# TECHNICAL UNIVERSITY OF CLUJ-NAPOCA 

ACTA TECHNICA NAPOCENSIS
Series: Applied Mathematics, Mechanics, and Engineering
Vol. 60, Issue IV, November, 2017

# STUDY OF THE INFLUENCE OF GEOMETRIC PARAMETERS ON THE DISPLACEMENT OF PISTON AND COMPRESSION RATIO FOR A VARIABLE COMPRESSION RATIO MECHANISM 

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#### Abstract

A variable compression ratio mechanism is presented and the constrained relations between different parameters are established. The study is performed in two ways: by geometrical and multibody approaches. Based on the formulae deduced in this study, the authors determine the extreme positions of the piston and the compression ratio for certain geometric parameters. The variations of extreme positions and compression ratio depending on the dimensions' variations of the elements are presented in graphical mode and the conclusions are highlighted. A special attention is paid to the tiller's curve of a specific point. Key words: variable compression ratio mechanism, extreme positions, compression ratio, lever's curve, multibody.


## 1. INTRODUCTION

Freudenstein and Maki [1] synthesize the constructive solutions of the mechanism of an engine with variable compression ratio with six elements and seven joints, and with eight elements and ten joints. Numerous patens were brevetted in this field [2]. For a spark engine the compression ratio is limited by the materials used in its construction and the phenomenon of knocking. The maximum value of the compression ratio is $13: 1$, usually being limited to $10: 1$. The prevention of knocking is made with the aid of the swirl phenomenon that creates a circular motion of the fuel in the combustion chamber in order to homogenize it. The usual methods for the obtaining of the variable compression ratio are:

- articulated engine's block. Such a method was used by Hara et al. [3], Clenci [4] obtaining a variation of the compression ratio from 8.5:1 to $12.5: 1$. Another solution is given by SAAB company [5], varying the compression ratio from 8:1 to 14:1;
- modification of the volume of the combustion chamber by adding a supplementary volume. The solution was
adopted by Ford [6] using a small piston acted by a cam;
- modification of the piston's geometry used by Daimler-Benz and developed by University of Michigan [7];
- tiller eccentrically assembled by inserting an eccentric between the tiller and the crankshaft. Another construction is based on the use of a worm gear, the compression ratio varying between $8.5: 1$ and 14:1 [8];
- eccentric crankshaft presented by FEV in 2007 and obtaining a compression ratio between $8: 1$ and 16:1 [9];
- a combination of the crank-shaft and gear mechanisms, used by PSA Group and leading the compression ratio between $6: 1$ and $15: 1$ [10];
- additional different kinematic joints of the crank-shaft mechanism. This solution is used by Nissan for compression ratios between $8: 1$ and $14: 1$ [11].

Some aspects concerning the transitory vibration for a variable compression ratio mechanism was studied by the authors in [12].

This paper determines the influences of different geometric parameters.


Fig. 1. The mechanism.

## 2. THE MECHANISM

The mechanism is presented in Fig. 1. It contains the shaft $O C$, intermediate triangular element $A B C$, crank $B D$, piston situated at the point $D$, and the control element $C E$. The crankshaft rotates uniformly with an angular velocity $\omega$. The control element has a determined motion which is considered to be known, $Y_{E}=Y_{E}(t)$. The system has five elements and, that is, it has maximum 15 possible degrees of freedom. The position of the mechanism is described by the positions of the centers of weight of the five elements, $X_{C_{i}}$, $Y_{C_{i}}$, and the rotational angles of them $\varphi_{i}$, $i=\overline{1,5}$. The displacement between the $Y$-axis and the direction of motion of the piston is equal to $e$; usually $e$ has small values and in the most cases $e=0$. The coordinate $X_{E}$, which is a constant of the mechanism is denoted by $d$.

## 3. GEOMETRIC APPROACH

Further on, we make the following notations:

- OXY - the fixed reference frame
- $C_{i}, i=\overline{1,5}$ - centers of mass of the elements, assumed to be homogeneous;
- $C_{1} x_{1} y_{1}$ - mobile reference system attached to the element $O A$ and having the $x_{1}$-axis along the line $O A$ and orientated from point $C_{1}$ to point $A$;
- $C_{2} x_{2} y_{2}$ - mobile reference system attached to the triangular element $A B C$ with the $x_{2}$-axis situated along the line $C_{2} C$ and orientated from point $C_{2}$ to point $C$;
- $C_{3} x_{3} y_{3}$ - mobile reference frame attached to the element $B D$ and having the $x_{3}$-axis along the line $B D$ and orientated from point $C_{3}$ to point $D$;
- $C_{4} x_{4} y_{4}$ - mobile reference frame attached to the element $C E$, with the $x_{4}$-axis along the line $C E$ and orientated from point $C_{4}$ to point E;
- $C_{5} x_{5} y_{5}$-mobile reference system attached to the piston (element 5) and having the axes parallel to the axes of the fixed reference system;
- $\varphi_{i}, i=\overline{1,5}$ - the rotational angles of the mobile reference systems. One may observe that always

$$
\begin{equation*}
\varphi_{5}=0 ; \tag{1}
\end{equation*}
$$

$-\left[\mathbf{A}_{i}\right], i=\overline{1,5}-$ the rotation matrices,

$$
\left[\mathbf{A}_{i}\right]=\left[\begin{array}{cc}
\cos \varphi_{i} & -\sin \varphi_{i}  \tag{2}\\
\sin \varphi_{i} & \cos \varphi_{i}
\end{array}\right]
$$

- $X_{P}, Y_{P}$ - the coordinates of a generic point $P$ relative to the fixed system of coordinates;
$-x_{P}^{(i)}, y_{P}^{(i)}$ - the coordinates of the generic point $P$ relative to the mobile system of coordinates $C_{i} x_{i} y_{i}, i=\overline{1,5}$;
- $m_{a}, m_{b}, m_{c}$ - the lengths of the medians of the triangle $A B C$;
- $\theta$ - the angle $B C_{2} C$.

The coordinates of the point $A$ are

$$
\begin{equation*}
X_{A}=O A \cos \varphi_{1}, Y_{A}=O A \sin \varphi_{1} . \tag{3}
\end{equation*}
$$

The coordinates of the point $E$ are

$$
\begin{equation*}
X_{E}=d, Y_{E}=h, \tag{4}
\end{equation*}
$$

where the dimension $h$ are considered to be known at each moment of time.

Point $C$ is obtained as the intersection between the circle with center at the point $A$ and radius $A C$, and the circle with the center at the point $E$ and radius $C E$, that is, the coordinates of the point $C$ are the solutions of the system

$$
\begin{align*}
\left(X-X_{A}\right)^{2}+\left(Y-Y_{A}\right)^{2} & =(A C)^{2}, \\
\left(X-X_{E}\right)^{2}+\left(Y-Y_{E}\right)^{2} & =(C E)^{2} . \tag{5}
\end{align*}
$$

It results

$$
\begin{equation*}
X_{C}=\frac{-e_{2}+\sqrt{e_{2}+d_{1} e_{1}}}{e_{1}}, Y_{C}=a X_{C}+b \tag{6}
\end{equation*}
$$

where

$$
\begin{gather*}
c=\frac{(C A)^{2}-(C E)^{2}}{2}  \tag{7}\\
+\frac{-X_{A}^{2}+X_{E}^{2}-Y_{A}^{2}+Y_{E}^{2}}{2}, \\
a=-\frac{X_{E}-X_{A}}{Y_{E}-Y_{A}},  \tag{8}\\
b=\frac{c}{Y_{E}-Y_{A}},  \tag{9}\\
e_{1}=1+a^{2},  \tag{10}\\
e_{2}=a b-a Y_{A}-X_{A},  \tag{11}\\
d_{1}=(C A)^{2}-X_{A}^{2}-\left(b-Y_{A}\right)^{2} . \tag{12}
\end{gather*}
$$

Similarly, point $B$ is the intersection between the circle with the center at the point $A$ and the radius equal to $A B$, and the circle with the center at the point $C$ and the radius equal to $B C$, that is, the coordinates of the point $B$ are the solution of the system

$$
\begin{align*}
& \left(X-X_{A}\right)^{2}+\left(Y-Y_{A}\right)^{2}=(A B)^{2}, \\
& \left(X-X_{C}\right)^{2}+\left(Y-Y_{C}\right)^{2}=(B C)^{2} . \tag{13}
\end{align*}
$$

We obtain

$$
\begin{equation*}
X_{B}=\frac{-e_{2}+\sqrt{e_{2}+d_{1} e_{1}}}{e_{1}}, Y_{B}=a X_{C}+b, \tag{14}
\end{equation*}
$$

in which the parameters $e_{1}, e_{2}, a, b$, and $c$ are given by

$$
\begin{gather*}
c=\frac{(A B)^{2}-(B C)^{2}}{2}  \tag{15}\\
+\frac{-X_{A}^{2}+X_{C}^{2}-Y_{A}^{2}+Y_{C}^{2}}{2}, \\
a=-\frac{X_{C}-X_{A}}{Y_{C}-Y_{A}},  \tag{16}\\
b=\frac{c}{Y_{C}-Y_{A}},  \tag{17}\\
e_{1}=1+a^{2},  \tag{18}\\
e_{2}=a b-a Y_{A}-X_{A},  \tag{19}\\
d_{1}=(A B)^{2}-X_{A}^{2}-\left(b-Y_{A}\right)^{2} . \tag{20}
\end{gather*}
$$

Point $D$ is situated at the intersection of the circle with the center at the point $B$ and radius $B D$, and the vertical line of equation $X_{D}=e$. We obtain the values

$$
\begin{gather*}
X_{D}=e  \tag{21}\\
Y_{D}-Y_{B}+\sqrt{(B D)^{2}-\left(X_{D}-X_{B}\right)^{2}}
\end{gather*}
$$

## 4. MULTIBODY APPROACH

Denoting by $\left\{\mathbf{R}_{P}\right\}$ and $\left\{\mathbf{r}_{P}^{(i)}\right\}$ the column matrices

$$
\begin{align*}
& \left\{\mathbf{R}_{P}\right\}=\left[\begin{array}{ll}
X_{P} & Y_{P}
\end{array}\right]^{\mathrm{T}},  \tag{22}\\
& \left\{\mathbf{r}_{P}^{(i)}\right\}=\left[\begin{array}{ll}
x_{P}^{(i)} & y_{P}^{(i)}
\end{array}\right]^{\mathrm{T}}, \tag{23}
\end{align*}
$$

where $P$ is a generic point, one may write the relation

$$
\begin{equation*}
\left\{\mathbf{R}_{P}\right\}=\left\{\mathbf{R}_{C_{i}}\right\}+\left[\mathbf{A}_{i}\right]\left\{\mathbf{r}_{P}^{(i)}\right\} . \tag{24}
\end{equation*}
$$

First of all, we have to determine the coordinates of the points $A, B$, and $C$ relative to the mobile reference system $C_{2} x_{2} y_{2}$.

We successively write

$$
\begin{gather*}
m_{a}=\frac{\sqrt{2\left[(C A)^{2}+(A B)^{2}\right]-(B C)^{2}}}{2},  \tag{25}\\
m_{b}=\frac{\sqrt{2\left[(A B)^{2}+(B C)^{2}\right]-(C A)^{2}}}{2},  \tag{26}\\
m_{c}=\frac{\sqrt{2\left[(B C)^{2}+(C A)^{2}\right]-(A B)^{2}}}{2},  \tag{27}\\
=\arccos \left(\frac{\left(\frac{2 m_{c}}{3}\right)^{2}+\left(\frac{2 m_{b}}{3}\right)^{2}-(B C)^{2}}{2\left(\frac{2 m_{c}}{3}\right)\left(\frac{2 m_{b}}{3}\right)}\right),
\end{gather*}
$$

$$
\begin{gather*}
=\arccos \left(\frac{\left(\frac{2 m_{a}}{3}\right)^{2}+\left(\frac{2 m_{c}}{3}\right)^{2}-(C A)^{2}}{2\left(\frac{2 m_{a}}{3}\right)\left(\frac{2 m_{c}}{3}\right)}\right),  \tag{29}\\
x_{A}^{(2)}=\frac{2 m_{a}}{3} \cos \varphi_{2}, y_{A}^{(2)}=\frac{2 m_{a}}{3} \sin \varphi_{2},  \tag{30}\\
x_{B}^{(2)}=\frac{2 m_{b}}{3} \cos \theta, y_{B}^{(2)}=\frac{2 m_{b}}{3} \sin \theta,  \tag{31}\\
x_{C}^{(2)}=\frac{2 m_{c}}{3}, y_{C}^{(2)}=0 . \tag{32}
\end{gather*}
$$

The following constraint functions may be written:

- the point $A$ belongs to the elements 1 and 2 ; hence

$$
\begin{align*}
& {\left[\begin{array}{l}
X_{A} \\
Y_{A}
\end{array}\right]=\left[\begin{array}{l}
X_{C_{1}} \\
Y_{C_{1}}
\end{array}\right]+\left[\begin{array}{cc}
\cos \varphi_{1} & -\sin \varphi_{1} \\
\sin \varphi_{1} & \cos \varphi_{1}
\end{array}\right]\left[\begin{array}{l}
x_{A}^{(1)} \\
y_{A}^{(1)}
\end{array}\right],}  \tag{33}\\
& {\left[\begin{array}{c}
X_{A} \\
Y_{A}
\end{array}\right]=\left[\begin{array}{c}
X_{C_{2}} \\
Y_{C_{2}}
\end{array}\right]+\left[\begin{array}{cc}
\cos \varphi_{2} & -\sin \varphi_{2} \\
\sin \varphi_{2} & \cos \varphi_{2}
\end{array}\right]\left[\begin{array}{c}
x_{A}^{(2)} \\
y_{A}^{(2)}
\end{array}\right],} \tag{34}
\end{align*}
$$

where

$$
\begin{equation*}
x_{A}^{(1)}=\frac{O A}{2}, y_{A}^{(2)}=0 . \tag{35}
\end{equation*}
$$

Equating the expressions (33) and (34), we obtain the first two constraint functions;

- point $B$ belongs to the elements 2 and 3; we have

$$
\begin{align*}
& {\left[\begin{array}{c}
X_{B} \\
Y_{B}
\end{array}\right]=\left[\begin{array}{l}
X_{C_{2}} \\
Y_{C_{2}}
\end{array}\right]+\left[\begin{array}{cc}
\cos \varphi_{2}-\sin \varphi_{2} \\
\sin \varphi_{2} & \cos \varphi_{2}
\end{array}\right]\left[\begin{array}{l}
x_{B}^{(2)} \\
y_{B}^{(2)}
\end{array}\right],}  \tag{36}\\
& {\left[\begin{array}{c}
X_{B} \\
Y_{B}
\end{array}\right]=\left[\begin{array}{c}
X_{C_{3}} \\
Y_{C_{3}}
\end{array}\right]+\left[\begin{array}{cc}
\cos \varphi_{3} & -\sin \varphi_{3} \\
\sin \varphi_{3} & \cos \varphi_{3}
\end{array}\right]\left[\begin{array}{c}
x_{B}^{(3)} \\
y_{B}^{(3)}
\end{array}\right],} \tag{37}
\end{align*}
$$

in which

$$
\begin{equation*}
x_{B}^{(3)}=-\frac{B D}{2}, y_{B}^{(3)}=0 . \tag{38}
\end{equation*}
$$

From the equations (36) and (37) one deduces another two constraint functions;

- point $C$ belongs to the elements 2 and 4 and therefore one may write

$$
\begin{align*}
& {\left[\begin{array}{c}
X_{C} \\
Y_{C}
\end{array}\right]=\left[\begin{array}{c}
X_{C_{2}} \\
Y_{C_{2}}
\end{array}\right]+\left[\begin{array}{cc}
\cos \varphi_{2} & -\sin \varphi_{2} \\
\sin \varphi_{2} & \cos \varphi_{2}
\end{array}\right]\left[\begin{array}{l}
x_{C}^{(2)} \\
y_{C}^{(2)}
\end{array}\right],}  \tag{39}\\
& {\left[\begin{array}{c}
X_{C} \\
Y_{C}
\end{array}\right]=\left[\begin{array}{c}
X_{C_{4}} \\
Y_{C_{4}}
\end{array}\right]+\left[\begin{array}{cc}
\cos \varphi_{4} & -\sin \varphi_{4} \\
\sin \varphi_{4} & \cos \varphi_{4}
\end{array}\right]\left[\begin{array}{l}
x_{C}^{(4)} \\
y_{C}^{(4)}
\end{array}\right],} \tag{40}
\end{align*}
$$

where

$$
\begin{equation*}
x_{C}^{(4)}=-\frac{C E}{2}, y_{C}^{(4)}=0 . \tag{41}
\end{equation*}
$$

Equating now the relations (39) and (40), we obtain another two constraint functions.

- point $D$ belongs to the elements 3 and 5 and, consequently, one gets

$$
\begin{gather*}
{\left[\begin{array}{c}
X_{D} \\
Y_{D}
\end{array}\right]=\left[\begin{array}{c}
X_{C_{3}} \\
Y_{C_{3}}
\end{array}\right]+\left[\begin{array}{cc}
\cos \varphi_{3} & -\sin \varphi_{3} \\
\sin \varphi_{3} & \cos \varphi_{3}
\end{array}\right]\left[\begin{array}{c}
x_{D}^{(3)} \\
y_{D}^{(3)}
\end{array}\right],}  \tag{42}\\
 \tag{43}\\
{\left[\begin{array}{c}
X_{D} \\
Y_{D}
\end{array}\right]=\left[\begin{array}{c}
X_{C_{5}} \\
Y_{C 5}
\end{array}\right]+\left[\begin{array}{c}
x_{D}^{(5)} \\
y_{D}^{(5)}
\end{array}\right],}
\end{gather*}
$$

where we kept into account that

$$
\left[\mathbf{A}_{5}\right]=\left[\mathbf{I}_{3}\right]=\left[\begin{array}{lll}
1 & 0 & 0  \tag{44}\\
0 & 1 & 0 \\
0 & 0 & 1
\end{array}\right],
$$

while

$$
\begin{align*}
& x_{D}^{(4)}=\frac{B D}{2}, y_{D}^{(4)}=0,  \tag{45}\\
& x_{D}^{(5)}=0, \quad y_{D}^{(5)}=0 . \tag{46}
\end{align*}
$$

Equations (42) and (43) lead to another two constraint functions;

- the coordinate $X_{E}$ is known,

$$
\begin{equation*}
X_{E}=d, \tag{47}
\end{equation*}
$$

and therefore we may write

$$
\left[\begin{array}{c}
X_{E}  \tag{48}\\
Y_{E}
\end{array}\right]=\left[\begin{array}{c}
X_{C_{4}} \\
Y_{C_{4}}
\end{array}\right]+\left[\begin{array}{cc}
\cos \varphi_{4} & -\sin \varphi_{4} \\
\sin \varphi_{4} & \cos \varphi_{4}
\end{array}\right]\left[\begin{array}{c}
x_{E}^{(4)} \\
y_{E}^{(4)}
\end{array}\right],
$$

wherefrom it results the relation

$$
\begin{gather*}
X_{E}=d \\
=X_{C_{4}}+x_{E}^{(4)} \cos \varphi_{4}-y_{E}^{(4)} \sin \varphi_{4}  \tag{50}\\
=X_{C_{4}}+\frac{C E}{2} \cos \varphi_{4}
\end{gather*}
$$

and the corresponding constraints function;

- the coordinate $X_{D}$ is also known,

$$
\begin{equation*}
X_{D}=e ; \tag{51}
\end{equation*}
$$

similarly, we have

$$
\left[\begin{array}{c}
X_{D}  \tag{52}\\
Y_{D}
\end{array}\right]=\left[\begin{array}{c}
X_{C_{3}} \\
Y_{C_{3}}
\end{array}\right]+\left[\begin{array}{cc}
\cos \varphi_{3} & -\sin \varphi_{3} \\
\sin \varphi_{3} & \cos \varphi_{3}
\end{array}\right]\left[\begin{array}{c}
x_{D}^{(3)} \\
y_{D}^{(3)}
\end{array}\right],
$$

wherefrom

$$
\begin{gather*}
X_{D}=e \\
=X_{C_{3}}+x_{D}^{(3)} \cos \varphi_{3}-y_{D}^{(3)} \sin \varphi_{3}  \tag{53}\\
=X_{C_{3}}+\frac{B D}{2} \cos \varphi_{3}
\end{gather*}
$$

and we get other constraints function;

- the coordinates $X_{C_{1}}$ and $Y_{C_{1}}$ are

$$
\begin{equation*}
X_{C_{1}}=\frac{O A}{2} \cos \varphi_{1}, Y_{C_{1}}=\frac{O A}{2} \sin \varphi_{1}, \tag{54}
\end{equation*}
$$

resulting another two constraints function;

- the rotation angle of the element 5 is always equal to zero, obtaining the constraints function given by equation (1).

The previous discussion shows that there exist at least 13 constraints functions. Taking into account this statement, it results that the mechanism has no more than two degrees of freedom.

The last constraints function is obtained from the condition

$$
\begin{equation*}
Y_{E}=h, \tag{55}
\end{equation*}
$$

which leads to (see equation (48))

$$
\begin{gather*}
Y_{E}=h \\
=Y_{C_{4}}+x_{E}^{(4)} \sin \varphi_{4}+y_{E}^{(4)} \cos \varphi_{4}  \tag{56}\\
=Y_{C_{4}}+\frac{C E}{2} \sin \varphi_{4} .
\end{gather*}
$$

Due to our assumption that the coordinate $Y_{E}$ is always known, it results that one knows the function

$$
\begin{equation*}
h=h(t) . \tag{57}
\end{equation*}
$$

If the relation (57) is not known, then the mechanism has two degrees of freedom, the expression (56) not leading to a constraints function.

We will consider that this degree of freedom is the rotation angle $\varphi_{1}$.

## 5. POSSIBILITIES TO DETERMINE THE EXTREME POSITIONS OF THE PISTON AND THE COMPRESSION RATIO

There exist the following ways in which one may determine the extreme positions of the piston and, consequently, the compression ratio:

- the formulae developed in the paragraph 3 may be written as

$$
\begin{equation*}
Y_{D}=Y_{D}\left(\varphi_{1}\right) . \tag{58}
\end{equation*}
$$

The extreme positions are obtained from the equation

$$
\begin{equation*}
\frac{\mathrm{d} Y_{D}}{\mathrm{~d} \varphi_{1}}=0 . \tag{59}
\end{equation*}
$$

The equation (59) is a very complicated one and may be solved only by numerical methods. Moreover, this equation has at least two real
roots in the variable $\varphi_{1}$, and the great challenge is the separation of these roots. In addition, the fast numerical methods in solving the equation may not be directly applied because of convergence conditions required by these methods (e.g. Newton's method [13]). For these reasons, combined numerical methods must be applied;

- the second approach uses the constraints functions presented in paragraph 4. Let us denote by $\{\boldsymbol{q}\}$ the column matrix formed with $X_{C_{1}}, Y_{C_{1}}, \ldots, X_{C_{5}}, Y_{C_{5}}, \varphi_{1}, \ldots, \varphi_{5}$. Each constraints function is an equation in the form

$$
\begin{equation*}
f_{i}(\{\mathbf{q}\})=0, i=\overline{1,14} . \tag{60}
\end{equation*}
$$

Considering the Lagrange function

$$
\begin{gather*}
F\left(\{\mathbf{q}\}, \lambda_{1}, \ldots, \lambda_{14}\right)=Y_{D}+\sum_{i=1}^{14} \lambda_{i} f_{i}(\{\mathbf{q}\}) \\
=Y_{C_{3}}+\frac{B D}{2} \sin \varphi_{3}+\sum_{i=1}^{14} \lambda_{i} f_{i}(\{\mathbf{q}\}) \tag{61}
\end{gather*}
$$

the solution is obtained from the system

$$
\begin{gather*}
\frac{\partial F}{\partial X_{C_{i}}}=0, \frac{\partial F}{\partial Y_{C_{i}}}=0, \frac{\partial F}{\partial \varphi_{i}}=0, i=\overline{1,5}  \tag{62}\\
\frac{\partial F}{\partial \lambda_{j}}=f_{j}(\{\mathbf{q}\})=0, j=\overline{1,14}, \tag{63}
\end{gather*}
$$

that is, a nonlinear system of 29 equations with 29 unknowns, which can be generally solved by numerical methods. The same discussion about the convergence on the numerical methods holds true in this situation too;

- by direct use of numerical methods. Recalling the formulae developed in paragraph 3 or 4 , and using a small incremental step $\Delta \varphi_{1}$ for the rotation angle $\varphi_{1}$, one may construct a sequence of values $Y_{D}=Y_{D}\left(\varphi_{1}\right)$. It is now an easy task to determine the maximum and the minimum values in this sequence.

The above discussion proves that the minimum and maximum values for the coordinate $Y_{D}$ can be determined only by approximates, and one has to set the required precision.

## 6. NUMERICAL STUDY

The following realistic values are selected for our numerical study: $(A B)_{0}=0.043 \mathrm{~m}$,
$(B C)_{0}=0.128 \mathrm{~m}, \quad(C A)_{0}=0.099 \mathrm{~m}$, $(C E)_{0}=0.103 \mathrm{~m}, \quad(O A)_{0}=0.030 \mathrm{~m}$, $(B D)_{0}=0.130 \mathrm{~m}, \quad d=0.086 \mathrm{~m}, \quad(e)_{0}=0 \mathrm{~m}$, $\left(Y_{E}\right)_{0}=(h)_{0}=0.108 \mathrm{~m}$, the angular step $\Delta \varphi_{1}=\frac{\pi}{1800} \mathrm{rad}=0.1^{0}$, height of the combustion chamber $h_{c c}=0,0158 \mathrm{~m}$, $\left(Y_{D}\right)_{0}=0.200 \mathrm{~m}$. The index 0 stands for the standard values.

Each parameter is varied with $\pm 0.01 \mathrm{~m}$ from the standard values.

The diagrams obtained by numerical simulation are given in the next figures.

The compression ratio was denoted by $i_{c}$ in the corresponding figures. The standard value for the compression ratio is $10: 1$.


Fig. 2. The variation $\left(Y_{D}\right)_{\text {min }}=\left(Y_{D}\right)_{\text {min }}(e)$


Fig. 3. The variation $\left(Y_{D}\right)_{\max }=\left(Y_{D}\right)_{\max }(e)$
Analyzing the Figures 2-4, one may observe that minimum and maximum values for the parameter $Y_{D}$ decrease when the eccentricity $e$ increases (negative values for $e$ signify that the piston is in the left part of the $Y$-axis). The variation are small (up to one or two millimeters).


Fig. 4. The variation $i_{c}=i_{c}(e)$


Fig. 5. The variation $\left(Y_{D}\right)_{\min }=\left(Y_{D}\right)_{\min }(A B)$


Fig. 6. The variation $\left(Y_{D}\right)_{\max }=\left(Y_{D}\right)_{\max }(A B)$


Fig. 7. The variation $i_{c}=i_{c}(A B)$


Fig. 8. The variation $\left(Y_{D}\right)_{\min }=\left(Y_{D}\right)_{\min }(B C)$


Fig. 9. The variation $\left(Y_{D}\right)_{\max }=\left(Y_{D}\right)_{\max }(B C)$


Fig. 10. The variation $i_{c}=i_{c}(B C)$

The same variation is characteristic to the compression ratio (it decreases when the eccentricity increases), the variation being again a small one.

The influence of the length $A B$ (Figs. 5-7)is more dramatically. The variation of the same parameters are situated in larger limits. The compression ratio may reach values of 50:1, which is impossible. In fact, we may conclude that the variation of $A B$ assures a raw adjustment of the mechanism.


Fig. 11. The variation $\left(Y_{D}\right)_{\min }=\left(Y_{D}\right)_{\min }(C A)$


Fig. 12. The variation $\left(Y_{D}\right)_{\max }=\left(Y_{D}\right)_{\max }(C A)$


Fig. 13. The variation $i_{c}=i_{c}(C A)$


Fig. 14. The variation $\left(Y_{D}\right)_{\min }=\left(Y_{D}\right)_{\min }(O A)$


Fig. 15. The variation $\left(Y_{D}\right)_{\max }=\left(Y_{D}\right)_{\max }(O A)$


Fig. 16. The variation $i_{c}=i_{c}(0 A)$


Fig. 17. The variation $\left(Y_{D}\right)_{\min }=\left(Y_{D}\right)_{\min }(C E)$


Fig. 18. The variation $\left(Y_{D}\right)_{\max }=\left(Y_{D}\right)_{\max }(C E)$


Fig. 19. The variation $i_{c}=i_{c}(C E)$


Fig. 20. The variation $\left(Y_{D}\right)_{\min }=\left(Y_{D}\right)_{\min }(B D)$


Fig. 21. The variation $\left(Y_{D}\right)_{\max }=\left(Y_{D}\right)_{\max }(B D)$


Fig. 22. The variation $i_{c}=i_{c}(B D)$


Fig. 23. The variation $\left(Y_{D}\right)_{\min }=\left(Y_{D}\right)_{\text {min }}(d)$


Fig. 24. The variation $\left(Y_{D}\right)_{\text {max }}=\left(Y_{D}\right)_{\text {max }}(d)$


Fig. 25. The variation $i_{c}=i_{c}(d)$
The rest of the figures may be similarly judged. One may observe that the length of the control lever $C E$ has no influence on the extreme positions of the piston and the compression ratio, the length $C E$ is set by constructive criteria.

Figure 26 presents the geometric locus of the point $B$ when the control moves on vertical direction. In this situation, the variation of $Y_{E}$ is $\pm 0.02 \mathrm{~m}$. This zone influences the constructive dimensions of the engine.


Fig. 26. The geometric locus of the tiller's curve of the point $B$ when $Y_{E}$ is varied

## 7. CONCLUSION

The variations of the extreme positions of piston and of compression ratio are important in the synthesis of the mechanism.

The tiller curves for different characteristic points and dimensions of the elements give information about the constructive dimensions of the mechanism, relative positions of the elements etc.

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# STUDIUL INFLUENȚEI PARAMETRILOR GEOMETRICI ASUPRA DEPLASĂRII PISTONULUI ŞI A RAPORTULUI DE COMPRIMARE PENTRU UN MECANISM CU COMPRIMARE VARIABILĂ 


#### Abstract

Se prezintă un mecanism cu raport de comprimare variabil şi se stabilesc relaţiile dintre diferiţi parametri. Studiul este realizat în două moduri: prin abordare geometrică şi abordare multicorp. Pe baza formulelor deduse în cadrul acestui studiu, autorii determină pozițiile extreme ale pistonului, precum şi raportul de comprimare pentru anumiţi parametri geometrici. Variaţiile poziţiilor extreme şi ale raportului de comprimare in funcţie de variatiile dimensiunilor elementelor sunt prezentate in mod grafic şi de aici se deduc concluziile. O atenţie deosebită este dată curbei de bielă pentru un punct specific al mecanismului.


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