -\frac{1}{2}u_j u_j + \nu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) - \frac{2}{3}\nu\frac{\partial u_k}{\partial x_k}\delta_{ij} +$$

$$+\nu_t\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) - \frac{2}{3}\left(\nu_t\frac{\partial u_k}{\partial x_k} + \rho k\right)\delta_{ij} + const$$

Hence the expression above suggests that the pressure depends on the speed's square.

3. MATHEMATICAL SUPPORT

The equations we are looking for are the classic conservation equation [5]:

$$\rho\left(u_i \frac{\partial}{\partial x_i}\right)\left(u_j \vec{e_j}\right)=\frac{\partial}{\partial x_i}\left[-p\delta_{ij}\vec{e_j}+ (\nu+\nu_t)\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)\vec{e_j}- \frac{2}{3}(\nu+\nu_t)\frac{\partial u_k}{\partial x_k}\delta_{ij}\vec{e_j}-\frac{2}{3}\rho k\delta_{ij}\vec{e_j}\right], \quad (1)$$

which by components becomes:

$$\rho u_i \frac{\partial u_j}{\partial x_i}=\frac{\partial}{\partial x_i}\left[-p\delta_{ij}+(\nu+\nu_t)\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)- \frac{2}{3}(\nu+\nu_t)\frac{\partial u_k}{\partial x_k}\delta_{ij}-\frac{2}{3}\rho k\delta_{ij}\right], \quad (2)$$

and equivalent

$$\rho u_i \frac{\partial u_j}{\partial x_i}=-\frac{\partial p}{\partial x_j}\delta_{ij}+\frac{\partial}{\partial x_i}(\nu+\nu_t)\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)+ (\nu+\nu_t)\frac{\partial}{\partial x_i}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)- \frac{2}{3}\frac{\partial}{\partial x_i}(\nu+\nu_t)\frac{\partial u_k}{\partial x_k}\delta_{ij}- \frac{2}{3}(\nu+\nu_t)\frac{\partial}{\partial x_i}\left(\frac{\partial u_k}{\partial x_k}\right)\delta_{ij}-\frac{2}{3}\frac{\partial}{\partial x_j}\rho k\delta_{ij},\ j=\overline{1,3}, \quad (3)$$

and the equation giving the product term P_k

$$P_k=\nu_t\left[\frac{\partial u_i}{\partial x_j}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)-\frac{2}{3}\left(\frac{\partial u_i}{\partial x_i}\right)^2\right]- \frac{2}{3}\rho k\frac{\partial u_i}{\partial x_i}, \quad (4)$$

where the notations are:

- u_i -velocity fluid field's components (SI unit: m/s),
- ρ -density of the fluid (SI unit: kg/m^3),
- p - pressure of the fluid (SI unit: Pa),
- k - kinetic energy (SI unit: m^2/s^3),
- ε - k' s rate of dissipation (SI unit: m^2/s^3),
- ν - dynamic viscosity (SI unit: $kg/(m\cdot s)$),
- ν_t - turbulent viscosity (SI unit: $Pa\cdot s$),
- ν_{eff} - effective viscosity (SI unit: $Pa\cdot s$), see[7].

4. THE INPUT DATA AND THE INTERACTION BETWEEN THE DEVICE BODY AND THE FLUID MOTION

We consider a millimeter device (fig.1., a, fig.1., b and fig.1., c)

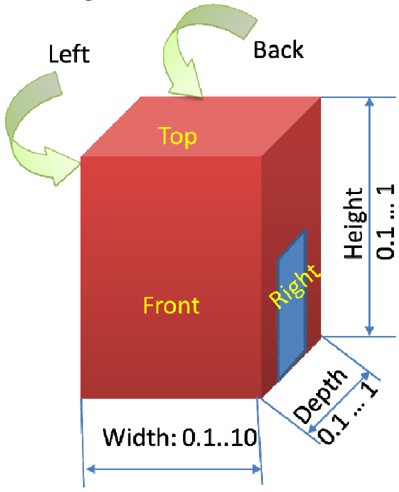

Fig. 1., a. Device body.

The sizes will vary as below ,as well as the fluid speed (tab.1).

We fixed the limits of the device body size and the focal area of the fluid speed, considering a low viscosity turbulent fluid motion.

Tab. 1. Initial data ($4^3\times3=192$ states)

Width [mm]	Depth [mm]	Height [mm]	Flow speed [m/s]
0.1	0.1	0.1	4
0.4	0.4	0.4	14
0.7	0.7	0.7	24
1.0	1.0	1.0	-

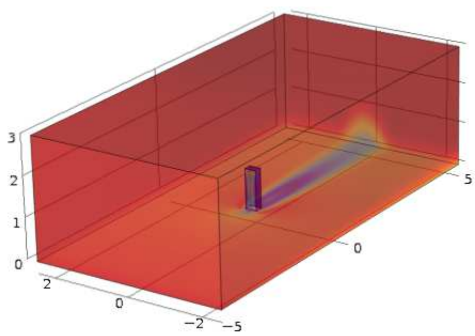

Fig.1. b. Device body in the volume unit.

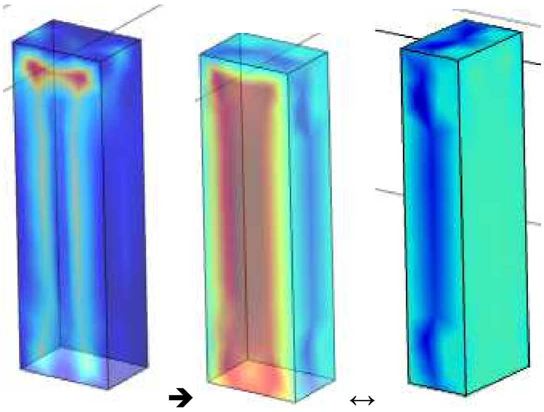

Fig.1. c. Device body.

5. THE NUMERICAL ALGORITHM

We follow the action of the fluid motion versus the device and the turbulent behavior of the flow.

This will have a direct effect on the device electric response (fig. 2).

In order to understand the importance of the pressure measurement on each side of the millimeter device, the stream lines of the turbulent motion are simulated in fig. 2.

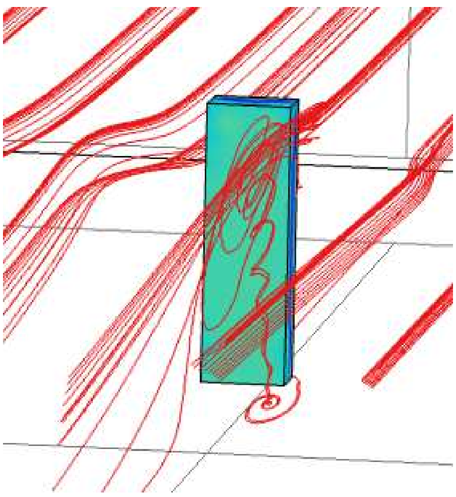

Fig. 2. Device body confronted with the stream lines of the turbulent motion - the continuous current interact with the device.

We will now define the pressure functions for each side (front, left ,right, top, back) and their independent variables.

Considering that the pressure of each face is different from another and it depends of the dimensions and the fluid speed, we have dimensionless relations:

$$p_{face} = p_{face}\left(D_x, D_y, D_z, S\right), \tag{5}$$

$$p\left(D_x^0, D_y^0, D_z^0, S\right) = c_2^S S^2 + c_1^S S + c_0^S, \tag{6}$$

$$p\left(D_x^0, D_y^0, D_z, S^0\right) = c_2^{Dz} D_z^2 + c_1^{Dz} D_z + c_0^{Dz}, \tag{7}$$

$$p\left(D_x^0, D_y, D_z^0, S^0\right) = c_2^{Dy} D_y^2 + c_1^{Dy} D_y + c_0^{Dy}, \tag{8}$$

$$p\left(D_x, D_y^0, D_z^0, S^0\right) = c_2^{Dx} D_x^2 + c_1^{Dx} D_x + c_0^{Dx}, \tag{9}$$

From (6), (7), (8), (9) result:

$$p_{face} = \sum_{i,j,k,l=0}^{2} c_{ijkl} D_x^i D_y^j D_z^k S^l . \tag{10}$$

There are 3^4 coefficients for each pressure. The total number of coefficients are 324, 81 for each face.

Running the algorithm we obtain the following coefficients (tab 2):

Tab.2. The evaluated error is about 1.2%

c_{ijkl}	i	0			1			2		
k	j / l	0	1	2	0	1	2	0	1	2
0	0	-1.4	17.9	53.0	87.2	-146.8	-39.8	35.9	10.2	20.3
	1	-48.6	26.8	-24.1	98.0	-15.1	-42.8	-13.3	36.7	24.7
	2	-2.7	60.2	-23.7	-33.2	4.3	24.9	23.5	-11.5	6.5
1	0	-1.6	-2.1	-20.1	-5.2	14.9	29.2	35.8	-16.5	29.6
	1	10.2	-6.7	-44.8	-15.5	2.1	20.4	-5.4	-12.1	45.8
	2	32.5	-23.3	21.1	10.9	18.9	48.8	-17.7	-34.7	56.7
3	0	-10.5	68.3	-9.7	30.7	5.4	24.9	18.5	1.7	-43.6
	1	8.0	10.6	5.8	64.3	4.4	-32.7	-40.7	-9.3	-13.6
	2	29.9	-17.3	-23.5	10.6	42.6	49.8	19.6	-21.0	-37.6

The effect of the pressure we analyze below.

6. CONCLUSIONS

The pressure on the device body depends of:

- the faces of the body device (front, top, back, left and right),
- the dimensions of the body device(width, depth and height),
- the fluid speed in the manner we proved.

Let' see this dependence of the pressure of each variable as we will show below (fig. 3, fig. 4, fig. 5 and fig. 6).

We tree to improve the shape of the device and the optimal position for the piezo component, in order to improve the electric response, things we will develop elsewhere.

We notice that the turbulence behavior in not negligible, so that it must be taken into consideration even if we work with air or water.

The massive calculus resources are no longer requested on this approach, thing that allow us to search for other optimal response.

We find that the electric signal device is based on the pressure field captured by the surface stress optimal converted.

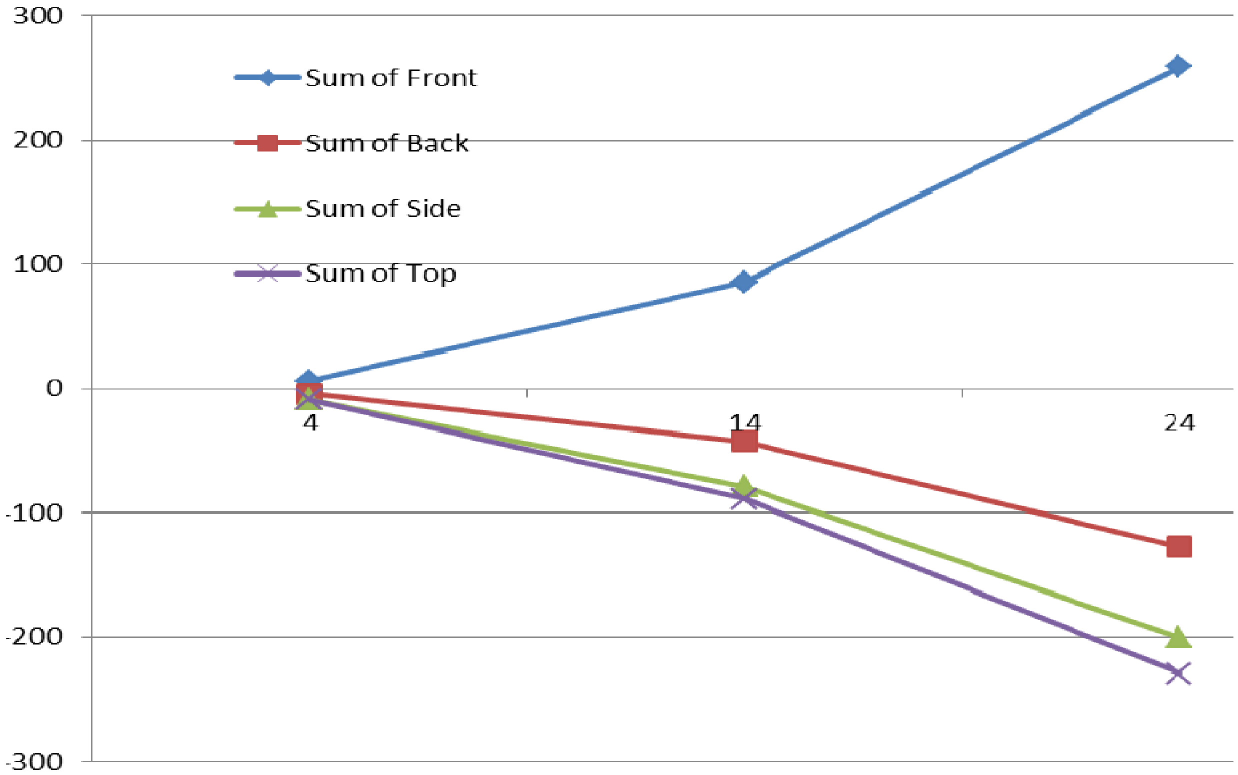

Fig.3. Pressure vs. fluid flow speed.

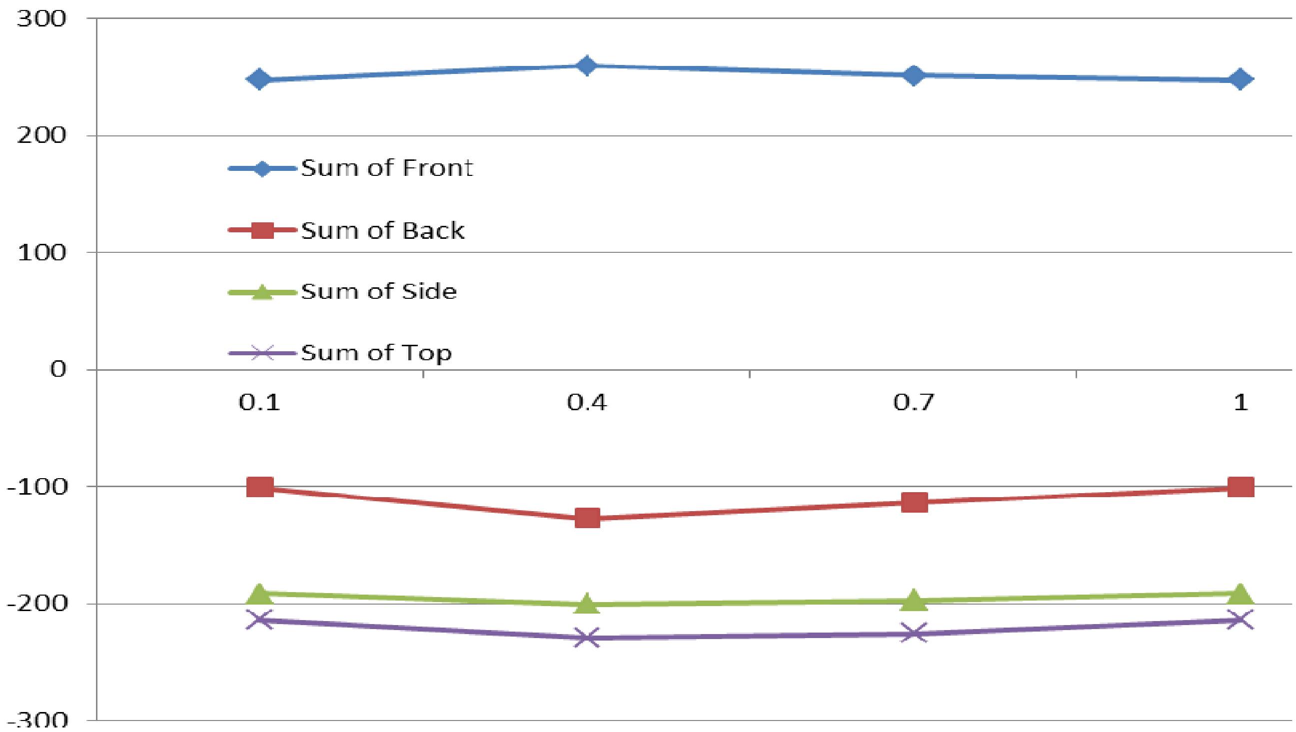

Fig.4. Pressure vs. device depth.

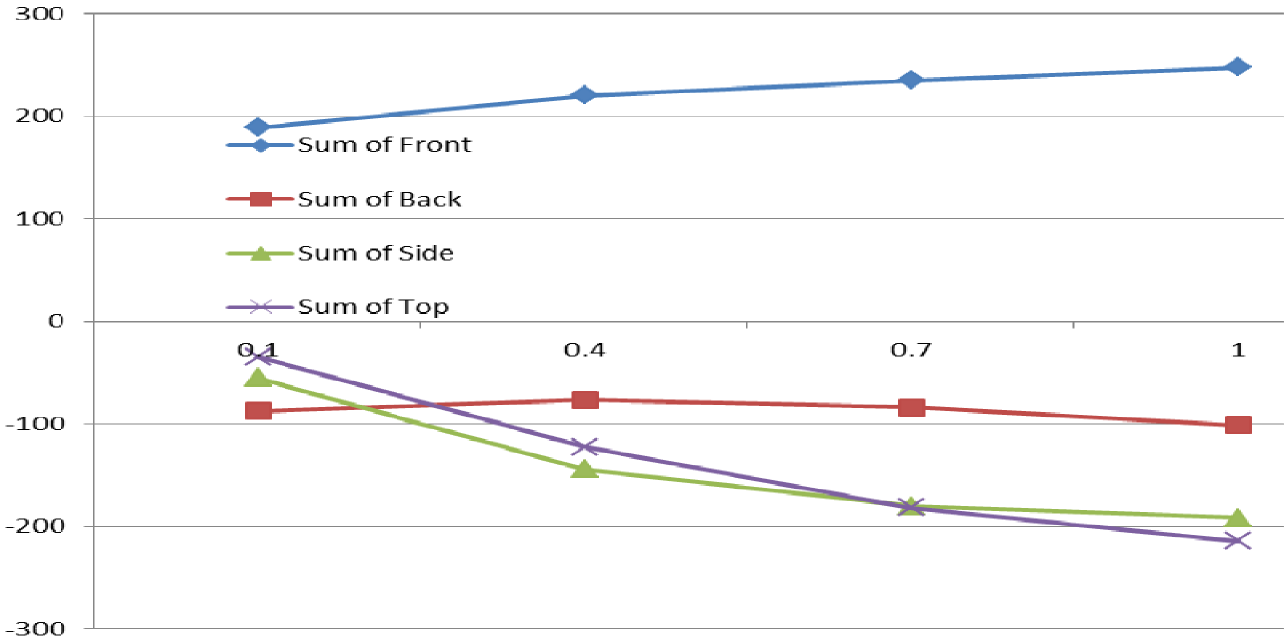

Fig. 5. Pressure vs. device width.

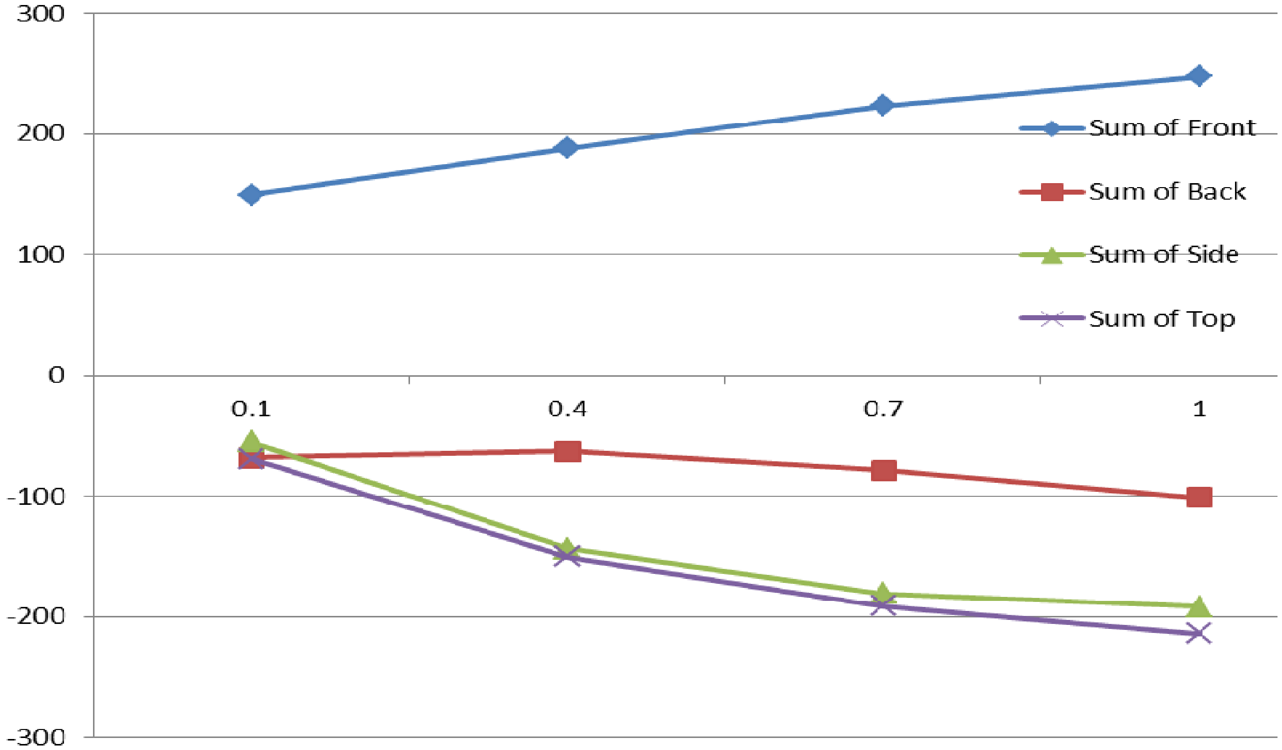

Fig. 6. Pressure vs. device height.

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MODEL NUMERIC BAZAT PE O EXTENDERE A INTEGRALEI BERNOULI PENTRU ANALIZA PRESIUNII PE UN DISPOZITIE DE ORDINUL MILLIMETRILOR ÎN FLUX TURBULENT

Abstract: *Aşa cum s-a determinat într-o lucrare anterioară sunt necesare resurse de calcul masive pentru a calcula o mişcare fluidă RANS şi un model piezoelectric. Ipoteza folosită şi verificată privind răspunsul dispozitivului şi comportamentul calitativ determină o nouă abordare. Prin urmare, unele observaţii teoretice şi experimentale asupra PDE-urilor, împreună cu algoritmii genetici aplicaţi, determină soluţia convergentă a problemei. Scopul lucrării este de a valida un model de calcul ce presupune o presiune a fluidului în funcţie de viteza fluidului, de dimensiunile şi limitele corpului dispozitivului în vederea obţinerii unui răspuns electric optimizat.*

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