TECHNICAL UNIVERSITY OF CLUJ-NAPOCA

ACTA TECHNICA NAPOCENSIS

Series: Applied Mathematics, Mechanics, and Engineerin
Vol. 61, Issue I, March, 2018

THE DYNAMIC ISOLATION PERFORMANCES ANALYSIS OF THE VIBRATING EQUIPMENT WITH ELASTIC LINKS TO A FIXED BASE

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Abstract: *The paper presents the results of researches carried out for constructions base isolation technical solutions with dissipative elastic devices rheological modelled as a viscous-elastic compound system. In this context, there are analysed the conditions for the achievement of the equivalent viscous-elastic connection, based on the coupling of elastic and viscous devices in a Newton -Voigt -Kelvin structure. On this basis, the dynamic analysis highlights the level of the dynamic response by the significant amplitudes of the instantaneous displacements to dominant spectral frequency of the seismic excitation. In the end, there are presented the parametric variation curves which allow the assessment of the dynamic isolation level and the efficiency of the viscous – elastic link, used as construction base isolation system (buildings, bridges, viaducts) located in seismic prone areas.*

Key words: *Base isolation, rheological dissipative elastic devices, Newton-Voigt-Kelvin structure, dynamic isolation level assessment.*

1. INTRODUCTION

The paper highlights the results of the applicative researches regarding the mitigation of the vibrations transmitted to the bearing support, by the industrial dynamic equipment. In this case, the industrial dynamic equipment for granular materials sorting or for the fine milling of the friable materials granules use, as a functional principle, the vibrations generated by a rotating system with eccentric masses of dynamic imbalance.

Thus, the rotating disturbing force is the inertial centrifugal force of the equivalent eccentric mass m_0 placed at the distance r in relation to the geometric rotation axis, in such a manner that the rotating disturbing force can be expressed as $P_0 = m_0 r\omega^2$, where ω is the angular frequency the mass m_0. The industrial equipment in this category are realized as rigid metallic structures, interconnected, so that the rigid body model can be adopted. The bearing system consists of four identical elastic linking groups in relation to the fixed supporting base of the equipment. Each elastic group consists of four elastomeric devices mounted two by two symmetrically at 60° in relation with the local vertical axis parallel to the axis C_z.

The conceptual problematic of the technological parameters correlation of the, expressed by the equipment amplitudes, with the vibrations isolation parameters transmitted to the base. This consists in the dynamic regime analysis with the identification of the resonance states, as well as of the functional states, with minimal transmissibility of vibrations at the base, simultaneously with the ensuring of the technological vibrations level for the same excitation frequency in the post-resonance.

In this context, there will be presented the calculus model, the motion equations and the computational relationships for the amplitudes, the forces transmitted to the base and the dynamic transmittance coefficients of the vibrations.

2. DYNAMIC RESPONSE ANALYSIS

In figure 1 it is schematically presented the calculus model, which is characterized as follows:

- The dynamic equipment is modelled as a rigid with two vertical symmetry planes, C_{xz} and C_{yz};
- The bearing consists of four elastic groups, each group being characterized by the orthogonal rigidities k_x and k_z , in relation with the axis C_x and respectively C_z;
- The dynamic excitation is of inertial type, expressed by the force $P_0 = m_0 r\omega^2$, placed in the median longitudinal plane of the equipment, with the origin in the point O(x_0, 0, z_0). The rotating force P_0 generates perturbatory excitations, expressed as: $F_x(t) = P_0 \cos\omega t$; $F_y(t) = 0$; $F_z(t) = P_0 \sin\omega t$; $M_{Cx}(t) = 0$; $M_{Cy}(t) = (z_0 \cos\omega t - x_0 \sin\omega t) P_0$; $M_{Cz}(t) = 0$.

The excitation moment $M_{Cy}(t)$ can be expressed as $M_{Cy}(t) = M_0 \sin(\omega t - \alpha)$, where $M_0 = m_0 r\omega^2 l$, with $l^2 = x_0^2 + z_0^2$ and $\alpha = arctg \frac{z_0}{x_0}$.

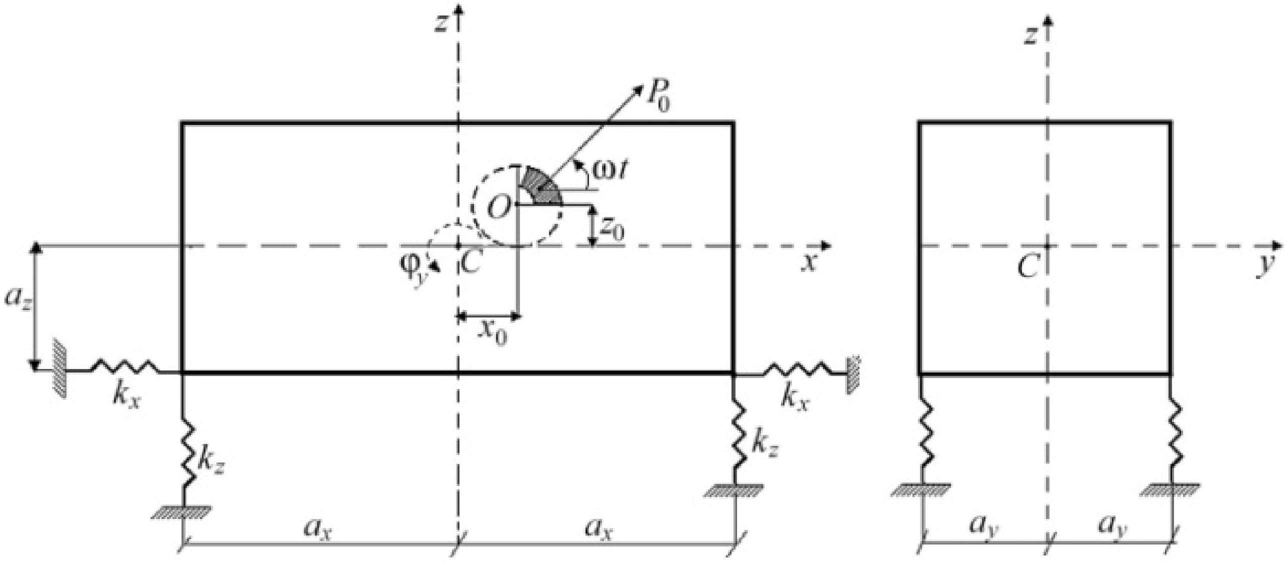

Fig. 1. The schematically model calculus

2.1 Parameters of dynamic response

The geometrical configuration and the mass distribution with the symmetries previously defined, as well as the elastic bearing, symmetric in relation with the longitudinal and transversal median vertical planes, ensure the motion in three degrees of freedom X, Z, φ_y. Of these, two movements are coupled X, φ_y), and the translation motion, on vertical axis Y is independent.

In this case, the differential equation of the motion can be expressed as:

$$\begin{cases} m\ddot{X} + \tilde{k}_x - \tilde{k}_x a_z \varphi_y = m_0 r\omega^2 \cos\omega t \\ \Im_y \ddot{\varphi}_y + (\tilde{k}_z a_x^2 + \tilde{k}_x a_z^2)\varphi_y - \tilde{k}_x a_z X = m_0 r\omega^2 l \sin(\omega t - \alpha) \\ m\ddot{Z} + \tilde{k}_z Z = m_0 r\omega^2 \sin\omega t \end{cases} \quad (1)$$

In stabilized regime, the dynamic response, depending on the angular frequency ω , is expressed by the linear amplitudes $A_x(\omega), A_z(\omega)$ and the angular amplitudes $A_{\varphi_y}(\omega)$. Thus, based on the expressions (1), and taking into account the shape of the excitation harmonic functions, in stabilized regime, results:

$$A_x(\omega) = \frac{m_0 r\omega^2}{\tilde{k}_z} \frac{B(\omega)}{D(\omega)} \quad (2)$$

where $A_x(\omega)$ is the linear vibrations amplitude, in the axis C_x direction.

$$A_{\varphi_y}(\omega) = \frac{m_0 r\omega^2}{\tilde{k}_z \rho_y} \frac{C(\omega)}{D(\omega)} \quad (3)$$

where $A_{\varphi_y}(\omega)$ is the angular amplitude of the sway vibrations, around the axis C_y.

$$A_z(\omega)=\frac{m_0 r\omega^2}{\tilde{k}_z}\frac{1}{1-\frac{\omega^2}{\omega_z^2}} \qquad (4)$$

$A_z(\omega)$ is the amplitude of the translational vibrations, in the axis C_z direction.

In the previous expressions, the following relationships were used:

$$\tilde{k}_x=4k_x;\tilde{k}_z=4k_z;\omega_z^2=\frac{\tilde{k}_z}{m} \quad \rho_y^2=\frac{1}{m}\Im_y$$

or $$\Im_y=m\rho_y^2=\tilde{k}_z\frac{\rho_y^2}{\omega_z^2}$$

where ρ_y is the mass m rotation radius in relation to the axis Cy:

$\Im_y$ - the moment of inertia in relation to the axis Cy

ω_z – the vibrations natural frequency in relation to the axis Cy.

The expressions $B(\omega)$, $C(\omega)$ and $D(\omega)$ are:

$$B(\omega)=\sqrt{\left[\frac{\tilde{k}_x}{\tilde{k}_z}\frac{a_z}{\rho_y^2}(a_z-z_0)+\frac{a_x^2}{\rho_y^2}-\frac{\omega^2}{\omega_z^2}\right]^2+\frac{\tilde{k}_x^2}{\tilde{k}_z^2}\frac{x_0^2 z_0^2}{\rho_y^4}} \qquad (5)$$

$$C(\omega)=\sqrt{\left[\frac{\tilde{k}_x}{\tilde{k}_z}\frac{1}{\rho_y^2}(a_z-z_0)+\frac{z_0}{\rho_y}-\frac{\omega^2}{\omega_z^2}\right]^2+\frac{x_0^2}{\rho_y^2}\left(\frac{\tilde{k}_x}{\tilde{k}_z}-\frac{\omega^2}{\omega_z^2}\right)^2} \qquad (6)$$

$$D(\omega)=\left(\frac{\omega}{\omega_z}\right)^2-\left[\frac{a_x^2}{\rho_y^2}+\frac{k_z}{k_x}\frac{a_z^2}{\rho_y^2}+\frac{k_x}{k_z}\right]\left(\frac{\omega}{\omega_z}\right)^2+\frac{\tilde{k}_x}{\tilde{k}_z}\frac{a_x^2}{\rho_y^2} \qquad (7)$$

For the coupled vibrations amplitudes $A_x(\omega)$, $A_{\varphi_y}(\omega)$ there are two states of resonance for $D(\omega)=0$, and the post-resonance amplitude $A_x(\omega)$ tends to a constant and stable value $\frac{m_0 r}{m}$, while the amplitude $A_{\varphi_y}(\omega)$ tends to zero. Also, the in post-resonance amplitude $A_z(\omega)$ has a constant and stable value $\frac{m_0 r}{m}$.

3. INSULATION CAPACITY OF THE TRANSMITED VIBRATIONS

The maximum transmitted force (forced amplitude) in the direction C_x is expressed as:

$$F_z^{max}(\omega)=\tilde{k}_x\sqrt{A_x^2+a_z^2A_{\varphi_y}^2-2a_zA_xA_{\varphi_y}\cos(\Psi_x-\Psi_{\varphi_y})} \qquad (8)$$

where Ψ_x and Ψ_{φ_y} are the dephasings resulting from the expressions:

$$\tan\Psi_x=\frac{\tilde{k}_z}{\tilde{k}_x}\frac{\rho_y}{a_x}\frac{\rho_y}{x_0}\left[\frac{\tilde{k}_x}{\tilde{k}_z}\frac{a_z}{\rho_y^2}(a_z-z_0)+\frac{a_x^2}{\rho_y^2}-\frac{\omega^2}{\omega_z^2}\right] \qquad (9)$$

$$\tan\Psi_{\varphi_y}=\frac{1}{x_0\left(\frac{\tilde{k}_x}{\tilde{k}_z}-\frac{\omega^2}{\omega_z^2}\right)}\left[\frac{\tilde{k}_x}{\tilde{k}_z}(a_z-z_0)+z_0\frac{\omega^2}{\omega_z^2}\right] \qquad (10)$$

The maximum transmitted force in the direction C_z is expressed as:

$$F_z^{max}(\omega)=\tilde{k}_zA_z \qquad (11)$$

The maximum torque transmitted to the fixed support (base) is expressed as:

$$M_y^{max}(\omega)=\tilde{k}_z a_x^2 A_{\varphi_y} \qquad (12)$$

The transmissibility coefficients associated to the forces which are transmitted to the base can be defined as:

$$T_x=\frac{F_x^{max}}{m_0 r\omega^2};T_z=\frac{F_z^{max}}{m_0 r\omega^2};T_{\varphi_y}=\frac{M_y^{max}}{m_0 r\omega^2 a_z}$$

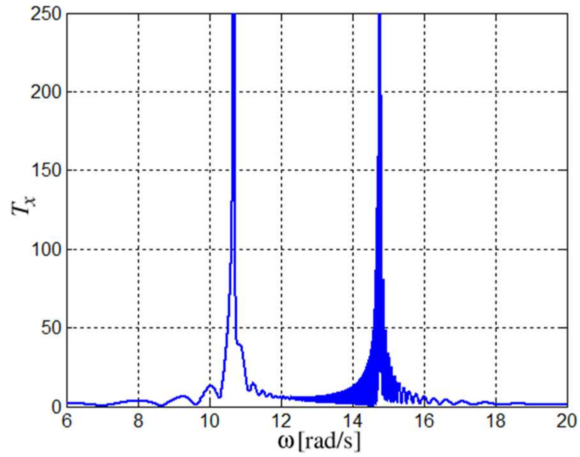

Fig.2. T_x variation with ω

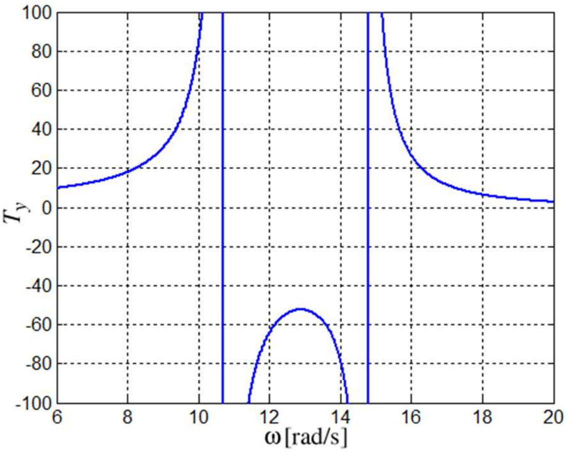

Fig.3. T_y variation with ω

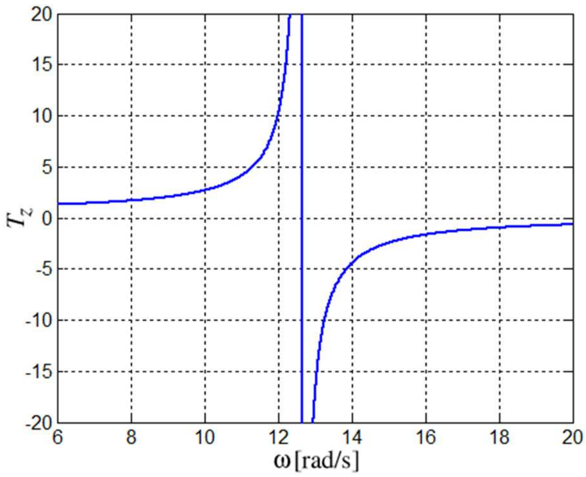

Fig.4. T_z variation with ω

For a dynamic equipment with the mass m = 6500 kg and the parameters $\tilde{k}_x = 1{,}22\cdot10^6$ N/m; $\tilde{k}_z = 10^6$ N/m ; $\rho_y = 2{,}38$ m; $a_x = 2{,}17$ m; $a_z =$ 0,6 m; $x_0 = 0{,}03$ m; $z_0 = 0{,}01$ m; $m_0 r = 26$ kgm; and the variation of $\omega \in (0{,}120)$ rad/s there were plotted the variation curves for the transmissibility coefficients (Figures 2, 3, 4).

The experimental results obtained for the dynamic equipment with the above presented characteristics were highlighted by the measured amplitudes of the vibrations, by the values of the forces measured at the bearing point and by the values of the frequency measured in post-resonance. The measured values were compared with those determined by the analytical method, with maximum deviations of 3%.

The bearing system is composed of identical elastomeric anti-vibration devices, grouped by four by four, for each bearing point. Based on the variation curves of the transmissibility coefficients T, respectively by determining the dynamic isolation coefficients $I = 1 - T$, there were designed and manufactured the anti-vibration elastomeric devices with the following parametric characteristics:

– Geometrical shape: cylinder
– Diameter/ height, mm: 170/170
– Shape coefficient s: 0,25
– Coefficient of elasticity, N/m, compression/shear: $8{,}2\cdot10^4/5{,}1\cdot10^4$
– Maximum admissible deformation, mm compression/shear: 25/68
– Maximum loading force, N, compression/shear: $2\cdot10^3/3{,}5\cdot10^3$
– Elastomer: hardness, ShA: 45
modulus G, Mpa: 0,54

4. CONCLUSIONS

The industrial needs and the current technical solutions highlight the dynamic equipment, harmonically excited with rotating inertial disturbing forces, whose resulting force has the direction contained in a longitudinal median plane. They have symmetrical geometric configuration and symmetrical distribution of the component elements masses, with identical elastic bearings, symmetrically disposed in longitudinal and transversal plans, in four linking points to the fixed base.

Based on the structural configuration of the dynamic model and the technological requirements, as well as on the dynamic isolation requirements, the following conclusions can be summarized:

a. Determination of the rigid model with the geometric, mass and elastic bearing symmetries, defined in accordance with the structural configuration of the vibratory dynamic equipment ensures the conformity with the real physical models;
b. Bearing system consists of four elastic groups, each group having of four elastomeric devices arranged two by two at an angle of 60^o with respect to the local vertical axis of symmetry;
c. Dynamic calculus model allows the determining the dynamic response in stabilized regime, in analytic mode, of the amplitudes of the excited movements (X,φ_y) and Z;
d. Transmissibility degree and, respectively the dynamic isolation degree, for each significant movement, can be determined analytically. The associated variation curves provide the dynamic isolation efficiency assessment for different angular frequency, by variation domains;
e. Parametric experimental verification on industrial vibratory equipment, which is mass production, in Romania, ensured the coherence of the analytical results with those obtained in the tests.

Taking into account the above statements, results that the dynamic model selected for the vibratory dynamical equipment, from the category of the presented ones, can be efficiently used in the designing, manufacturing and maintenance activities.

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Analiza performanţei de izolare dinamică a echipamentelor vibratoare cu legături elastice la baza fixă

REZUMAT. Lucrarea prezintă rezultatele cercetărilor efectuate pentru soluţiile de izolare a bazei a construcţiilor realizate cu dispozitive elastice disipative reologice modelate ca un sistem compozit vâscos-elastic. În acest context, sunt analizate condiţiile de realizare a unei conexiuni elastice vâscoase echivalente, bazate pe cuplarea dispozitivelor elastice şi vâscoase într-o structură Newton -Voigt-Kelvin. Pe această bază, analiza dinamică evidenţiază nivelul răspunsului dinamic prin amplitudinile semnificative ale deplasărilor instantanee la frecvenţa spectrală dominantă a excitaţiei seismice. În final, sunt prezentate curbele parametrice de variaţie care permit evaluarea nivelului de izolare dinamică şi a eficienţei legăturii vâsco-elastice utilizate ca sistem de izolare a bazelor de construcţie (clădiri, poduri, viaducte) situate în zonele predispuse la seism.

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